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Idealized Bilinear Moment-Curvature Curves of Slender Rectangular RC Wall Sections

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ABSTRACT

Methods are proposed to develop idealized moment-curvature response curves of slender rectangular RC wall sections with uniformly distributed longitudinal reinforcement, using limit states, namely, cracking of concrete in tension, tensile yielding of reinforcement layers, and compression failure of concrete. It is recommended that tensile yielding of an inner layer of reinforcement is considered to develop idealized moment-curvature response curve of RC wall sections in the absence of compressive axial load, as against tensile yielding of extreme layer of reinforcement in the presence of compressive axial loads. Distance of the critical inner layer of reinforcement from highly compressed edge depends on percentage of longitudinal reinforcement in the section; the distance varies from 0.5D to 0.98D with an increase in percentage of longitudinal reinforcement, where D is the length of the wall, but does not depend on plan aspect ratio. Furthermore, axial-flexure interaction envelope can also be developed from the idealized moment-curvature response curves.

Keywords:
Flexural rigidity;
Curvature ductility;
Compression failure;
Limit state design;
RC walls

1. Introduction

Slender reinforced concrete (RC) structural walls form the main lateral load resisting system (LLRS) in low to mid-rise buildings, located in high seismic regions. RC walls have high flexural and shear strengths and can be expected to efficiently resist strong earthquake shaking through inelastic actions. Proper design and detailing of longitudinal and transverse reinforcements helps in achieving the required strength and ductility. Two common ways of detailing longitudinal reinforcement in RC walls are: (a) uniform distribution of reinforcement along the length of the wall, and (b) uniform distribution of reinforcement along the length of the wall with more reinforcement concentrated at the two ends. Although, in-plane flexural strength of walls with the two distributions are generally not too different for the same amount of total reinforcement, the former often results in enhanced shear strength and improved shear behaviour [1]. Flexural strength and curvature ductility of rectangular RC wall sections can be determined from their nonlinear moment-curvature (M-φ) curves. In general, curvature φ of a RC section is defined as the ratio between strain at highly compressed edge to the depth of neutral axis, while the ratio between ultimate curvature (φ_u) to yield curvature (φ_y) is termed as curvature ductility of the section. Curvature ductility is the general measure of ductile response of a structure that significantly depends on the ultimate compressive strain capacity of concrete, compressive strength of concrete, yield strength of reinforcement bars, percentage of tension and compression reinforcements, and level of axial load.

RC structural walls can be modelled as *mid-pier frame elements* in wall-frame systems [2-3]. Inelastic regions in frame elements are defined in the form of idealized bilinear M-φ curves to undertake nonlinear analysis in commercial structural analysis programs. The idealized M-φ response curve must represent the effective (cracked) flexural rigidity, flexural strength, and curvature ductility of the RC section. Strain levels in concrete and reinforcement at the onset of critical damage states like cracking of concrete, yielding of reinforcement bars in tension, and compression failure of concrete are used in the estimation of flexural strength and curvature ductility. This paper proposes methods to arrive at idealized bilinear M-φ curves of RC wall sections with uniformly distributed reinforcement along the length of the wall, at different levels of axial loads, which can be used as an input to perform nonlinear analysis of RC walls modelled as mid-pier frame elements.

2. Estimation of Flexural Strength and Curvature

Flexural strength of RC walls can be estimated basic principles of using mechanics, considering equilibrium of forces, compatibility of strains and constitutive relations of materials, Figure (1). These are given in Eqs. (1) to (4):

Force equilibrium equation:

$$\sum(f_{sci} - f_{\mathrm{csc}\,i})A_{sci} + f_{c,avg}bx_u - \sum f_{sti}A_{sti} = P \qquad (1)$$

Compatibility conditions:

$$\frac{\varepsilon_{sci}}{x_u - d''} = \frac{\varepsilon_c}{x_u} = \frac{\varepsilon_{sti}}{d - x_u} = \frac{\varepsilon_{\mathrm{csc}\,i}}{x_u - d''} \qquad (2)$$

