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Research Paper

On Importance of Seismic Phase Weighting in Earthquake Location Problem

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ABSTRACT

Accurate seismic source location is a critical endeavor in seismology, essential for precisely determining the origin of seismic events. Various methods are available for pinpointing seismic sources, each exhibiting varying degrees of accuracy and reliability. In this study, our objective is to assess the influence of seismic phase weighting on earthquake location by comparing the outcomes of three distinct earthquake location programs. To achieve this, we generated two synthetic earthquake catalogs employing a sophisticated 3D velocity model and a seismic network comprised of 15 stations. We intentionally introduced two synthetic faults, each with different orientations, situated to the south and north of the network. On these fault surfaces, we distributed 100 synthetic earthquakes. We then utilized the phase arrivals of these synthetic events to construct both unweighted and weighted catalogs. Weighting was assigned based on error calculations and local noise models unique to each station. Our findings reveal that the incorporation of appropriate data weighting during the phase reading stage significantly enhances the accuracy of earthquake location, resulting in reduced errors and uncertainties. Among the three programs we compared, HypoDD demonstrated the best performance, successfully determining earthquake locations with minimal localization errors. In the comparison between Hypocenter and Hypo71, the latter's robust outlier detection algorithm proved advantageous in estimating hypocentral errors, even in the presence of outliers. When examining the results obtained from the weighted catalogs, both programs yielded similar outcomes when the data were appropriately weighted, underscoring the crucial role of proper data weighting in achieving consistent performance.

Keywords:
Seismic phase; Weighting; Earthquake location; Location accuracy; Synthetic test

1. Introduction

Seismic source location is a crucial task in seismology as it helps to determine the location of earthquakes, explosions, and other seismic sources. Accurate location of an earthquake is important for reliable magnitude estimation. Both parameters are crucial for decision makers and disaster risk management organizations for rapid response after strong earthquakes. The reliable location and magnitude of earthquakes are also critical parameters in seismic hazard assessment studies. There are various methods for locating seismic sources, some of which are more accurate and reliable than others. Most earthquake location programs employ a fundamental concept that involves creating a system of equations to determine the hypocenters of earthquakes. These earthquakes are recorded by seismic stations located within a regional or local distance cluster. The determination of earthquake location is primarily based on analyzing the arrival times of the first P-wave and/or S-wave. Initially, an estimate of the hypocenter is made, and then it is refined using Geiger's method (Geiger, 1912). This method aims to minimize the sum of the squared differences between the observed and the computed arrival times based on known earth velocity model (Eaton, 1969). The earth model used typically consists of layers with constant velocities. The adjustment process continues until a set of criteria for adequacy are satisfied or until an iteration limit is reached. During the adjustment process, different factors are taken into account to assign weights to individual arrivals. These factors include the quality or clarity of the arrivals, the epicentral distance of the station, the azimuthal position of the stations, and the time residual of the arrivals. These weights play a role in subsequent adjustments of the hypocenter estimate.

2. Methodology

The problem of determining the best hypocentral parameters appears straightforward in principle. However, due to the nonlinear nature of the travel-time function T (Equation 1) with respect to the model parameters, it is not possible to solve the relevant equation analytically.

$$t_i^c = T\left(x_i, y_i, z_i, x_0, y_0, z_0\right) + t_0 \qquad (1)$$

t_i^c: Calculated arrival time in station i, t_0: seismic event origin time,
$T(x_i, y_i, z_i, x_0, y_0, z_0)$: calculated travel time between the source (x_0, y_0, z_0) and station i,

Despite the relative ease of computing travel-time function, particularly in the case of 1-D Earth models or pre-calculated travel-time tables, the nonlinearity of travel-time function significantly increases the complexity of the problem of inverting for the optimal hypocentral parameters. In the following, some of the methods of solving this problem will be discussed.

2.1. Linearized Iterative Methods

Linearized iterative methods are commonly used for determining seismic source locations instead of grid search, despite advancements in computational power (Eaton, 1969; Lee & Lahr, 1979; Lienert et al., 1986; Klein, 2002). These methods involve linearizing the inversion problem by making an initial guess of the hypocenter and origin time (x_0, y_0, z_0, t_0). By assuming the true hypocenter is close to the initial guess, travel-time residuals $r_i = t_i^o - t_i^c$ at the trial hypocenter can be expressed as a linear function of the corrections $(\Delta x, \Delta y, \Delta z, \Delta t)$ applied to the hypocentral distance (Equation 2):

$$r_i = \left(\partial T \big/ \partial x_i \right) \Delta x + \left(\partial T \big/ \partial y_i \right) \Delta y + \left(\partial T \big/ \partial z_i \right) \Delta z + \Delta t \qquad (2)$$

This linearization allows the problem to be represented in matrix form, where the vector of travel-time residuals can be related to the vector of corrections in hypocentral parameters using the Jacobian matrix. Standard linear algebra methods, like least squares or singular value decomposition, are then applied to solve for the corrections. The Geiger method, based on iterative least squares, is widely used but may converge to a local minimum if the initial guess is far from the true solution or if the data is poorly configured. Tests and considerations of uncertainties in input data are essential for accurate results.

