Research Note

Regarding a Continuum Approach Towards Predicting Earthquakes

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ABSTRACT

The paper develops a mathematical model that can produce effective predictions on the possibility of an inter-plate earthquake in a particular region. A hypothetical model based on the dilatancy-diffusion principle which, by its experimental studies, has proven to be an important feature towards earthquake prediction. The purpose of the study is to develop a model backed up by mathematics to provide a comprehensive way of predicting earthquakes. Theoretical methods involving continuum mechanics are used to determine the critical viscosity between the plates at which the movement of the plates is susceptible to causing an earthquake. Such a prediction methodology based on mathematical techniques is thus dealt in the paper to predict all three important parameters of an earthquake - time, magnitude and place. The interpretations of the work might prove to take one step closer at solving the question of predicting earthquakes.

Keywords:
Plate tectonics;
Inter-plate compression;
Viscosity; Pressure;
Fault lines

1. Introduction

Learning to predict earthquakes is the final goal of any seismologist and that certainly has been an incredibly difficult goal to achieve. Since earthquakes are effects caused due to tectonic pressures developed deep within the depths of Earth, it is difficult to say what actually happens underneath. In the following paper, a hypothetical model to predict compressional inter-plate earthquakes is discussed. Compressional inter-plate earthquakes are dealt in particular since they are the most recurring and devastating among the various types of earthquakes. Tectonic movement can rather be mapped but the right instant and place or energy of an earthquake has never been predicted on a substantially consistent basis.

The dynamic source theory and theories about rupture along fault line and subsequent models like finite source models and crack models of seismic sources are very useful in understanding the kinetics and resultant effects of the tectonic plate motion. Though these theories and models are extensive beautiful pieces of mathematics involving tensors etc., they haven't yet been developed enough to generate predictions on the time and location (on a global scale) of a particular earthquake. However, it is for a fact that the implications of these theories have been observed experimentally, so I don't mean to say they are wrong. Yet I have chosen not to involve dynamic theory into the current work. This is so, to create an alternative theory of what happens underneath and to create a model from scratch that could at least hypothetically predict earthquakes. Therefore, the huge field of dynamic source theory is not ignored, but an attempt has been made to create another theory that could be used to create earthquake prediction models. Furthermore, it is also an attempt in using a different concept rather than solid mechanics (as in most part of dynamic source theory). Continuum mechanics is used as the basis of the discussed earthquake mechanism.

Since no other known mechanism is the most probable one that accounts for the onset of an earthquake, a mechanism based on dilatancy-diffusion and continuum mechanics is proposed. Experimental analysis shows that dilatancy in rocks could prove to be a way of predicting earthquakes. The steep fall in the value of V_p/V_s has been suspected to be a precursor to an earthquake. The current research aims to take the very same concept to be able to not only predict earthquakes, but also pin-point its location, time and magnitude. The model is based on the assumption that due to dilatancy-diffusion, the boundary between the plates subjected to enormous pressure behaves as an undersaturated fluid of a finite viscosity. The proposed model is developed based on this assumption. The implications of the model are discussed in the paper as well. The mathematical equations and graphs of the model to generate the predictions and the methods used are discussed in the following work. By such means, I hope to achieve the aim of the paper - to develop at least a hypothetical model to predict the time, place and magnitude of an earthquake.