Constitutive relations:

$$f_{c,avg} = \begin{cases} 0.67\,f_{ck}\left[\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right) - \dfrac{1}{3}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] & 0 \le \varepsilon_c \le \varepsilon_{co} \\ 0.67\,f_{ck}\left[1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_{co}}{\varepsilon_c}\right)\right] & \varepsilon_{co} < \varepsilon_c \le \varepsilon_{cu} \end{cases} \qquad (3)$$

and

$$f_{\mathrm{csc}} = \begin{cases} 0.67\,f_{ck}\left[2\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right) - \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] & 0 \le \varepsilon_c \le \varepsilon_{co} \\ 0.67\,f_{ck} & \varepsilon_{co} < \varepsilon_c \le \varepsilon_{cu} \end{cases} \qquad (4)$$

where A_{sti} is the area of reinforcement bars in i^{th} layer under tension, A_{sci} the area of reinforcement bars in i^{th} layer under compression, d'' the effective cover on compression side, $f_{c,avg}$ the average compressive stress in concrete, f_{sti} the stress in i^{th} layer of reinforcement bars under tension, f_{sci} the stress in i^{th} layer of reinforcement bars under compression (both estimated from stress-strain characteristics of reinforcement bar), f_{csci} the stress in concrete at the level of i^{th} layer of reinforcement bars under compression, x_u the depth of neutral axis, f_{ck} the characteristic strength of concrete, ε_c the compressive strain in concrete, ε_{co} the strain in concrete at highly compressed edge at peak stress, and ε_{cu} the ultimate strain in concrete at highly compressed edge at peak stress.

Depth of the neutral axis x_u is estimated through iterations to satisfy the force equilibrium given by Eq. (1), and the curvature φ for a given strain

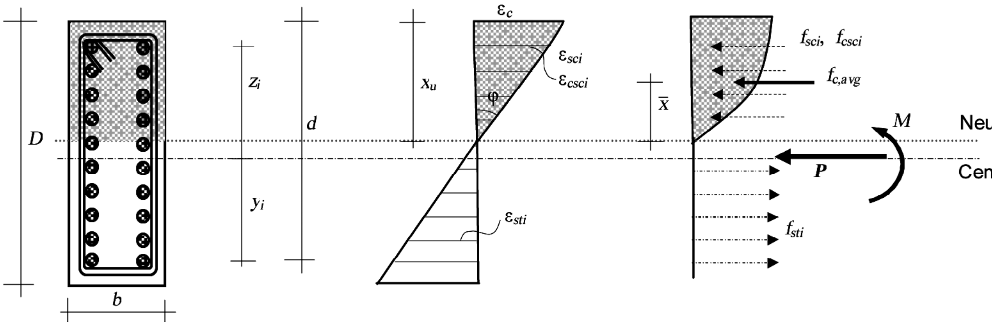


Figure 1. Typical strain and stress distributions across a rectangular RC wall section under flexure with salient geometric, strain and stress quantities

distribution, Figure (1), is obtained as:

$$\varphi = \frac{\varepsilon_c}{x_u} = \frac{\varepsilon_{st}}{d - x_u} = \frac{\varepsilon_c + \varepsilon_{st}}{d} \tag{5}$$

where ε_{st} is the strain in extreme layer of tension reinforcement. Besides, flexural strength is estimated by considering moments of compressive and tensile forces about the centroidal axis, as:

$$M = \sum f_{sti} A_{sti} y_i + \sum (f_{sci} - f_{csc\,i}) \times A_{sci} z_i + f_{c,avg}\, b x_u \left(\frac{D}{2} - \left(x_u - \bar{x} \right) \right) \tag{6}$$

where,

$$\bar{x} = \begin{cases} \left[\dfrac{\dfrac{2}{3} - \dfrac{1}{4}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)}{1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)} \right] x_u & 0 \le \varepsilon_c \le \varepsilon_{co} \\ \left[\dfrac{\dfrac{1}{2} - \dfrac{1}{12}\left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^2}{1 - \dfrac{1}{3}\left(\dfrac{\varepsilon_{co}}{\varepsilon_c}\right)} \right] x_u & \varepsilon_{co} < \varepsilon_c \le \varepsilon_{cu} \end{cases} \tag{7}$$

Here, y_i is the distance of centroid of i^{th} layer of reinforcement bars in tension from centroidal axis, and z_i the distance of centroid of i^{th} layer of reinforcement bars in compression from centroidal axis.

3. Idealized Moment-Curvature Curves

Idealized bilinear M-φ curve of RC wall sections can be developed based on *limit states* of strain in concrete and reinforcement bars in the section, Figures (2) and (3). These are:

1) *Cracking of concrete*, represented by maximum tensile strain at the extreme tension fibre of concrete reaching limiting tensile strain of concrete ε_{cr} of 0.00008 [4];
2) *Yielding of critical layer of reinforcement bars on tension side*, represented by tensile strain in the critical layer of reinforcement bars reaching limiting strain ε_y of 0.002 + (f_y/ E_s); and
3) *Compression failure of concrete*, represented by maximum compressive strain at the highly compressed edge of concrete reaching limiting strain of ε_{cu}.