2.2. Grid Search Methods

Given sufficient computational power, a commonly employed method for calculating travel times of seismic phases involves conducting a grid search over all possible locations and origin times, computing the arrival time at each station, comparing them with the observed arrival times (Sambridge, 1999; Lomax, 2005). The optimal hypocentral location and origin time are determined based on the best agreement between observed and calculated travel times. The agreement is quantitatively measured using the least squares solution, which minimizes the sum of squared residuals. The root mean squared residual (RMS) is often provided as a measure of location precision, indicating the average residual when residuals are of comparable magnitudes. However, it is important to note that a low RMS value does not necessarily guarantee an accurate hypocenter determination. Precision should not be confused with accuracy, as computational solutions can still introduce uncertainties and errors due to factors like velocity model errors, data quality, and incomplete understanding of wave propagation characteristics. To address these potential errors and establish confidence in the results, additional approaches such as uncertainty analysis, sensitivity analysis, and statistical testing should be employed.

2.3. Relative Methods

This is one of the most powerful approaches for determining earthquake location builds upon the concept of relative arrival time differences (Double Difference - DD) between pairs of earthquakes and stations, rather than relying solely on absolute arrival times. By comparing the differences in arrival times between seismic events recorded at multiple stations, the DD method seeks to identify and exploit the commonalities in the seismic wave propagation patterns. This approach effectively reduces the impact of errors introduced by inaccuracies in the seismic velocity model. The well-known program HypoDD (Waldhauser, 2001), as the most popular earthquake location program which incorporated the DD criteria, uses an iterative algorithm that adjusts the hypocenter locations of earthquakes by minimizing the overall misfit between observed and predicted arrival time differences. The method's ability to identify and correct systematic errors makes it particularly valuable for resolving closely spaced earthquakes and accurately relocating seismic clusters, thus contributing to a more comprehensive understanding of Earth's seismic activity.

2.4. Inversion

In almost all earthquake location programs, the most important step is the inversion or minimization process. Generally, different programs utilize a similar approach in this step. The least squares approach is commonly used to quantify misfit since it yields simple formulations of the minimization problems. It is also effective when the residuals are generated by uncorrelated Gaussian noise.

However, this ideal scenario is often not realized in real-world problems. A major challenge arises from the presence of outliers, which are individual residuals with abnormally large values. In such cases, a single residual of 4, for instance, can contribute 16 times more to the misfit than a residual of 1, which can significantly affect the optimization results. After computing the misfits (e.g., RMS), the trial point with the lowest RMS value may be designated as the "solution".

However, in practice, real data may exhibit multiple final points with comparable RMS values, even if these points are far apart. Consequently, it is necessary to estimate the likely uncertainties associated with the solution. Based on this, many inversion algorithms embedded in earthquake location programs are capable of considering the uncertainties of input data in their calculations. Additionally, during the inversion process, after determining a new hypocentral candidate, phase residuals are recalculated. If outliers are detected, the location program applies appropriate weighting to mitigate their influence on the inversion process, reducing their impact on the final results.

2.5. Outliers and Solution

The commonly used least squares fit can give undue weight to the largest residuals, which can lead to inaccurate earthquake locations. To address this issue, most location programs use a weighting scheme that assigns lower or no weight to observations with large residuals. This can be done after a few iterations, when the residuals are already close to their final values. However, if there are individual large residuals, residual weighting may not be effective in obtaining accurate solutions, even if the majority of the data is good. In these cases, it is advisable to scan the data for gross errors, such as minute errors, before beginning the iterative procedure (Lee & Lahr, 1972; Lienert et al., 1986). Additionally, a Wadati diagram can be used to identify obvious outliers, which can help to improve the accuracy of the earthquake location.

Another weighting scheme is distance weighting, which implies that data from closer stations are given more weight in the location process than data from more distant stations. The choice of the appropriate weighting function depends on various factors, including the geometry of the seismic network, the distribution of the stations, and the characteristics of the seismic source. In general, using distance weighting functions can improve the accuracy and reliability of seismic locations, especially for local seismic events. However, it is important to note that the use of distance weighting functions can also introduce some biases, particularly if the seismic network is not homogeneous or if there are systematic errors in the data. Therefore, it is important to carefully evaluate the effectiveness of any distance weighting function used in seismic location studies.