2. The Proposed Mechanism of an Earthquake

As said earlier, only compressional inter-plate earthquakes (thrust tectonics) are dealt in this paper. The theories of dynamic models of seismic faulting, basically, are concerned about what happens to a fault line with reference to slip rate, friction, etc. to study the rupture pattern and subsequent motion. The big part of the theory is based on "frictional instability". Since no other known confirmed mechanism is in place, I hypothesize a mechanism in this section to explain the instability that triggers two plates to slip. Considering compression movement, the two plates move towards one another. This movement between two plates, against each other, causes a heavy build up of pressure along the boundary of contact between the two plates. This pressure continues to mount until it is high enough to create cracks in the rocks. This experimentally is known to occur at about or more than half the breaking strength of the rocks. The rocks begin to dilate by virtue of the cracks, or in other words they expand. This enables the permeability of water into the medium (diffusion), thus causing the rocks to become an undersaturated solution. At this point, the layers of rock that are in contact develop a finite viscosity. This viscosity allows the rocks to slip past each other releasing the great energy developed. This mechanism, I suggest, accounts for what finally triggers an earthquake after the massive build up of pressure. This sudden release of energy creates powerful seismic waves that surface in the form of earthquakes as a result of the focal mechanism. Thereby, at this stage, the heavy pressures stacked up are suddenly released due to the freer movement after a certain threshold between the plates. This hypothetical change in viscosity, wherein the solid plates behave as an undersaturated fluid at the boundary between the plates is assumed to trigger an earthquake since it is the only theoretical plausible cause. Therefore, all further mathematics in the model is based on this assumption. The idea is to pinpoint the moment at which the viscosity is at its critical point when the plates can give in to each other causing an earthquake. Thus we can predict the time, location and magnitude of an earthquake. This is the thought upon which the prediction model is developed in the paper. To find the viscosity of the undersaturated solution of rock mathematical methods, continuum mechanics, are used.

3. Continuum mechanics and Critical Viscosity

Let us consider a tectonic plate system consisting of two plates moving in accordance with the compression movement to be studied. Let the plates be made of a material with density ρ moving with a net relative velocity v into each other. Let η be the hypothetical viscosity between the plates. Then by continuum mechanics, viscosity is given via the Power Law equation:

$$\eta = K\gamma^{n-1}$$

where K is a material based constant and γ is the shear rate. n is greater than one for a dilatant behavior and since dilatancy-diffusion occurs in rocks constituting the tectonic plates we will use a positive value a in place of $n-1$.

$$\frac{\eta}{K} = \gamma^{a}$$

Since a simple case of collateral shearing is considered, the shear rate is equal to the velocity gradient. Thus the equation becomes:

$$\left(\frac{\eta}{K}\right)^{\frac{1}{a}} = \frac{dv}{dx}$$

Let x be the relative distance, the plates move towards each other due to the acting pressures. Multiplying on both sides by the mass of the plate system m:

$$m\left(\frac{\eta}{K}\right)^{\frac{1}{a}} dx = m dv$$

Dividing on both sides by the area of contact between the plates and time differential dt, Pressure P is obtained on the right hand side,

$$\frac{m}{A}\left(\frac{\eta}{K}\right)^{\frac{1}{a}} \frac{dx}{dt} = P$$

Replacing mass by density and volume V of the system, the equation can be written as:

$$\frac{\rho V}{A}\left(\frac{\eta}{K}\right)^{\frac{1}{a}} dx = P dt$$

Now, by analyzing the plate sections in Figure (1), we can conclude that the area of contact A is yz and the volume V of the plate system is xyz.

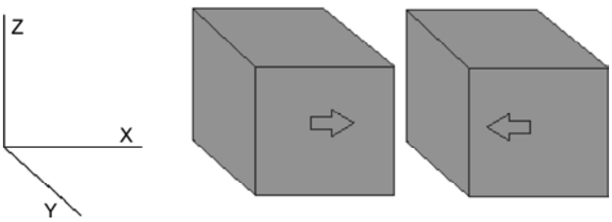

Figure 1. A vertical section of the two opposing plates is expressed. The arbitrary dimensions x, y, z are marked along the respective axes. x is the relative distance by which the plates push into each other.

x is the intrusion distance by which the plates move into each other; therefore, it is the only horizontal distance that will be used, as long as the "compressional two plate system" is analysed. Therefore, the equation becomes:

$$\rho x\left(\frac{\eta}{K}\right)^{\frac{1}{a}} dx = P dt$$

Integrating on both sides,

$$\rho\left(\frac{\eta}{K}\right)^{\frac{1}{a}} x^2 = 2\int P dt$$

Keeping the viscosity term alone on the left hand side,

$$\frac{\eta}{K} = \left(\frac{2\int P dt}{x^2 \rho}\right)^{a}$$

Replacing t and shifting K we get,

$$\eta = K\left(\frac{2\int P(t) dt}{x^2 \rho}\right)^{n-1}$$

This is the final equation obtained. On closer examination, this equation is very similar to the Power law equation. The difference being, that viscosity is now expressed in terms of pressure and time. $P(t)$ represents that pressure is a function of time and so is $x(t)$. In view of compressional movement $P(t)$ is an increasing function since the plates keep coming closer and closer causing the pressure to build up.