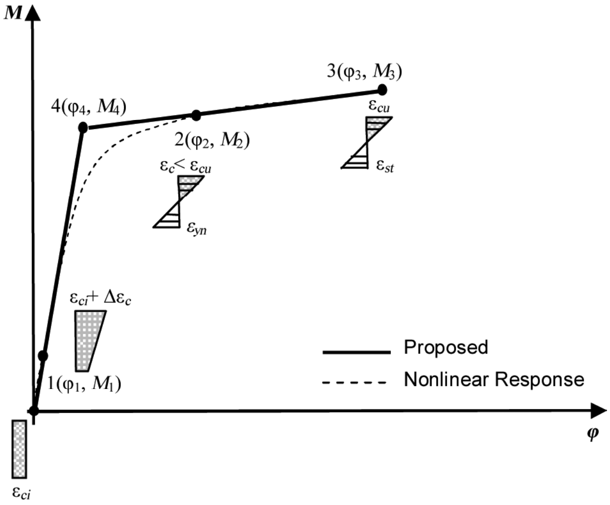


Figure 2. Proposed idealized M-φ curve of RC wall sections at axial load P >0.

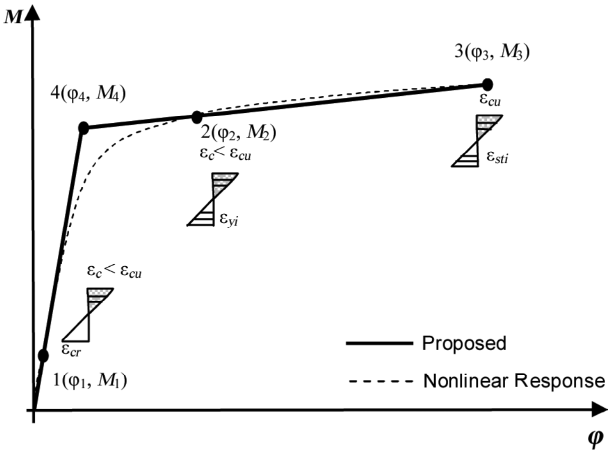


Figure 3. Proposed idealized M-φ curve of RC wall sections at axial load P = 0.

The above three limit states are always observed in walls with no or low levels of axial compressive load. However, in presence of significant compressive axial loads, the first limit state of cracking of concrete in tension will not be observed. Furthermore, the axial-flexure (*P-M*) interaction envelope of RC sections can be viewed as comprised of two distinct regions on either side of the balanced failure point where the limit states of yielding of extreme layer of reinforcement bars in tension and compression failure of concrete occur simultaneously. In the region above the balanced failure point, known as the compression failure region, the limit state of yielding of critical layer of reinforcement will not be observed [5-6]. Hence, simple methods are proposed using the above limit states to develop idealized *M*-φ curves of RC wall sections for two distinct cases of axial load levels, namely, (a) compressive axial load, as in columns, and (b) zero axial load, as in beams.

3.1. RC Wall Sections with Axial Load $P > 0$

In the presence of compressive axial loads, but less than the balance load (P_{bal}), the idealized M-φ curve can be developed using four salient points determined using principle of basic mechanics, corresponding to the strain profiles at limit states shown in Figure (2). By assuming the strain at salient locations across depth of RC wall section at these four points (i.e., extreme tension fibre of concrete, highly compressed edge of concrete, or a yielded layer of reinforcement), strains at other locations across depth can be estimated. Coordinates of point 1 are estimated by assuming a small strain increment $\Delta\varepsilon_c$ at the extreme compression fibre (assumed to be 0.0001 in this study) to the uniform compressive strain ε_{ci} corresponding to the axial compressive load P acting alone [6]. Thereafter, point 2 corresponds to tensile yielding of extreme layer of reinforcement, and point 3 to compression failure of concrete. Finally, the coordinates of an additional point 4 is obtained using coordinates of points 1, 2 and 3, to represent idealized yield by extrapolation as, Figure (2):

$$\varphi_4 = \frac{M_2 - \left(\dfrac{M_3 - M_2}{\varphi_3 - \varphi_2}\right)}{\dfrac{M_1}{\varphi_1} - \left(\dfrac{M_3 - M_2}{\varphi_3 - \varphi_2}\right)} \tag{8}$$