Phase picking weighting is another weighting technique, which mostly implies by cross-correlated-based or AI-based earthquake location methods (Chamberlain et al., 2018; Zhu et al., 2019; Mousavi et al., 2020). Phase weighting involves assigning different weights to seismic phases based on their reliability and importance. The weights reflect the confidence level of each phase and are used to prioritize their contribution in the location calculation. Typically, phases with higher signal-to-noise ratios and better arrival time measurements are assigned higher weights, indicating their greater influence on the final location determination. By appropriately weighing the phases, these programs can mitigate the impact of erroneous or less reliable phase measurements, resulting in more precise and reliable earthquake locations.

In this paper, we investigate the impact of outliers and proper data weighting on earthquake location results in the presence and absence of noise. To accomplish this, synthetic earthquake catalogs (noisy and noise-free) are generated, and the results of earthquake location will be examined using three selected location programs.

3. Generating Synthetic Data

This section undertakes an investigation of the seismic location results, emphasizing the crucial significance of appropriate weighting during the phase picking process. It explores into the profound impact that proper weighting has on the outcomes of earthquake location and the precise estimation of location errors. To facilitate this analysis, two distinct datasets are employed: one involving weighted data and the other utilizing unweighted data. The examination is facilitated by the generation of synthetic travel time data, enabling a thorough exploration of the aforementioned factors.

In this case, we focus on a designated area spanning 150 square kilometers. Within this region, 100 earthquakes are randomly distributed (Figure 1), employing a uniform distribution method across two fault planes characterized in Table (1). To accurately locate these hypothetical earthquake events, a seismic network comprising 15 stations, with mid-distance of 50.0 km is strategically deployed within a radius of 100 square kilometers surrounding the fault zones. Furthermore, a comprehensive three-dimensional velocity model

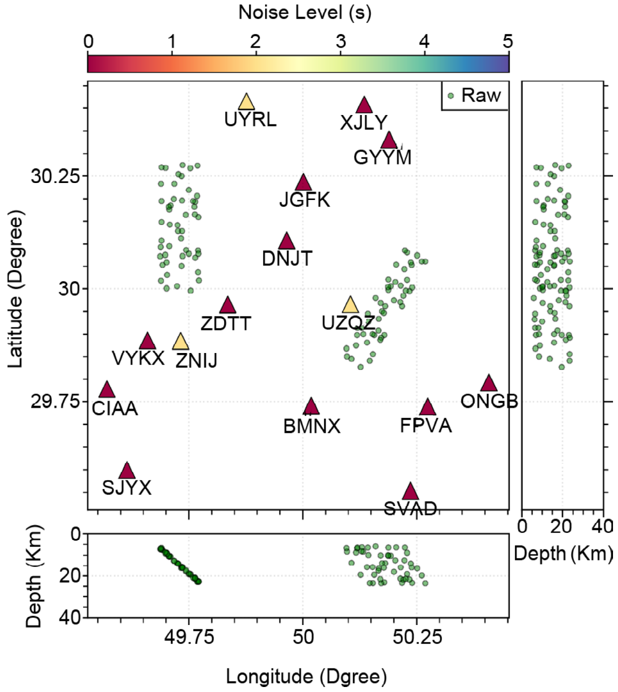


Figure 1. The seismic network employed in synthetic test (triangles) with two clusters of earthquakes distributed over two pre-assumed faults. Horizontal and vertical cross sections are plotted in right and down panels.

Table 1. Specifications of pre-assumed faults used for the generating synthetic events.

Fault No.	Geometry (km)	Depth (km)	Dip (°)	Strike (°)	N
1	10 x 15	15	60	0	50
2	10 x 15	15	70	30	50

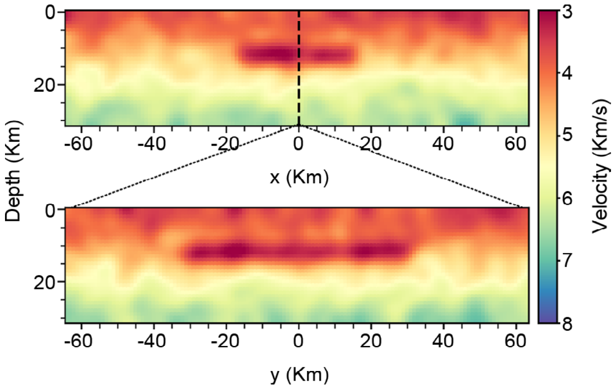


Figure 2. The 3D velocity model used for generating synthetic travel times.

(Figure 2) is constructed, incorporating depth-dependent linear velocity variations according to Equation (3).

$$v(z) = V_0 + r\,z \tag{3}$$

In Equation (1), the constant V_0 represents the velocity in the initial layer, while r denotes the rate of velocity increase with depth and z represents the depth. To improve the resemblance of the velocity model to the actual structure of Earth's crustal velocity, we introduce lateral velocity anomalies. This is accomplished by applying a Gaussian filter with $\sigma = 2$ km/s to the initial velocity model. This filtering process generates lateral variations in velocity between adjacent velocity nodes, with a maximum change of the two standard deviation relative to the mean velocity of them. Finally, we incorporate a negative anomaly with 2% perturbation located in central part of the region and with the dimension of 32 km in longitude, 64 km in latitude and 5 km in depth, into the velocity model (Figure 2).

