Time-Dependent Scaling Patterns in Sarpol-e Zahab Earthquakes

Yasaman Maleki[1*] and Mostafa Allamehzadeh[2]

1. Assistant Professor, Faculty of Mathematical Sciences, Alzahra University, Tehran, Iran,
* Corresponding Author; email: y.maleki@alzahra.ac.ir
2. Assistant Professor, Earthquake Prediction National Center, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran

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ABSTRACT

In this paper, the dynamics seismic activity and fractal structures in magnitude time series of Sarpol-e Zahab earthquakes are investigated. In this case, the dynamics seismic activity is analyzed through the evolution of the scaling parameter so-called Hurst exponent. By estimating the Hurst parameter, we can investigate how the consecutive earthquakes are related. It has been observed that more than one scaling exponent is needed to account for the scaling properties of earthquake time series. Therefore, the influence of different time-scales on the dynamics of earthquakes is measured by decomposing the seismic time series into simple oscillations associated with distinct time-scales. To this end, the empirical mode decomposition (EMD) method was used to estimate the locally long-term persistence signature derived from the Hurst exponent. As a result, the time-dependent Hurst exponent, H(t), was estimated and all values of H>0.5 was obtained, indicating a long-term memory exists in earthquake time series. The main contribution of this paper is estimating H(t) locally for different time-scales and investigating the long-memory behavior exist in the non-stationary multifractal time-series. The time-dependent scaling properties of earthquake time series are associated with the relative weights of the amplitudes at characteristic frequencies. The superiority of the method is the simplicity and the accuracy in estimating the Hurst exponent of earthquakes in each time, without any assumption on the probability distribution of the time series.

Keywords:
Empirical mode decomposition; Time-dependent Hurst parameter estimation; Long-memory

1. Introduction

Self-similarity is related to the occurrence of similar patterns at different time-scales. In this sense, probabilistic properties of self-similar processes remain invariant when the process is viewed at different time-scales [1]. In mathematical expression, a stochastic process $\{X(t),\ t \in R^+\}$ is scale invariant (or self-similar, denoted 'H-ss'), with Hurst parameter $0 < H < 1$, if for all $\lambda > 0$, it follows the scaling law:

$$X(\lambda t) \equiv \lambda^H X(t),\ t \in R^+ \tag{1}$$

where $\equiv$ means equality in all finite dimensional distributions [2]. The index H characterizes the self-similar behavior of the process, and a very large variety of methods has been proposed in the literature for estimating it [3-5]. An example of self-similar processes is fractional Brownian motion (*fBm*), a Gaussian process with stationary increments characterized by a positive scaling exponent $0<H<1$ [6]. For $0<H<0.5$, the increments of fBm show negative autocorrelation. The case $0.5 < H < 1$ corresponds to *fBm* with increment process exhibiting long-range dependence, i.e., the autocorrelation of the increment process decreases as a power law. When $H = 0.5$, the *fBm* is reduced to Brownian motion (BM), a process with independent increments [7].

Sometimes it happens that the self-similar processes are not quite adequate for real world phenomenon, and it would be useful to consider more general classes of stochastic processes, which are characterized by more than one parameter, but still preserve some of the good properties of scale invariant processes. For example, it has been observed that more than one scaling exponent is needed to account for the scaling properties of the real world processes. Such processes are called multi-scaling (or multifractals) and reflect the occurrence of different dynamics at different time-scales, and the local variations of roughness can be described by allowing the Hurst exponent to vary with time [8]. Cavanaugh et al. [9], Coeurjolly [5], Goncalves and Abry [10], Kent and Wood [11], Stoev et al. [12] and Wang et al. [13] studied this family of self-similar processes, and estimated the local Hurst parameter, $H(t)$.