and

$$M_4 = \frac{M_1}{\varphi_1}\varphi_4 \tag{9}$$

For compressive axial loads more than the balance load (P_{bal}), tensile yielding of reinforcement layers do not occur, and hence, point 2 cannot be estimated. In such cases, point 4 is directly obtained using coordinates of points 1 and 3 alone as:

$$M_4 = M_3 \tag{10}$$

and

$$\varphi_4 = \frac{M_3}{M_1}\varphi_1 \tag{11}$$

3.2. RC Wall Sections with Axial Load $P = 0$

In the absence of any axial load, again the idealized M-φ curve can be developed using four points determined using principle of basic mechanics, corresponding to the strain profiles at limit states shown in Figure (3). Here, points 2 and 3 represents the same limit states as in the previous case of $P > 0$ (i.e., of tensile yielding of critical layer of reinforcement and compression failure of concrete, respectively), while point 1 now represents the case of cracking of concrete in tension. Thereafter, point 4 is obtained using coordinates of points 1, 2 and 3, to represent idealized yield as before, Eqs. (8) and (9). However, tensile yielding of a critical inner layer is to be considered here for point 2 as against yielding of extreme layer of reinforcement used for cases with $P > 0$ to maintain better energy balance of idealized M-φ curve with actual nonlinear M-φ curve. A separate study is presented to identify this critical layer of tension reinforcement to be considered to obtain idealized M-φ curve of rectangular RC wall sections at zero axial loads.

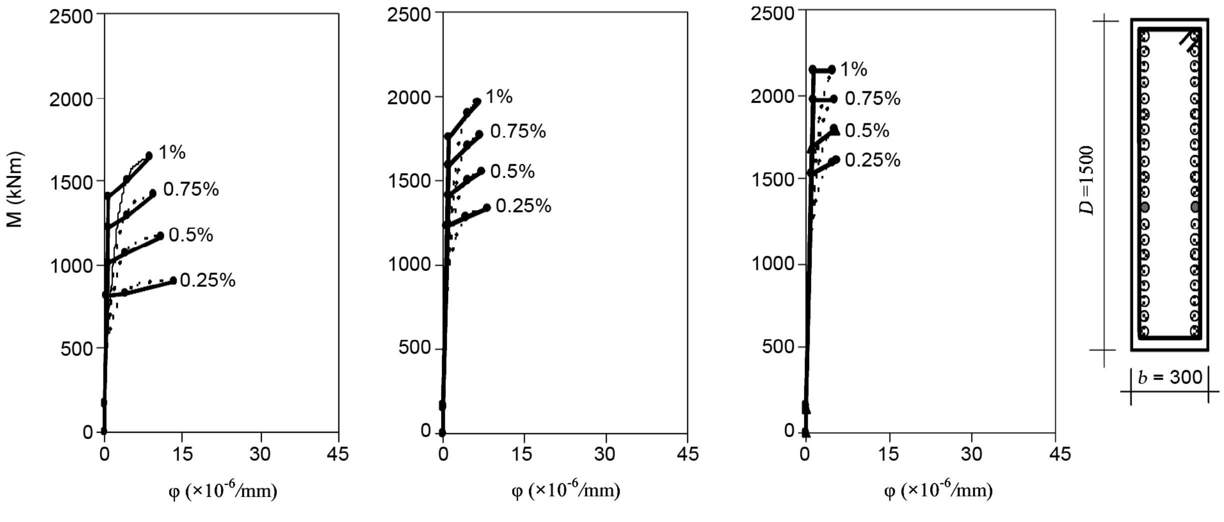
4. Numerical Study

Two sets of numerical study are presented to demonstrate the proposed idealization methods for RC wall sections of 300 mm width with longitudinal reinforcement spacing of 100 mm, but with varying plan aspect ratio (D/b), percentage of longitudinal reinforcement (ρ_{sl}), and axial load ratio, Table (1). The first set of study demonstrates the proposed idealization of M-φ curve, considering yielding of extreme layer of reinforcement and other limit states, for $P > 0$. The second set of study helps identify the critical layer of tensile reinforcement to be considered in arriving at a reasonably accurate idealized M-φ curve, at $P = 0$. Grades of concrete and reinforcement bars considered are M30 and Fe415, respectively, and stress-strain curves given in Indian Standard are used in the study [7].