Any mathematician who looks at the final viscosity equation will wonder why $P(t)dt$ hasn't been integrated. Strictly speaking, it cannot be done theoretically. Though $P(t)$ is a function of time, it cannot be said mathematically what kind of a function it will be. Let us consider the other two inter plate movements transform and extensional, the functions can be as illustrated, Figure (2).

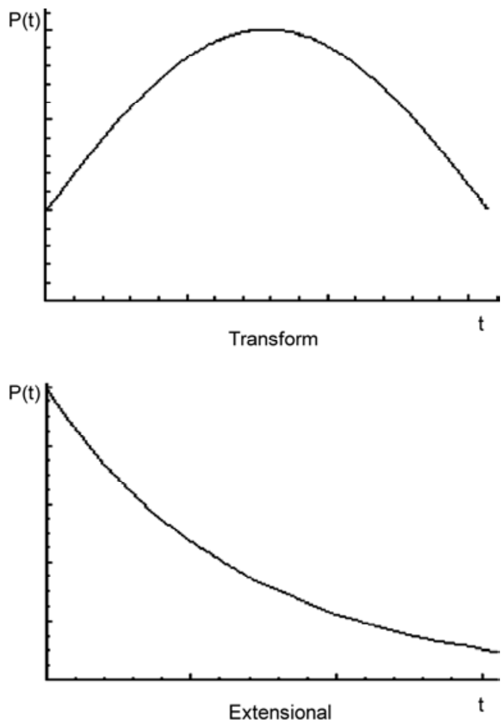

Figure 2. In extensional movement the plates draw away from each other causing the pressure to go below the normal value. In transform movement, the pressure increases as the plates draw closer and then drops as the plates pass each other.

Even in the case of a compressional movement, where the pressures will increase for sure how the function increases cannot be determined theoretically. The increase can be linear, exponential or simply any increasing function. Not always are the plates going to move in the same way. If that was the case, one would observe the same earthquake recurring in the same place again and again, which is clearly not observed. Therefore, since the function of pressure with respect to time cannot be determined, the integration cannot be carried out. Intrusion distance, x is also a function of time as we'll see later.

4. The Model and Predictions

The model is based on the viscosity equation derived in the previous section. The equation still contains terms that have to be explained on an experimental basis.

$$\eta = K\left(\frac{2\int P(t)\,dt}{x^2\rho}\right)^{n-1}$$

η is the viscosity to be calculated between the plates, K and n are a material constants of continuum mechanics that has to be experimentally determined, ρ is the material density and x, the distance moved by the plates into each other (which must be of the order of a few centimeters), is determined by geodetic methods using modern state of the art. GPS system can be used to find the total relative movement between the two plates. The remaining parameter to be found is pressure.

The prediction module consists of a pressure sensor planted at some depth along the fault lines. At this moment, it is unclear to me how exactly that (pressure sensor) can be achieved, but we are considering that pressure is known in some method. The depth at which the pressure is measured is critical since density depends on depth. Thus, for a particular sensor used in the model, the density used in the equation must be equal to the density of the material at that particular depth. Obviously, this requires a lot of pressure sensors, something similar to how buoys are used to monitor oceans. A grid of such pressure sensors would be necessary at different places and different depths along the fault line. Density of the material at that particular depth is found by any one of the known methods. As pressure increases, the material becomes more closely packed causing the density to increase. However, dilatancy counteracts that, so to practical terms density is reasonably a constant. The measurement of these variables is not discussed in the paper and is left to engineering and modern science solutions.