Earthquakes are examples of complex phenomena that are scale invariant and fractal in their collective properties. These properties are revealed both in nature and laboratory experiments where the spatial, temporal and size distributions of earthquakes or laboratory acoustic emissions display structures that are invariant in scale [14-16]. The emergence of these properties is indicative of complexity and nonlinear dynamics in the earthquake generation process, such that concepts like fractals and multifractals are becoming increasingly fundamental for understanding geophysical processes and estimating seismic hazard more efficiently. Therefore, time series of earthquakes are widely used to characterize the main features of seismicity and to provide useful insights into the dynamics of the seismogenic system. However, based on the multifractal structure of seismic data, more than one scaling exponent is needed to account for the scaling properties of such processes.

Thus, the Hurst parameter estimation is an important problem in earthquake studies, because the seismicity is a complex natural phenomenon studied from its time-space characteristics, which is manifested according to certain statistical laws governing its occurrence. The correlations that may arise between the dynamics earthquakes occurrence have been the subject of many studies conducted around the world, especially in the most affected countries. Recently, worldwide research activity concerning the seismic phenomenon has focused on the study of correlations viewed in the time series space diagram. The correlations arises between the dynamics earthquakes can be measured by the Hurst parameter. Besides, by considering this parameter locally, the correlation existance between consecutive earthquakes can be described. To this end, the Hilbert-Huang transform [17] for estimating the time-dependent Hurst exponent, H(t), of earthquakes is applied. In this method, at first, the analysed time series decomposes into several oscillatory modes by means of the empirical mode decomposition (EMD). Secondly, the Hilbert transform is applied to these oscillations to obtain time varying attributes. The time-dependent scaling properties of seismic data are associated with the relative weights of the amplitudes at characteristic frequencies.

The paper is organized as follows: In Section 2, the Hilbert-Huang transform and the methodology used for estimation of $H(t)$ are reviewed. In Section 3, the performance of the method is evaluated for simulated data, and then the method to estimate the time-dependent Hurst exponent for Sarpol-e Zahab Seismic data during November 12, 2017 to January 20, 2018 is applied to find a pattern that exists between consecutive earthquakes. By this method, the negative and positive autocorrelation exists between consecutive earthquakes is investigated by evolution of the time-dependent Hurst parameter for all times.

2. Hilbert-Huang Transform

The Hilbert-Huang transform (HHT) [17] is a method proposed in analysis of non-linear and non-stationary processes. It was originally introduced in studying water wave evolution, but it has proven to be a useful tool for other complex signals [18-23].

The HHT consists of two steps: namely, the empirical mode decomposition (EMD) and the Hilbert transform (HT). The EMD decomposes the time series into a set of intrinsic mode functions (IMFs) and the Hilbert transformation of these IMFs provides local frequency and amplitude attributes [1]. The EMD is a full adoptive decomposition that does not require any a priori basis systems. The purpose of the method is to identify oscillating components of the process with the scales defined by the local maxima and the minima of the data itself. Hence, given a time series $X(t)$, $t = 1, 2, ..., N$, the EMD decomposes it into a finite number of IMFs denoted as $c_k(t)$, $k = 1, ..., n$ and a residue function, $r(t)$. The IMFs are components oscillating around zero and obtained through a sifting process which uses the local extrema to separate oscillations starting with the highest frequency. At the end of the sifting process, the time series $X(t)$ can be expressed as:

$$X(t) = \sum_{k=1}^{n} c_k(t) + r(t) \quad (2)$$

where the residue function, $r(t)$, is the non-oscillating drift of the data [17].

First, the EMD method pre-processes the time series and then the Hilbert transform is applied. This method, generates components of the time series whose Hilbert transform leads to physically meaningful definitions of instantaneous amplitude and frequency. The Hilbert transformation of each function c_k is defined as:

$$\widehat{c_k}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{c_k(t)}{t - \tau} d\tau \quad (3)$$

where the integral has a singular point at $\tau = t$ and it is defined as a Cauchy principal value [19]. In this case, a complex function $\tilde{C}_k$, would be defined as $\tilde{C}_k(t) = c_k(t) + \widehat{c_k}(t)$, with amplitude $a_k(t)$ and phase $\phi_k(t)$ that are defined as follows:

$$a_k(t) = \sqrt{c_k^2(t) + \widehat{c_k}^2(t)} \quad (4)$$

$$\phi_k(t) = \tan^{-1} \frac{\widehat{c_k}(t)}{c_k(t)} \quad (5)$$

Besides, the instantaneous frequency is defined as the derivative of the phase, $\phi_k(t)$, with respect to the time

$$\omega_k(t) = \frac{\phi_k(t)}{dt} \quad (6)$$

3. Time-Dependent Hurst Parameter Estimation

The time-dependent Hurst estimation method proposed in [1] was constructed by observing how the local amplitudes $a_k(t)$, Equation (3), change with respect to the local periods $\tau_k(t) = \omega_k^{-1}(t)$, Equation (4), for all $k = 1, 2, ..., n$. The estimation method was first applied to fBm, and empirically observed that the amplitude function obtained through the HHT follows a power-law behavior with respect to the instantaneous period as:

$$a_k(t) \propto \tau_k^{H(t)}(t) \quad (7)$$

where the time-dependent Hurst exponent describes the local scaling properties of the IMF amplitudes and takes values distributed around the Hurst parameter of fBm [1]. When we fit a linear regression line, between $\log a_k(t)$ and $\log \tau_k(t)$ for four randomly chosen times instances of a fractional Brownian motion with length $T = 10000$ and Hurst parameter $H = 0.6$, in Figure (1), the time-dependent

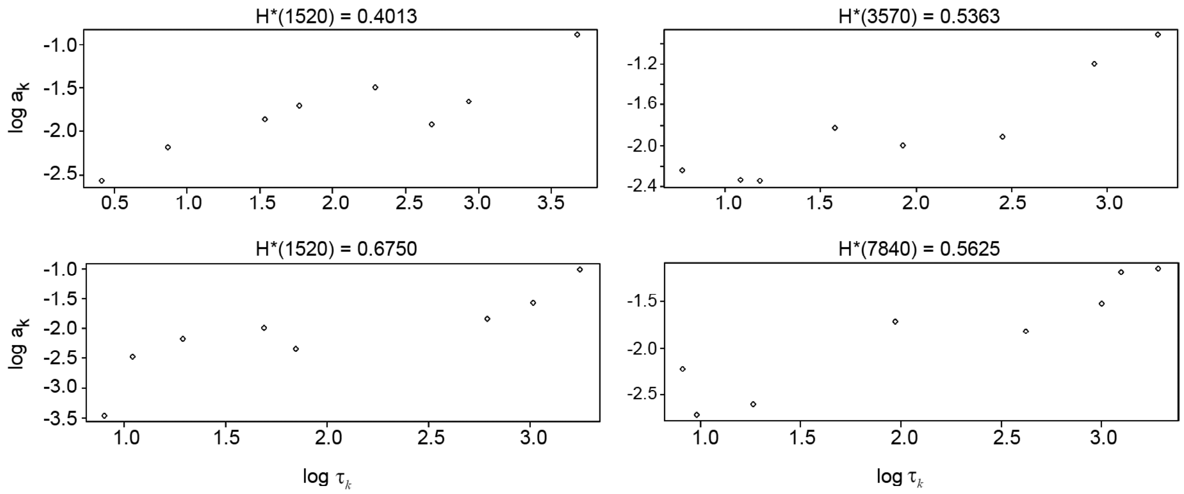

Figure 1. The local amplitudes $a_k(t)$ and the periods $\tau_k(t)$ are computed for a simulated fractional Brownian motion with Hurst parameter H=0.6 and length 10000 from Equation (6) $a_k(t) \propto \tau_k^{H(t)}(t)$. In these plots, the instantaneous amplitude is a function of period for four times t =1520, 3570, 5630 and 7840.