The idealized M-φ curves of the example cross-section (with $D/b = 5$, and sl of 0.25, 0.5, 0.75 and 1) are shown at $P = 0.1P_u$, $0.2P_u$ and $0.3P_u$ along with nonlinear M-φ curves in Figures (4a), (4b), and (4c), respectively. In the presence of axial loads, only few layers of reinforcement yield, unlike yielding of majority of reinforcement layers as in zero axial load case. Here, as discussed in Section 3.1, yielding of extreme layer of reinforcement is considered to determine the coordinates of point 2 of idealized M-φ curve, which gives reasonably

Table 1. Details of RC wall sections considered for numerical study.

	D/b	Percentage of Longitudinal Reinforcement (ρ_{sl})	Axial Load (As a Fraction of Axial Load Capacity P_u)
Set I	5	0.25, 0.5, 0.75, 1	0.1, 0.2, 0.3
Set II	5, 6, 7, 8, 9, 10, 15, 20, 25	0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2.0, 2.5, 2.75, 3	Zero



(a) (b) (c)

Figure 4. M-φ curves of example RC wall section of *D/b* = 5, with percentages of longitudinal reinforcement of 0.25%, 0.5%, 0.75% and 1%, at axial load levels (a) 0.1P_u (b) 0.2P_u and, (c) 0.3P_u.

good estimates of initial flexural rigidity, flexural strength, and curvature ductility of rectangular RC wall sections. It is evident from the *M*-φ curves, Figure (4) that curvature ductility of RC wall sections significantly reduces with increase in axial compressive loads. Thus, it is prudent to design RC wall sections to have axial load below the balance point (i.e., less than P_{bal}) in the P-M interaction diagram, under the combined action of dead, live and earthquake loads. This helps increase ductility capacity of the section and in turn of the structure. Moreover, considering yielding of extreme layer of reinforcement is appropriate to arrive at the idealized *M*-φ curve of RC wall sections in the presence of relatively low levels of axial loads, maintaining reasonable energy balance with actual nonlinear curve, due to the delayed yielding of reinforcement layers. A comparison of, bilinear *M*-φ curves developed using the proposed method considering yielding of extreme layer of reinforcement with nonlinear *M*-φ curves obtained from experimental investigations of RC wall sections reported in literature, at $P > 0$ (but, less than P_{bal}), is also presented [8]. Idealized *M*-φ curves obtained using the proposed methodology closely represents flexural rigidity, strength and curvature ductility of the wall sections, Figure (5).

The idealized *M*-φ curves of the example cross-section (with *D/b* = 5, and sl of 0.25, 0.5, 0.75 and 1) are shown at $P = 0$, along with nonlinear *M*-φ curves in Figure (6a). In the absence of axial load, many layers of reinforcement yield. Here, two cases of idealizations are shown, first considering the yielding of extreme layer of reinforcement (as considered for $P > 0$), and second, considering the yielding of a critical inner layer of reinforcement, to determine the coordinates of point 2 of idealized *M*-φ curve. It is seen that considering the yielding of a critical inner layer of reinforcement, located at a distance x_y from the highly compressed edge, leads to more appropriate idealization of *M*-φ curves and provides reasonably good estimates of initial flexural rigidity, flexural strength, and curvature ductility of rectangular RC wall sections with $P = 0$ [9]. The variation of the distance xy of the critical layer normalized by the length *D* of the wall, for different percentage of

