Inverse Statistics Method: Spatial and Temporal Dependence in Earthquake

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ABSTRACT

The aim of this work is to understand the relation between time and place that an earthquake takes place. In order to answer this question, the Modified Level Crossing (MLC) technique has been implemented. By studying two earthquakes, one in Iran and one in California we came to the conclusion that there is a relation between time and place of an earthquake occurrence. As a matter of fact, this relation is quite decisive. By performing MLC analysis and comparing the two regions, we can state that geographical effects play an effective role due to geophysical differences between Iran and California. Indeed, by comparing the readings of Iran and California, one could come to understand the geophysical differences between the two domains.The so-called level crossing analysis has been used to investigate the spatial and temporal fluctuations of earthquake form time series. In this paper, we calculated the average frequency of up-crossing for original and shuffled data of Iran and California earthquakes in spatial and temporal series. This analysis showed a significant difference between the original data and shuffled data. By introducing the relative change of the total number of up-crossings for original data with respect to the so-called shuffled data, R, and calculate the Hurst exponent, Iran and California earthquakes are compared.

Keywords:
Stochastic processes; Level crossing; Inverse statistics; Earthquake

1. Introduction

The internal motions of the Earth cause earthquakes. An earthquake is the result of a sudden release of energy in the Earth's crust that creates seismic waves. The question is that whether its behavior can be easily predicted? When there are large numbers of variables influencing the system, the main factors to consider its behavior cannot be simply tagged. This feature is called the system complexity. As a result of current and past events, resultant of all factors is efficacious that ultimately is observed. Can the study of the history and behavior of the system be used to achieve (for achieving) a method to describe and predict the behavior of the system?

In recent years, a vast majority of researches have been devoted to the study of seismic data that contain information about the complex events that lead to an earthquake [1-5].

We live in a world where random processes are ubiquitous. Although the random values of a stochastic process at different times may be independent random variables, in most cases they are considered to indicate complicated statistical correlations. Therefore, over the past decade, several different methods have been introduced to study the properties of the process. Spatial and temporal fluctuations of earthquake form time series. The purpose of the application of statistical mechanics is to describe the behavior of the time series that can help to better understanding of the stochastic processes. After that, this study lies in an effort to reproduce or predict some experimental facts with extraction of useful information. An important question is what the probability of obtaining or losing a certain level of return at different time intervals is. For the first time, Jenson presented the inverse method in turbulence [6], and Simonsen et al. presented the inverse statistics to apply on similar financial data to answer the similar questions [7]. Inverse statistics suggest the inverting of the structure function equation, and instead of considering the average moments of distance between two points, gives the difference value between these two points. Inverse statistics method is used for another popular technique called the level crossing method (LC). In the level crossing method no scaling feature is explicitly required [8-14], and this is the main advantage of this technique for estimating the statistical information of the series. Level crossing based on stochastic processes that grasp the scale dependence of time series.

What is the reason for the formation of level crossing? This method was developed for the study of a series of different insight. The memory, non-Gaussianity and waiting time (length) (an average time (length) interval that we should wait for an event to take place again [15-19]) could be measured by level crossing method. Since the fractional Gaussian noises are well-known examples, their comparison with empirical data can be used as a criteria to better understanding the results obtained from the level crossing method applied to unknown empirical data.

Here, the total amount that is designated as N_{tot}^{+}, which represents the total number of up-crossings of a series, reflects how memory plays role. To better assess the effects of memory, shuffled counterparts of each underlying time series were calculated and compared their associated total number of so-called crossings, N_{sh}^{+}, with that given by their original time series N_{tot}^{+} to obtain the change percentage in the system. The auto-correlations are destroyed by the shuffling procedure. Using this method is to answer such questions as: Will an earthquake have effect on another earthquake? Will happening an earthquake in a certain time have an effect on the time of next ones? Will happening an earthquake in a place have an effect on the place of next ones? Are there any relation between where it happens and when it happens?

2. Level Crossing Analysis

For better understanding, we begin with a summary of the analysis of LC [9-14]. Consider the $\{x(t)\}$ series from time intervals between earthquake events as n_{α}^{+} represents the number of positive difference crossings (up-crossings) at the level of $x(t)-\bar{x}=\alpha$ in time interval (Figure 1). For all time intervals, the mean value of n_{α}^{+} is equal to $N_{\alpha}^{+}(T)$ [11]:

$$N_{\alpha}^{+}(T)=\left\langle n_{\alpha}^{+}(T)\right\rangle \tag{1}$$

where $\langle , \rangle$ represent the ensemble average. For a homogeneous process (constant) the average up-crossings is in accordance with time interval T. As a result:

$$N_{\alpha}^{+}(T)=v_{\alpha}^{+}(T) \tag{2}$$

where v_{α}^{+} is the average frequency of crossing with positive slope at the same level of $y=x(t)-\bar{x}=\alpha$. Frequency parameter v_{α}^{+} could be deduced from the underlying probability density functions (PDF) of $y=x(t)-\bar{x}$, and $y'=(y(t+\Delta t)-y(t)/\Delta t=\Delta y/\Delta t$, which is named as $P=(y,y')$ [6, 10].