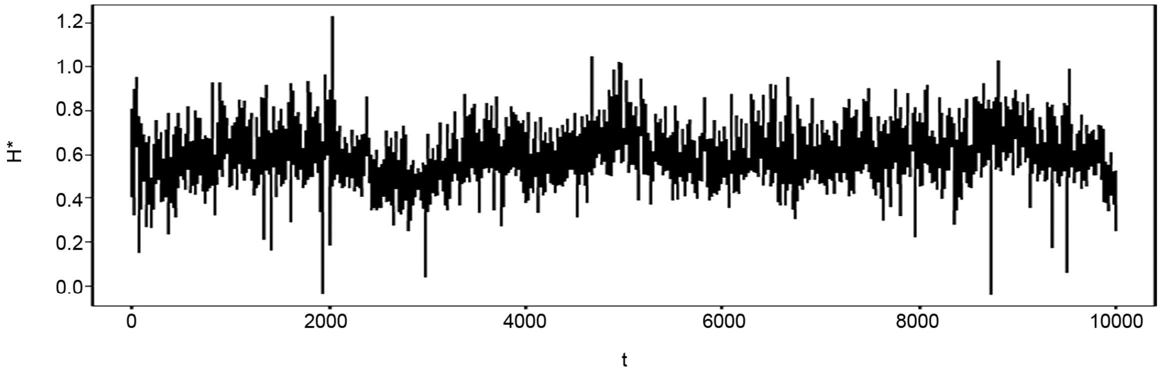

Figure 2. The time-dependent Hurst exponent, $H^*(t)$, estimated by the EMD method for a simulated fractional Brownian motion with Hurst parameter $H = 0.6$ and length 10000. The exponent $H^*(t)$ is on average, close to the Hurst parameter 0.6. The mean and standard deviation of estimated $H^*(t)$ in this figure are 0.5957 and 0.0079, respectively.

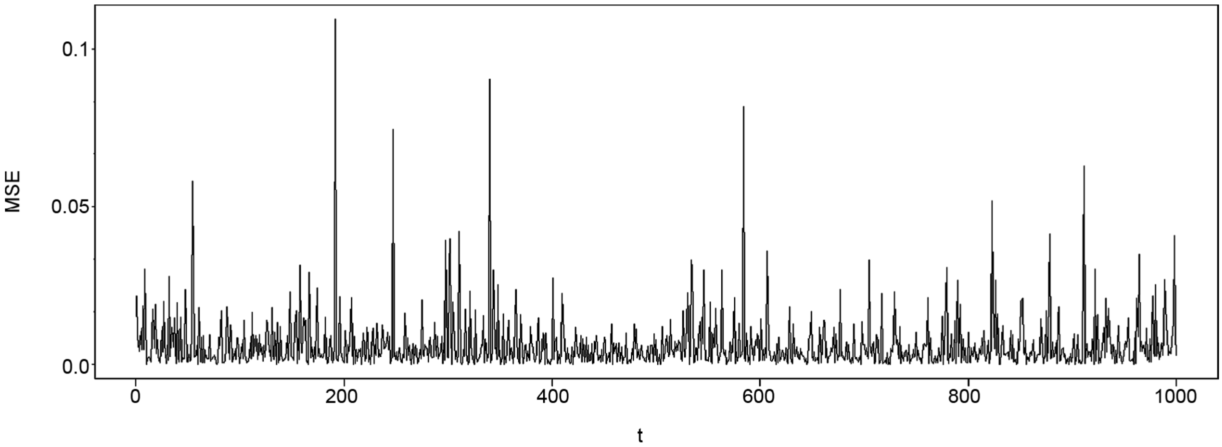

Figure 3. The mean-square of errors of the estimation method for m=100 iteration of the estimation algorithm for sample paths of fractional Brownian motion with length 10000 and Hurst parameter H=0.6, for the first 1000 observations.

Hurst exponent, $H^*(t)$, is estimated from the slope of the regression line. As it can be seen, all estimated Hurst parameters are consistently close to the value $H = 0.6$.