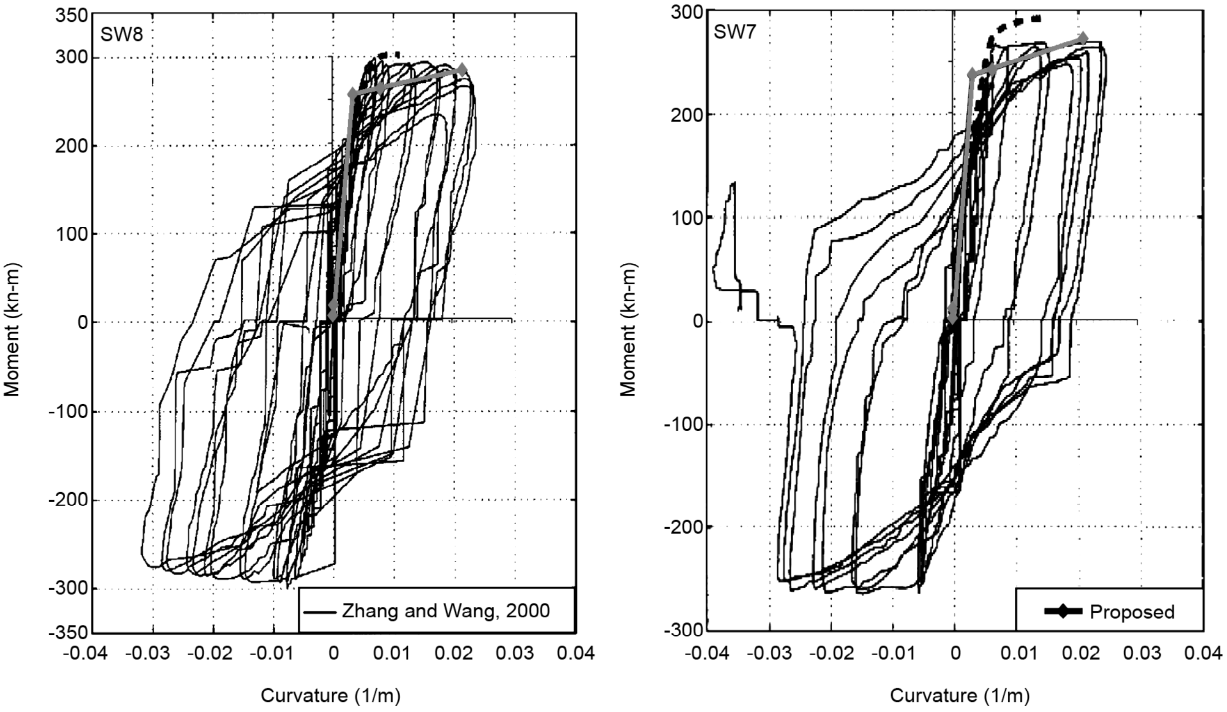
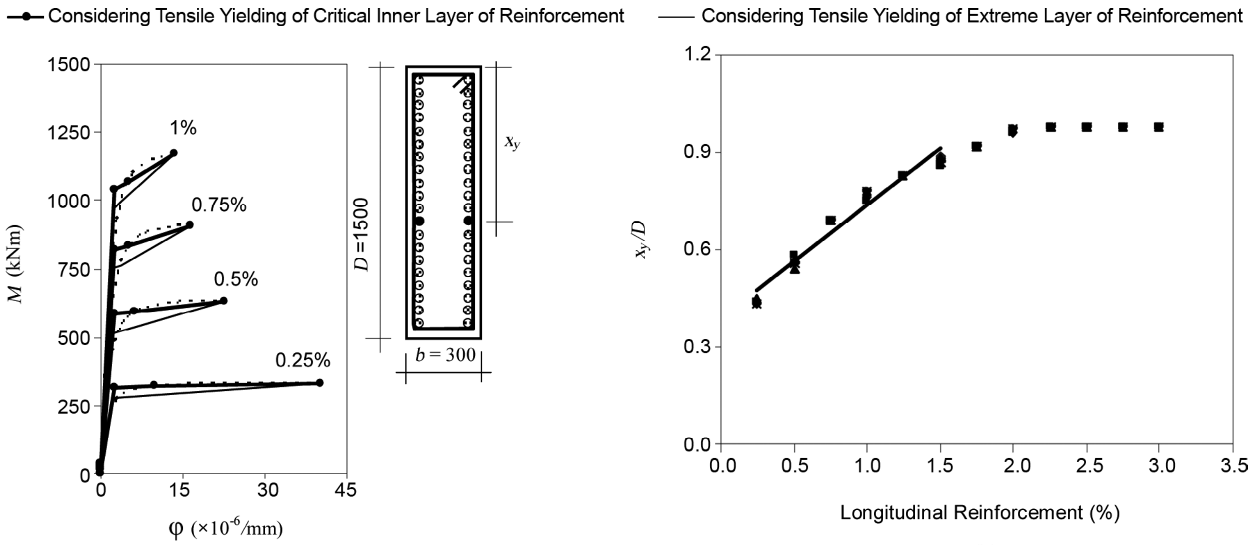


Figure 5. Comparison of M-φ curves of RC wall sections with axial loads.