In the next step, we generate synthetic travel times for the P phases, ranging from 90% to 50%, and for the S phases, with a maximum of 50%, across all stations. To accomplish this, we utilize a travel-time calculator code developed by White et al. (2020), which takes into consideration the three-dimensional velocity model. To account for station-specific noise characteristics, we employ the station noise model outlined in Table (2). Three stations, UZQZ, UYRL and ZNIJ, were chosen strategically within the context of seismic event clusters. Station UZQZ was positioned in close proximity to the epicenter of southern earthquake cluster, while station UYRL played a pivotal role in reducing the azimuthal gap within northern cluster. Station ZNIJ, on the other hand, was selected at random. Subsequently, an error component was introduced to the travel time measurements recorded by these selected stations. This error was generated through a normal distribution function characterized by a mean of 2 s and a standard deviation of 0.5 s. Notably, no such error perturbed the travel time measurements obtained from the other stations. This model allows us to generate noise characteristics specific to each station. It is important to note that the accuracy of phase readings for seismic phases varies based on the noise level of each station. Higher noise levels result in a greater phase reading error. Consequently, we implement an appropriate weighing scheme to assign the correct weight to each synthetic phase. The correct weight is determined based on the amount of noise added to each phase, as specified in Table (3). To compute the appropriate weight corresponding to the generated travel time error, our default approach entailed taking the maximum allowable error as 1 s (Table 3). Subsequently, if the randomly generated error fell below 25% of this maximum value (i.e., 0.25 s), an equivalent weight of 0 was assigned. When the error ranged between 25% and 50% of the maximum error (i.e., from 0.25 to 0.50 s), a weight of 2 was designated. Likewise, error levels of 75% and 100% of the maximum error yielded weights of 3 and 4, respectively.

Table 2. Noise model utilized to generate random error with calculated arrival times.

Noise Model (m, σ)	Station Code
(2.0s, 0.5s)	UYRL
(2.0s, 0.5s)	UZQZ
(2.0s, 0.5s)	ZNIJ
(0.0s, 0.0s)	Others

Table 3. Assigned arrival time weights according to travel time errors. The errors scaled to 1 seconds.

Assigned Weight	Error Relative to 1s (%)
100	0-25
75	25-50
50	50-75
25	75-100
0	>100

Finally, we end up, with two catalogs of earthquakes, each containing 100 seismic events. The first catalog (Cat-1), includes seismic phases without applying any weighing and in the second catalog, (Cat-2), seismic phases are provided along with their corresponding weights. The seismicity map illustrating the distribution of epicenters and depths of the synthetic catalog, are depicted in Figure (1).

4. Results

The earthquake location analysis using three distinct programs, namely Hypocenter (Lienert et al., 1986), Hypo71 (Lee & Lahr, 1972), and HypoDD (Waldhauser et al., 2001), coupled with two different catalogs, Cat-1 (unweighted) and Cat-2 (weighted), has provided valuable insights into the accuracy and precision of event locations. The synthetic data approach, which incorporated known true event locations, facilitated a thorough assessment of the errors calculated by the programs and the corresponding true errors, allowing for a comprehensive evaluation of the seismic location methodologies.

The earthquake location results obtained from the Hypocenter program (Figure 3) were analyzed to assess both the calculated errors ($error_{unw}$ and $error_{w}$ for unweighted and weighted errors, respectively) and the true errors ($raw\text{-}rel_{unw}$ and $raw\text{-}rel_{w}$ for unweighted and weighted errors, respectively). The histograms illustrate the distribution of hypocentral and depth errors for both the calculated and true values. The utilization of the weighted catalog led to significant improvements in events location with the true horizontal errors less than 2 km and 5 km by 42.4% and 17.2%, respectively. Moreover, the events with true depth errors less than 2 km and 5 km experienced enhancements of 7.5% and 18.9%, respectively. These results clearly demonstrate the positive impact of employing a weighted catalog on the accuracy and reliability of earthquake location analyses using the Hypocenter program. Table (4) summarizes the key findings of the comparison between Cat-1 and Cat-2 events for the Hypocenter program results.