Figure (2) shows the time-dependent Hurst parameter that is estimated locally for all times in t = 1 to 10000, for a simulated *fBM* with H = 0.6 and length 10000. Then, the EMD method is applied and $H^*(t)$ is estimated for different times. The mean and standard deviation of estimated $H^*(t)$ in Figure (2) are 0.5957 and 0.0079, respectively. As it can be seen in simulated data, combination of EMD and HHT methods, estimated the Hurst parameter accurately, where the mean is close to the value 0.6 that the process was generated from, and the standard deviation is very small.

To investigate the accuracy of the estimation method, the method for simulated data should be applied. We have simulated $m = 100$ sample paths of fractional Brownian motions with length T = 10000 and Hurst parameter $H = 0.6$. Then, the time-dependent Hurst exponent, $H(t)$, is estimated for each time. The time-dependent sample mean for each t is calculated as $\langle H(t)\rangle = \frac{1}{m}\sum_{j=1}^{m} H_j(t)$. Besides, we the mean square of errors (MSE) of the estimation is calculated as:

$$MSE = \frac{1}{mT}\sum_{j=1}^{m}\sum_{t=1}^{T}(H_j(t) - H)^2 \tag{8}$$

where H is the Hurst parameter of a fBm that is simulated, and $H_j(t)$ is the Hurst exponent estimated from the jth sample path simulated from fBm. Figure (3) shows the MSEs of $H^*(t)$ for Hurst parameter H = 0.6. The small values of MSEs, show the accuracy of the estimation method.

3.1. Analysis of the Hurst Exponent for Seismic Data

In this section, we investigate the evolution of the time-dependent Hurst exponent in time series of seismic events (by magnitude) occurred in Sarpol-e

Zahab during November 12, 2017 to January 20, 2018 with a total of 295 seismic events with magnitude $M \geq 2.2$, the data period is about two months, beginning with the earthquake occurred on 12/11/2017 at 21:05:36 and ending with the earthquake on 20/01/2018 at 05:35:51. The time series of seismic events is depicted in Figure (4) and logarithm of frequency vs. magnitude for the earthquakes is shown in Figure (5).

Using the empirical mode decomposition method on the time series of seismic events, and applying the Hilbert-Huang transform on IMFs, the Hurst exponent is estimated locally for all times. The result is depicted in Figure (6) that shows the evolution

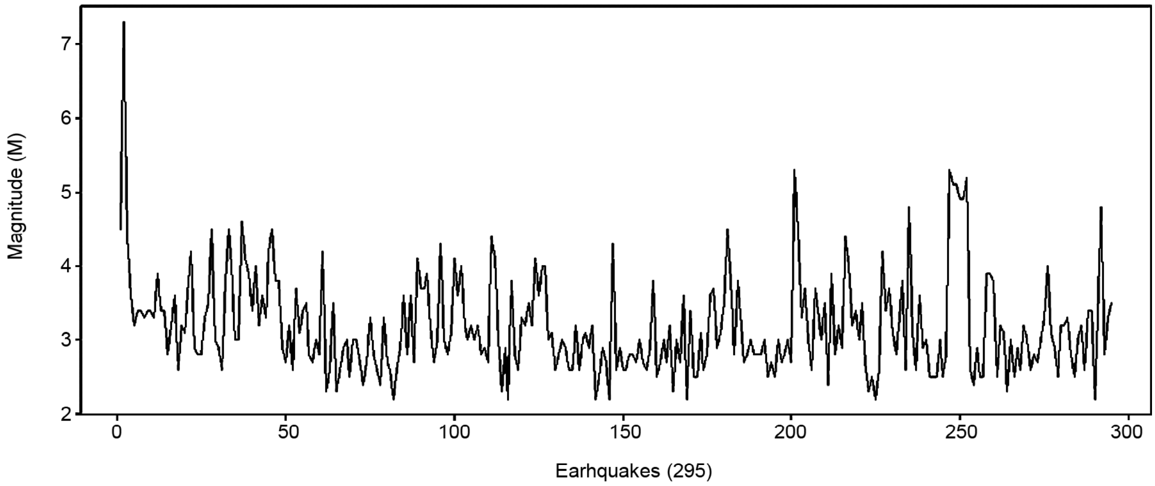

Figure 4. Time-series of magnitudes of the Sarpol-e Zahab seismic activities during November 12, 2017 to January 20, 2018, (295) earthquakes.