(a)

(b)

Figure 6. M-φ curves of example RC wall section of D/b = 5 at axial load level of P = 0 (b) Variation of x_y / D with percentage of longitudinal reinforcement (x_y / D shown for D/b 5, 6, 7, 8, 9, 10, 15, 20 and 25).

longitudinal reinforcement is presented in Figure (6b) and Table (2). The distance xy increases from 0.5D to 0.98D with an increase in the percentage of longitudinal reinforcement, but does not depend on the plan aspect ratio of walls. Besides, x_y / D remains constant at higher percentages of reinforcement (more than 2%), when yielding of extreme layer of reinforcement bars in tension forms the basis to develop the idealized *M*-φ curve. This is because the compression failure of concrete governs the behaviour without yielding of the majority of layers of reinforcement. The values of x_y / D given in Table (2) can be used as a guideline to identify the critical inner layer of reinforcement to be considered to develop idealized *M*-φ curve of RC wall sections at zero axial load. A comparison of, bilinear *M*-φ curves developed using the proposed method considering the yielding of critical inner layer of

Table 2. Location of critical inner layer of reinforcement bars in tension represented by x_y/D in rectangular RC wall sections with varying D/b and percentage of longitudinal reinforcement.

D/b	**Longitudinal Reinforcement (%)**												
	0.25	0.50	0.75	1.0	1.25	1.50	1.75	2.0	2.25	2.5	2.75	3.0	
5	0.43	0.56	0.69	0.76	0.83	0.89	0.92	0.96	0.96	0.98	0.98	0.98	x_y/D
6	0.45	0.56	0.69	0.75	0.83	0.86	0.92	0.96	0.96	0.98	0.98	0.98	
7	0.45	0.54	0.69	0.78	0.83	0.88	0.92	0.97	0.97	0.98	0.98	0.98	
8	0.43	0.56	0.68	0.78	0.83	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
9	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
10	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
15	0.43	0.56	0.68	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
20	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.97	0.98	0.98	0.98	
25	0.43	0.56	0.69	0.78	0.84	0.87	0.92	0.97	0.98	0.98	0.98	0.98	

reinforcement, Table (2), with nonlinear *M*-φ curves obtained from experimental investigations of RC wall sections reported in literature, at $P = 0$, is also presented [10]. Idealized *M*-φ curves obtained using the proposed methodology closely represents the flexural rigidity, strength and curvature ductility of the sections, Figure (7). Finally, P-M interaction curve of an example section (with $D/b = 5$ and $\varepsilon_{sl} = 0.25\%$), and the idealized *M*-φ curves at $P=0$, $0.1P_u$, $0.2P_u$, $0.3P_u$, $0.5P_u$, $0.7P_u$, and $0.9P_u$ are shown in Figure (8); the variation in flexural capacity and reduction of curvature ductility with increase in axial compressive load are evident.

5. Conclusions

The salient conclusions drawn from the study are:

- Normal strain-based bilinear idealization proposed of actual nonlinear *M*-φ curves of rectangular RC wall sections effectively represents the initial flexural rigidity, flexural strength and curvature ductility.
- Under compressive axial loads, considering the yielding of extreme layer of reinforcement is recommended to arrive at the idealized moment-curvature curve. The distance xy of the critical inner layer of tension reinforcement from highly compressed edge to be considered in developing idealized bilinear moment-curvature curve of RC wall sections at zero axial load depends on the percentage of longitudinal reinforcement bars in the section and varies from 0.5D at low percentage of the reinforcement of 0.25% to 0.98D at high percentage of reinforcement of 2%, but does not vary with *D/b* of section.

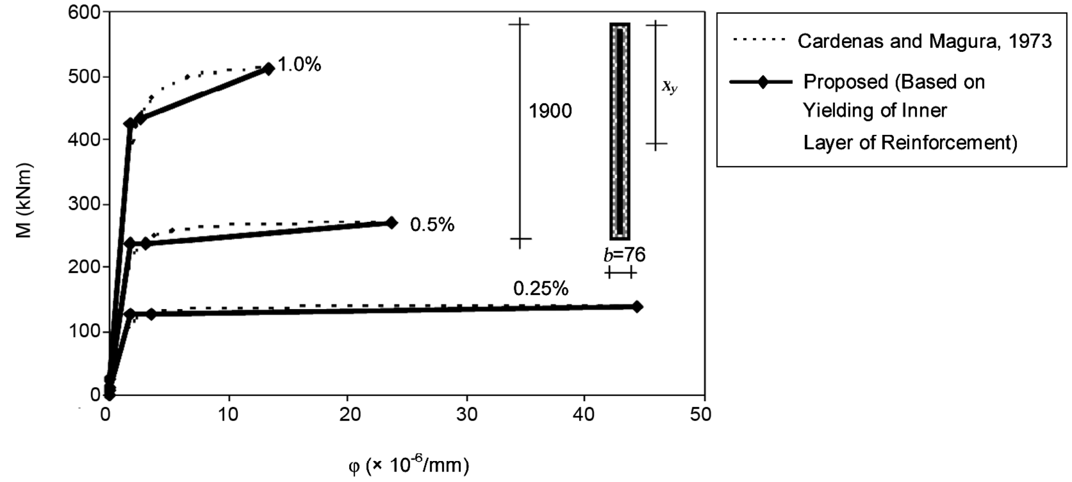


Figure 7. Comparison of M-φ curves of RC wall section at axial load P=0.