As shown in Table (4), Cat-1 events analyzed using the Hypocenter program exhibited significant differences between the true and calculated horizontal and depth errors. The unweighted approach of the Hypocenter program resulted in considerable discrepancies, with the calculated

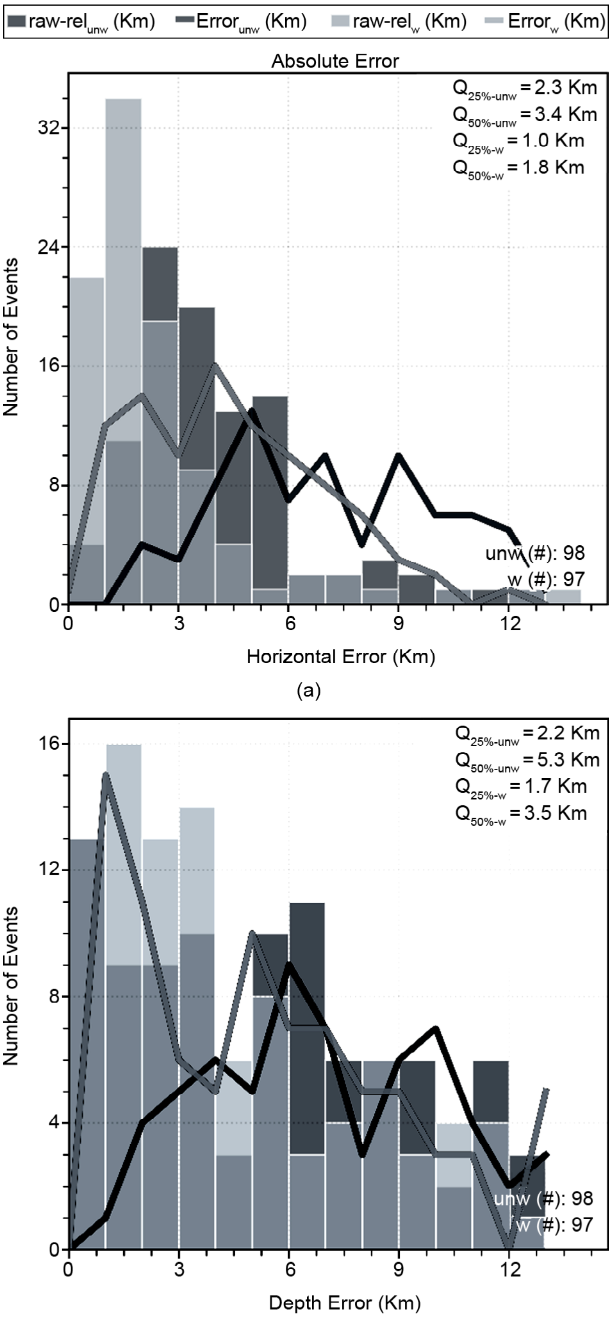


Figure 3. Histograms of horizontal (a) and depth (b) errors for Cat-1 and Cat-2 events located using the Hypocenter program. Curved lines denote the calculated horizontal and depth errors reported by Hypocenter program. The symbols $error_{unw}$ and $error_{w}$ represent the unweighted and weighted errors calculated by the Hypocenter program respectively. The symbols $raw\text{-}rel_{unw}$ and $raw\text{-}rel_{w}$ represent the unweighted and weighted true errors respectively. $Q_{(25\%\text{-}unw)}$ and Q_(25%-w)show the mean of horizontal/depth errors for the first quarter of Cat-1 and Cat-2, and $Q_{(50\%\text{-}unw)}$ and $Q_{(50\%\text{-}w)}$ show the mean of horizontal/depth errors for half of the Cat-1 and Cat-2 events, respectively.

Table 4. Statistical results obtained by three earthquake location programs used in this study. Here the Terms "true" refereed as the difference between actual and located hypocenters, and "calculated" means the location error reported by each program.