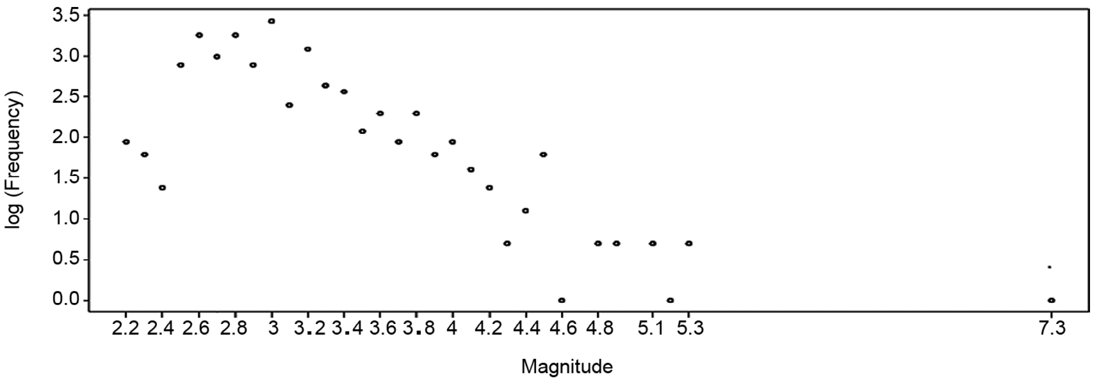

Figure 5. Logarithm of frequency vs. magnitude for the seismic events occurred since November 12, 2017 to January 20, 2018 in the Sarpol-e Zahab region.

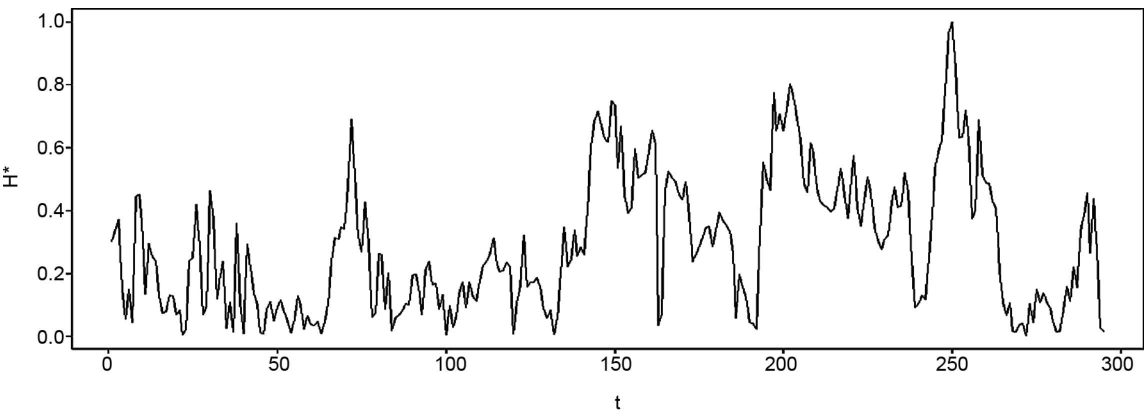

Figure 6. Time-dependent Hurst parameter $H^*(t)$ for seismic events occurred in the Sarpol-e Zahab region, since November 12, 2017 to January 20, 2018

model of the time-dependent Hurst parameter calculated for seismic activities occurred in the Sarpol-e Zahab region for a period of two months, from November 12, 2017 to January 20, 2018. As mentioned before, by estimating the Hurst parameter locally, we can investigate how consecutive earthquakes are related, and we can find a pattern between seismic activities. To this end, we have to separate the estimated Hurst parameters which are greater than 0.5, which shows the long memory pattern that exists between consecutive earthquakes. The trends and variation of the Hurst parameter reveals the time-dependent Hurst evolution for the considered period. The case $0 < H < 0.5$ corresponds to negative autocorrelation and the case $0.5 < H < 1$ exhibit long-range dependence, or long memory behavior, i.e., the autocorrelation of the process decreases as a power law. By separating the local Hurst parameters that are greater than 0.5 in Figure (6) and also compare this parameter with the dates that earthquakes occurred, we understand that, the long-memory behavior exists in consecutive earthquakes during