The model assumes that the terms are known in some way, thus pressure can be plotted as a function of time depending on the values returned by the sensor, and a curve fitting would return pressure as a function of time. An illustration of such a function for compressional movement is observed in Figure (3).

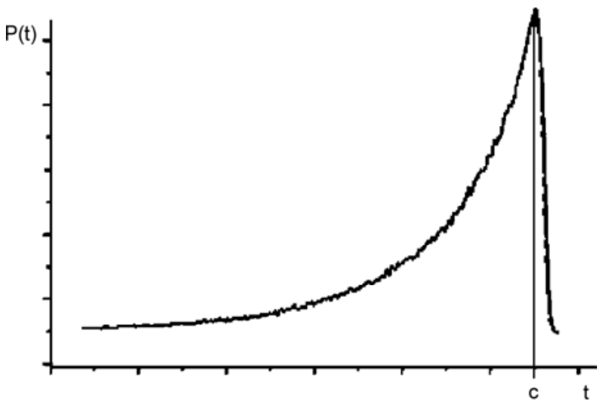

Figure 3. The pressure continues to build up until a particular point, after which there is a sudden drop when the pressure energy accumulated till then is released. c refers to that critical point.

At the critical point, the viscosity of the material comes to a certain level where the plates can literally move past each other. It is at this pressure that the plates give way to each other releasing the massive amount of energy build up. Therefore, an earthquake occurs only after that critical point. Hence, an earthquake can be known by monitoring the changes in pressure. Though an earthquake prediction can be done just by plotting pressure over time, viscosities are used to generate a much better prediction model as follows.

In the viscosity equation, though density is fairly a constant x is not a constant. It is a function of time as well. Since in the case of compression movement, as time increases the relative distance moved by the plates also increases. The initial value of pressure may not be zero but the initial value of x has to be zero because theoretically, it is only after $t = 0$ that the plates start to move towards each other. The viscosity field based on the equation looks as the one in Figure (4) with pressure along the y-axis and intrusion distance x along x-axis and viscosity is along the z-axis. The relation between pressure and time is known from the curve fitting and it is then plugged into the equation to result in the following. Figure (4) represents only an expected 3D plot.

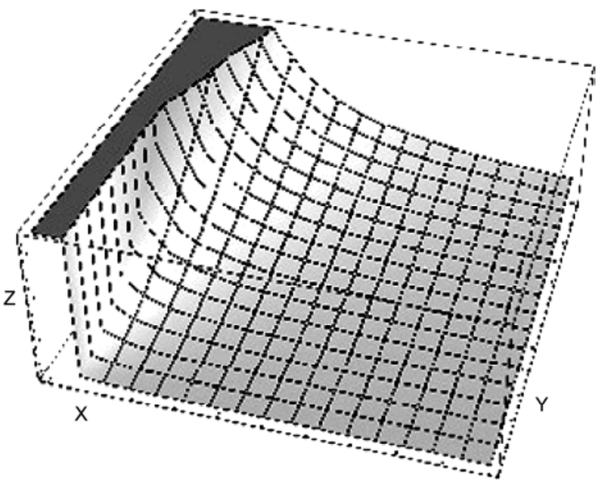

Figure 4. A 3D plot indicating the trend of viscosity based on the viscosity equation with pressure and intrusion distance along y and x axis respectively.

The interpretation of the 3D graph obtained is as follows. The flat patch on the top corresponds to viscosities of the plate system that are greater than the critical viscosity. Therefore, as seen from the equation, when $x = 0$ the viscosity is infinite, indicating that the plates are solid at $t = 0$. As time increases, due to compression movement the pressure and intrusion distance increases and thus the value of velocity begins to drop. At a particular point of time, the viscosity goes below the critical viscosity (indicated by the slide in the graph), during which the plates slip past each other resulting in the release of enormous amount of energy.

In real time application of the model, the values of pressure and x are plugged into the equation and such a 3D graph is built. Using the experimentally known value of critical viscosity at a particular depth between two particular plates, the time at which the most prominent earthquake occurs can be determined.