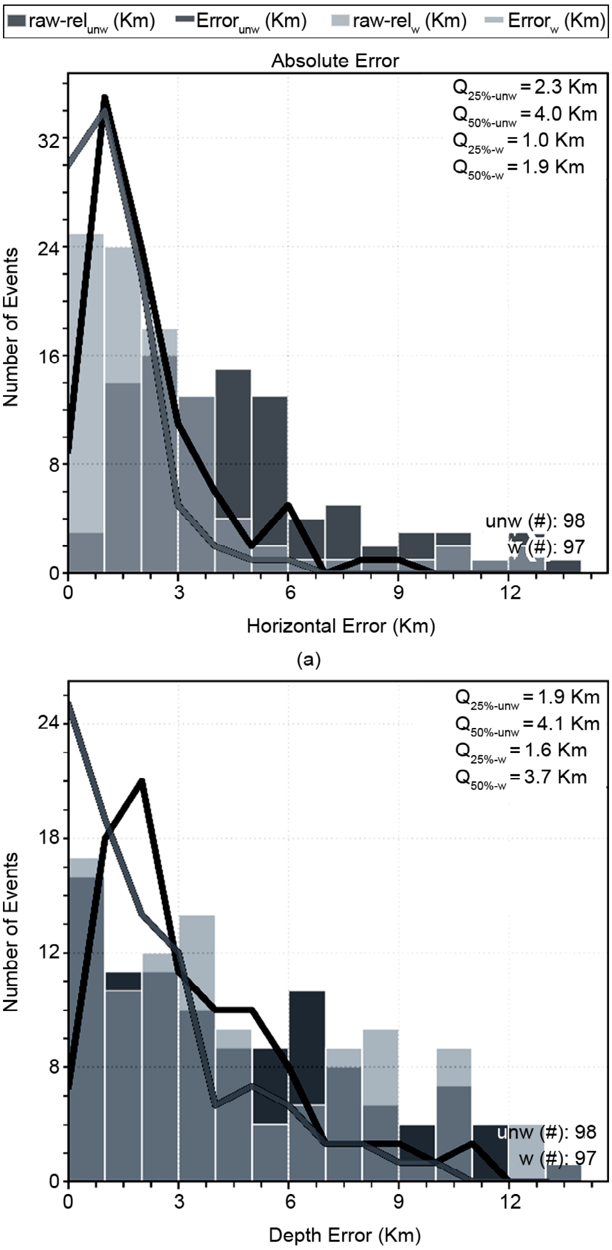
Error Type		Hypocenter		Hypo71		HypoDD	
		Calculated	True	Calculated	True	Calculated	True
Horizontal Error (< 2 km)	Cat-1	0%	15.3%	46.3%	17.5%	94.8%	95.7%
	Cat-2	13.4%	57.7%	67.4%	51.6%	100%	99%
Horizontal Error (< 5 km)	Cat-1	15.5%	73.5%	89.5%	62.9%	100%	100%
	Cat-2	54.6%	90.7%	97.9%	88.4%	100%	100%
Depth Error (< 2 km)	Cat-1	1%	22.4%	24.2%	27.8%	93.8%	78.7%
	Cat-2	15.5%	29.9%	46.3%	28.4%	100%	87.5%
Depth Error (< 5 km)	Cat-1	16.5%	45%	67.4%	55.7%	100%	97.9%
	Cat-2	38.3%	63.9%	77.9%	64.2%	100%	100%

errors often underestimating the true errors. In contrast, Cat-2, which incorporated a weighted catalog based on the local noise model for each station, demonstrated significantly improved accuracy, with the calculated errors aligning more closely with the true errors.

The Hypo71 program results (Figure 4 and Table 4) showed that Cat-1 events experienced larger errors in the horizontal and depth calculations compared to Cat-2 events. Employing the weighted catalog resulted in reduced errors, leading to more accurate event locations.

Specifically, the events with true horizontal errors of less than 2 km and 5 km were improved by 34.1% and 25.5%, respectively, while the true depth errors for the same error range witnessed enhancements of 0.6% and 8.5%, respectively. These findings underscore the efficacy of employing a weighted catalog in the Hypo71 program, corroborating its potential to enhance the accuracy of event locations.

Moving on to the results obtained using the HypoDD program, as presented in Figure (5) and Table (4). Notably, both Cat-1 and Cat-2 events analyzed using the HypoDD program demonstrated remarkable accuracy. The calculated errors closely matched the true errors for the vast majority of the events, indicating that the HypoDD program accurately estimated the event locations and associated errors. However, the weighted catalog demonstrated a slight advantage, achieving 8.8% improvement for true depth errors of less than 2 km and 2.1% improvement for true depth errors of less than 5 km. While the true horizontal errors remained relatively unchanged, the weighted catalog ensured that all events were accurately



(a)

(b)

Figure 4. Histograms of horizontal (a) and depth (b) error result from locating of Cat-1 and Cat-2 using Hypo71 program. Curved lines denote the calculated horizontal and depth errors reported by Hypo71 program. Other parameters are similar to Figure (3).