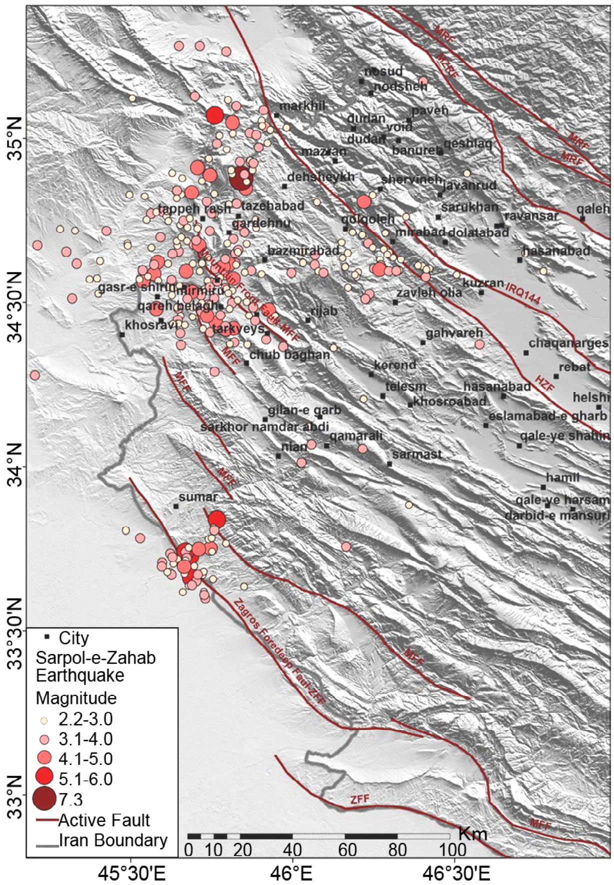

Figure 7. The map shows the events in Sarpol-e Zahab earthquakes in Iran.

25/11/2017 and 13/1/2018. Besides, there is a strong positive autocorrelation between earthquakes during 7/1/2018 to 13/1/2018. Moreover, Figure (6) shows that, in spite of the small values of the local Hurst parameter for foreshocks that shows a stochastic behavior in seismic activities, there is an increasing trend that exists between foreshocks, which leads to the main shock, and also between the aftershocks that goes to zero through seven days after the main shock occurred.

4. Conclusion

In this paper, we investigated the negative and positive autocorrelations exist between consecutive seismic activities by estimating the time-dependent Hurst parameter, $H^*(t)$. To this end, the empirical mode decomposition and the Hilbert-Huang transform are applied. Using this method, the seismic activities are studied locally, and the autocorrelation between consecutive earthquakes are estimated in each time. We have investigated the superiority of the estimator by simulation. Furthermore, the method is applied in estimating $H^*(t)$ of earthquakes occurred in Sarpol-e Zahab during November 12, 2017 to January 20, 2018. By estimating the Hurst parameter locally, and considering the values of $H^*(t)$ that are greater than 0.5, we identify that the long-memory behavior exist in consecutive earthquakes during 25/11/2017 and 13/1/2018. It is also seen that, in spite of small values of $H^*(t)$ for some times, which shows the stochastic behavior in earthquakes, the local Hurst parameters are tending to be greater than 0.5, and after this patterns, a peak in magnitude is seen. This paper shows that the combination of the EMD and its associated Hilbert spectral analysis offers a powerful tool to uncover the time-dependent scaling patterns of consecutive seismic activities data.

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