Though prediction can be much simpler if only pressure is taken into account, it does not predict consistently the magnitude and time of an earthquake. Besides, by factoring in the critical distance x by which the plates press into each other, the dilatancy-diffusion principle can be used to predict the viscosity of the system. The critical viscosity at which an earthquake is triggered is a constant and independent of the applied pressure. Therefore, by extrapolation of the values of pressure and intrusion distance that are experimentally known, the time at which an earthquake will occur can be predicted by referring to the critical point.

If pressure alone was assumed to cause an earthquake, then only an earthquake of particular magnitude will occur at a particular place, but experimentally that is not observed and therefore, the model presented in this paper would be a better option for predicting earthquakes.

The place where the earthquake will occur can be localized to the region where the pressure sensors are returning critical pressure values. Thus, place can be said for certain within the accuracy of a few kilometers if sufficient sensors are in place (like in a grid along fault lines). Time can be predicted to satisfactory standards using the extrapolation of the equation, from the P and x values that are applied, and comparing it to the critical viscosity. By comparing the trend of viscosity, the time at which the decreasing viscosity reaches the critical viscosity can be calculated. Based on that time and pressure, the magnitude can be calculated. This is because magnitude of an earthquake depends on the time for which the pressure keeps building up. General pressure energy equation is given by:

$$E = \frac{dp}{\rho}$$

where E is energy and dp is change in pressure. In our case, the change in pressure is the pressure accumulated over time; therefore, the earthquake energy becomes:

$$E = \int_{t=0}^{t=t_r} \frac{P(t)\,dt}{\rho}$$

Here, the limits are from time $t = 0$ to $t = t_r$ that corresponds to the time when viscosity was still infinity and the release time when the plates slip past each other due to their viscous nature respectively. Pressure is again a function of time. The experimental values returned by the sensor will be discrete in nature, but a curve fitting can be done to find $P(t)$.

Thus, by virtue of the model, the time and magnitude of an imminent earthquake can be predicted, at least theoretically. The total energy of the plate system interaction can be thus calculated. By using the classical elastic wave equation, the energy of P and S waves of the earthquake can also be suitably calculated. However, this energy doesn't always correspond to the energy of the main shock, it also accounts for the fore shocks or aftershocks (if any). But the highest possible magnitude of the earthquake can be predicted, which would be a safer option since some earthquakes show no fore shocks such as the M8.6 earthquake of 1950 in India-China. Therefore, the two important equations can also be written as follows:

$$\lim_{\delta\eta \to \eta_c} \eta = K\left(\frac{2\int P(t_r)\,dt}{x(t_r)^2 \rho}\right)^{n-1} \qquad E = \int_{t=0}^{t=t_r} \frac{P(t)\,dt}{\rho}$$

where η_c refers to the critical viscosity and the equation tells the release time tr of an earthquake. The energy (in Joules) of the earthquake is calculated by the integration of pressure till the release time. The magnitude of an earthquake can be predicted on the Richter scale to a reasonable extent using the following energy conversions, Table (1).

Table 1. Equivalents between energy in joules and corresponding magnitude on the Richter scale.

Richter Scale	Earthquake Energy (J)
0 – 1	63K – 2 M
1 – 2	2 M – 63 M
2 – 3	63 M – 2 G
3 – 4	2 G – 63 G
4 – 5	63 G – 2 T
5 – 6	2 T – 63 T
6 – 7	63 T – 2 P
7 – 8	2 P – 63 P
8 – 9	63 P – 2 E
9 – 10	2 E – 63 E

In a nutshell, the model can be described as follows: The values of pressure and x are applied to the viscosity equation to calculate the trend of viscosity between the two plates in compressional movement. The viscosity is extrapolated to find the time at which the trend of viscosity reaches the critical value, thus the time parameter of an earthquake can be predicted. Using that time and the pressure energy equation, the energy and thus magnitude of an earthquake can be predicted. Depending on the place where the pressure and x values were originally observed, the place of an earthquake can be calculated. Therefore, the three most important parameters can be predicted.