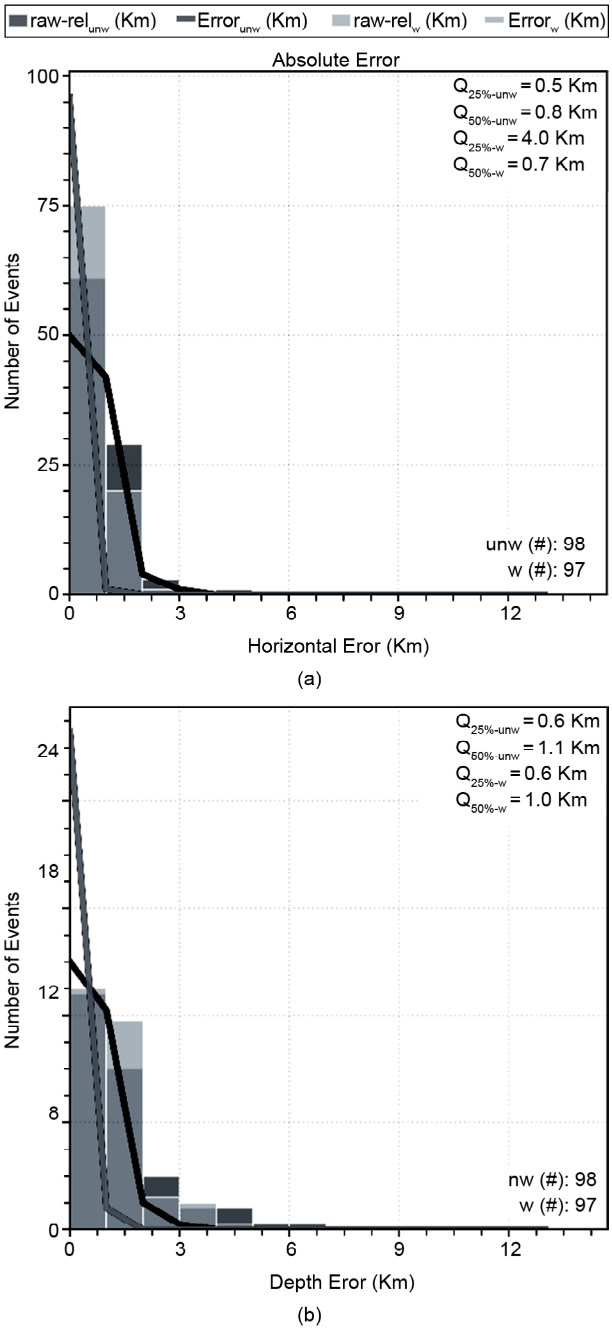


Figure 5. Histograms of horizontal (a) and depth (b) error result from locating of Cat-1 and Cat-2 using HypoDD program. Curved lines denote the calculated horizontal and depth errors reported by HypoDD program. Other parameters are similar to Figure (3).

identified within the specified error ranges. These results reaffirm the benefits of using a weighted catalog, even in a sophisticated program like HypoDD, which inherently yields highly accurate event locations.