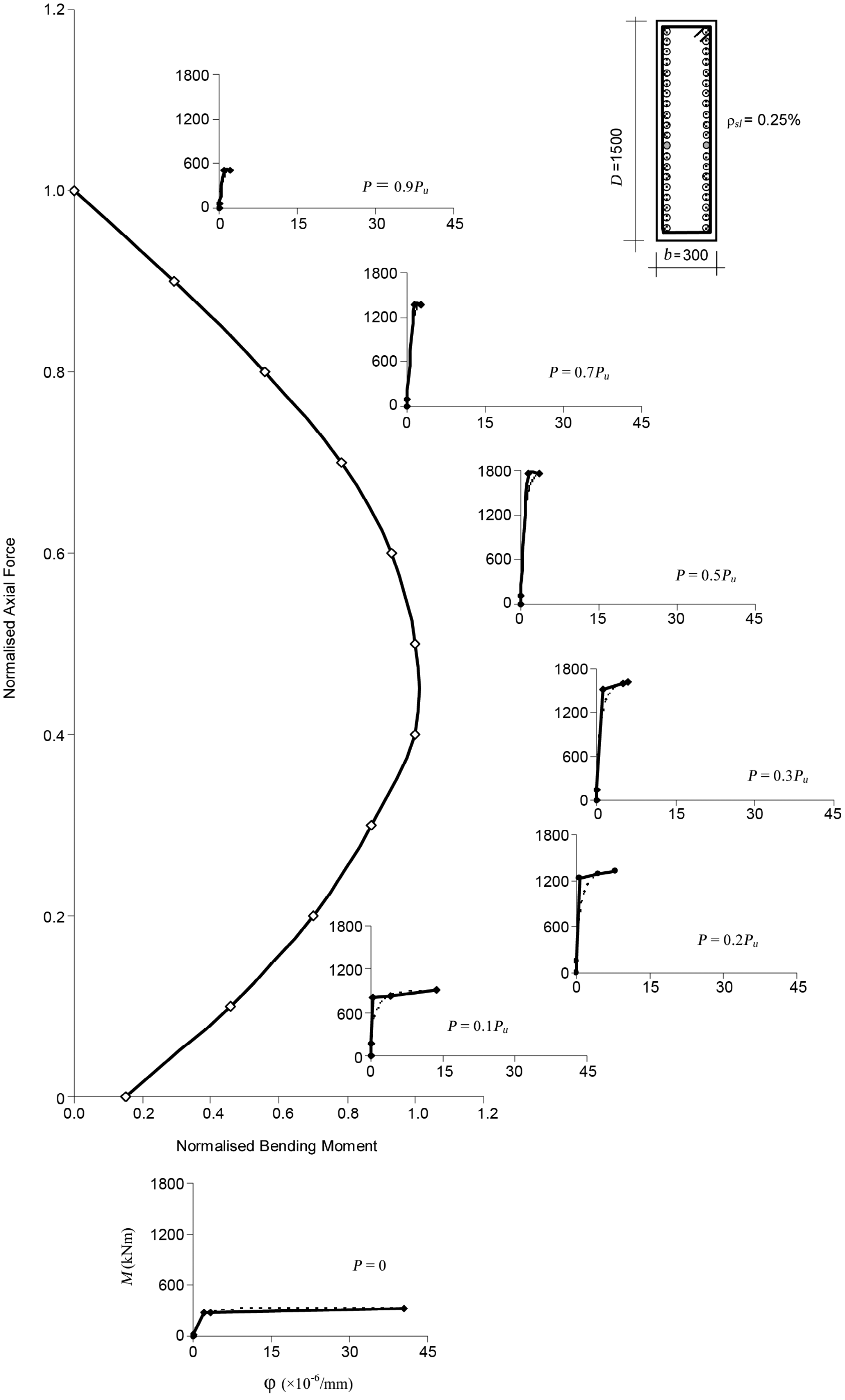


Figure 8. P-M interaction envelope and M-φ curves of example RC wall section at various axial load levels (axial load level P shown as a fraction of axial load capacity P_u).

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