The critical point, however, has to be experimentally determined. It is intrinsic to the tectonic plate's relation, and therefore, once that is known and the model is initialized, effective predictions can be made. Thus, hypothetically at least, all three major parameters of an earthquake - place, time and magnitude can be reasonably predicted.

5. Challenges Ahead and Conclusion

The model is purely hypothetical since there is no known mechanism of what triggers an earthquake, from which alone such a prediction model can be built. Therefore, I hypothesized a mechanism and further made derivations to make earthquake predictions. This model takes a different approach from that of the dilatancy-diffusion model that was suggested to predict earthquakes based on the change in V_p/V_s. The model takes dilatancy-diffusion to a literal level by suggesting that the viscosity at the plate boundary changes at high pressure, causing the boundary to behave as a fluid allowing the plates to slide past each other after a critical point causing the release of massive energy (the earthquake).

An expression to that critical viscosity based on pressure and interaction distance x is derived in the paper based on which the entire prediction module is based. An expression to the energy of an earthquake is also discussed.

Challenges ahead are how does one find the variables in the equation? Is this entire thing possible, and if possible is it reliable or is it all hypothetical as presented in the paper? Somehow experimentally, if this really can be done, then earthquakes can be predicted well in advance by analyzing the trend of some critical values. The model though backed up by some mathematics isn't any match to the existing theories, but as said earlier, the paper is targeted at expressing some alternate seismic content towards earthquake prediction other than conventional theories and models.

The model can't be guaranteed to be an experimental success as there could be other factors that weren't considered during the derivation. The model being a theoretical one, I cannot give convincing evidence since laboratory simulations will not completely simulate the conditions underneath. For example, the vertical plate boundary studied in the paper is actually pressurized in all directions. Therefore, though theoretically the most important pressure to be analyzed is in the horizontal direction, experimentally vindication would require pressure of high level applied in all directions with maximum pressure along the horizontal direction alone. Such a setting of equipment is by itself only hypothetical. Therefore, as of now, I cannot vindicate the model which is possibly the biggest challenge ahead.

The model is basically a deterministic one, thus the effective predictions can be used to create evacuation plans in case of an imminent earthquake. However, the technological, economical and statistical reliability of the model are places where heaps of experimental work has to be done to prove the validity of the model before applying it to generate predictions.

As a result, a basic outline of a dilatancy-diffusion based earthquake prediction model is discussed in the paper. Though its viability is undetermined, I hope its theoretical basis gives it some scope. But if the model is experimentally verified, it would be a great leap in EP studies and maybe one day we can effectively predict earthquakes, predict the magnitude, epicenter and maybe even the time to a greater deal of accuracy, saving countless lives.

References

1. Bolt, B. (2005). Earthquakes: 2006 Centennial Update - The 1906 Big One (Fifth ed.), W. H. Freeman and Company, p. 150, ISBN 978-0716775485.

2. Earthquakes: http://www.usgs.gov/faq/index.php?action=show&cat=113.

3. Earthquake Dynamics: http://kulikow.com/project/Literature/Cracks_Self_Healing/chap12.pdf.

4. Scholz et al. (1973). Earthquake Prediction: A Physical Basis, *Science*, **181**(4102).

5. Griskey, R. and Green, R. (1971). Flow of Dilatant (Shear-Thickening) Fluids, *AlChE Journal*, **17**(3), 725-728.

6. GPS-Measuring Plate Motion: http://www.iris.edu/hq/files/programs/education_and_outreach/aotm/14/1.GPS_Background.pdf.

7. Kayal, J.R. (2008). Microearthquake Seismology and Seismotectonics of South Asia, Springer, p. 15. ISBN 978-1-4020-8179-8.

8. Madariaga, R. et al. (1998). Modeling Dynamic Rupture in a 3D Earthquake Fault Model, Bulletin of the Seismological Society of America, **88**(5), 1182-1197.

9. Sykes, L.R. (1967). Mechanism of Earthquakes and Nature of Faulting on the Mid-Oceanic Ridges, *Journal of Geophysical Research*, **72**, 5-27.