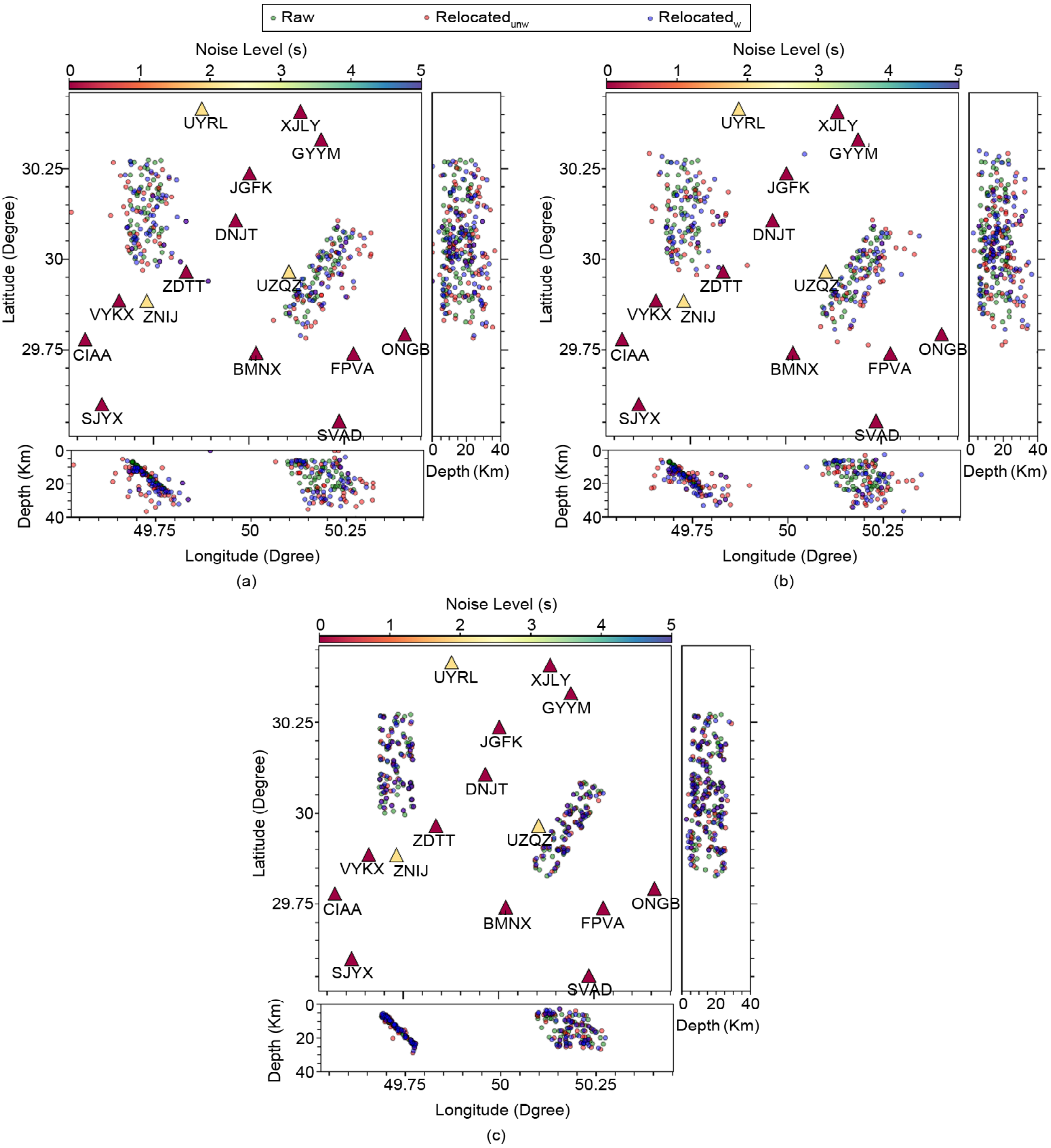
5. Discussion and Conclusion

The main focus of this study was to investigate the impact of weighting on earthquake location accuracy using three well-known earthquake location programs: Hypocenter, Hypo71, and HypoDD. Two synthetic catalogs of earthquakes, Cat-1 and Cat-2, were generated to represent different scenarios of adding noise and implementing the weighting scheme. In the first catalog (Cat-1), the phase arrivals at three random stations were contaminated with noise and the rest was left without any noise and no weighting scheme is applied in Cat-1. In the second catalog (Cat-2), the phase arrivals were weighted, considering the local noise model for each station. The results of locating both catalogs were extensively analyzed to assess the effects of weighting on earthquake location accuracy, error estimation, and uncertainty reduction.

The analysis of the results revealed that incorporating proper weighting during the phase reading stage had significant positive effects on improving earthquake location accuracy, reducing location errors, and minimizing uncertainties. Among the three programs used, the HypoDD program demonstrated the best performance in accurately determining earthquake locations and reducing localization errors. When applied to Cat-2, the HypoDD program successfully located over 94% of the events with a horizontal error of less than 2 km, surpassing the performance of the Hypocenter and Hypo71 programs, which achieved values close to 58% and 51%, respectively. The superiority of the HypoDD program was also evident in accurately determining the depth of events, further highlighting its superior performance compared to the other two programs (refer to Table 4 and Figure 6).

The Hypo71 program, with its robust outlier detection algorithm, outperformed in estimating the error of earthquake location. Even in the presence of outliers, the Hypo71 program demonstrated an average improvement of 25% in correctly estimating the hypocentral error for Cat-1. Furthermore, when comparing the location results of the two programs on Cat-2, it was evident that proper weighting of data led to consistent performance between the Hypocenter and Hypo71 programs (Figure 6).

In conclusion, this study showcases the remarkable benefits of using a weighted catalog



(a)

(b)

(c)

Figure 6. Relocation map obtained by three location programs, Hypocenter (a), Hypo71 (b) and HypoDD (c). Raw epicenters are plotted as green, unweighted catalog as red and weighted catalog as blue circles, respectively. Among all relocated catalogs, HypoDD performance is the best as both unweighted and weighted catalogs show high correlation with raw data. Notably, the Hypocenter unweighted catalog shows significant deviations between relocated and raw catalogs.

in earthquake location analyses. The consistent improvements in accuracy and precision observed in all three programs (Hypocenter, Hypo71, and HypoDD) highlight the potential of the weighted catalog to improve seismic research practices.

As a future extension, real-data applications could be explored to validate the findings obtained from synthetic data. Additionally, sensitivity analyses, hybrid methods, and uncertainty quantification could be investigated to further optimizing the utilization of weighted catalogs in practical seismic studies. Also, successful combination of automatic phase reading methods with accurate phase weighting offers a powerful approach to improving earthquake location accuracy and has broad implications for seismic hazard assessment, earthquake source characterization and engineering seismology.

The findings derived from this study pertain to a simulated seismic network, and thus, it is crucial to acknowledge that alterations in the network's geometry can potentially lead to some variations in the final results and the efficacy of the earthquake location programs employed under the new conditions. Consequently, we anticipate that future investigations will not only explore the impact of network geometry but also delve into the influence of other significant physical parameters that play a pivotal role in the determination of earthquake location problem.

The improved accuracy in earthquake location results is particularly relevant for seismic hazard assessment and earthquake source characterization. Accurate earthquake locations are fundamental for understanding the seismicity patterns in a region, identifying active fault zones, and assessing earthquake potential in a given area. Moreover, precise event locations can help to validate earthquake forecasting models, improving our ability to anticipate seismic events and their potential impact on populated regions. In conclusion, this study highlights the value of incorporating weighting schemes in earthquake location methodologies. The results indicate that accurate determination of phase weights can lead to consistent performance across different earthquake location programs. The adoption of proper weighting in earthquake location analyses can significantly enhance the reliability and accuracy of earthquake catalogs, ultimately contributing to more informed decision-making and improved seismic risk assessment and enhancing public safety.

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