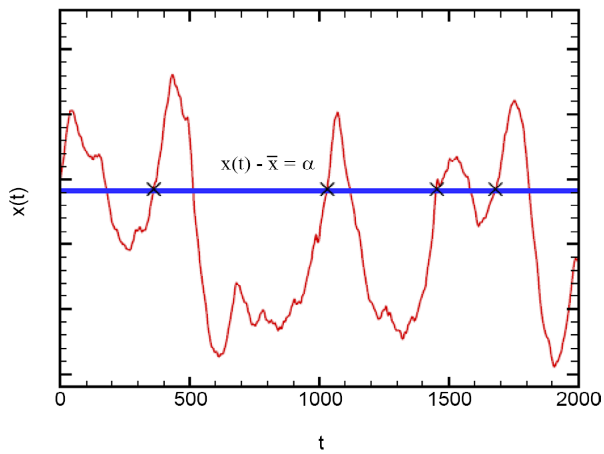

Figure 1. Schematic of up-crossing for an arbitrary level $x(t)-\bar{x}=\alpha$.

Within the time interval Δt, the sample can only pass with positive slope at the level of $x(t)-\bar{x}=\alpha$ provided that it has the $x(t)-\bar{x}<\alpha$ property at the beginning of the interval. Furthermore, there is a minimum difference at time *t*, if the level $x(t)-\bar{x}=\alpha$ is to be crossed in interval Δt depending on the value of $x(t)-\bar{x}$ at time *t*. Thus, it would be a positive crossing of $x(t)-\bar{x}=\alpha$ in the next Δt, interval if at time *t*:

$$x(t)-\bar{x}<\alpha \quad \frac{\Delta[x(t)-\bar{x}]}{\Delta t}<\frac{\alpha-[x(t)-\bar{x}]}{\Delta t} \tag{3}$$

As it was shown, v_α^+ can be defined as the probability density function $P(y=\alpha, y')$ as follows:

$$v_\alpha^+=\int_0^\infty P(\alpha, y')\,y'dy' \tag{4}$$

where $P(\alpha, y')$ is the joint probability density function $P(y, y')$ evaluated at $y=\alpha$.

Besides, let us define the $N_{tot}^+(q)$ quantity as:

$$N_{tot}^+(q)=\int_{-\infty}^{+\infty} v_\alpha^+\left|\alpha-\bar{\alpha}\right|^q d\alpha \tag{5}$$

where zero moment (with respect to v_α^+) $q=0$ shows the total number of crossings with positive slope for return to earthquake magnitude. The moments $q<1$ will give information about the frequent events while moments $q>1$ are sensitive for the tail of events.

To investigate the effect of correlation and memory, N_{sh}^+ is calculated that represents the total number of up-crossings in the time series when it is shuffled. Here, random permutation is used for shuffling the data. The auto-correlations are destroyed by the shuffling procedure. Hence, by comparing N_{tot}^+ of the original data with that computed for the shuffled data set, N_{sh}^+, we can obtain the magnitude of correlations in the time series and this gives useful information about the time series. By comparing the difference between N_{tot}^+ and N_{sh}^+ (after shuffling), the memory of the time series can be determined. Smaller relative difference suggests that the time series is less correlated (anti-correlated). Under the influence of shuffling, the total number of crossings with positive slope N_{tot}^+ increases (decreases) indicating the correlation in the underlying data (anti-correlation). In order to measure the value of memory (correlation and anti-correlation) in the data, the relative changes in the total number of up-crossings for original and shuffled data defined using N_{tot}^+ as follows:

$$R\equiv\left|N_{sh}^+-N_{tot}^+\right|/N_{tot}^+ \tag{6}$$

By using the PDF and the correlation function of the data, a stationary series can be pictured. Fractional Gaussian noises which are generalizations of ordinary discrete white Gaussian noise are characterized by their Hurst exponent. The Hurst exponent, *H*, gives a quantitative measure of the long-term persistence of a signal. In particular, the exponents $0<H<0.5$ and $0.5<H<1$ correspond to negative (anti-correlation) and positive correlation, respectively, while $H=0.5$ corresponds to an uncorrelated Gaussian process [20-28].

3. Application on Earthquake

Iran and California Earthquakes from 1/1/1971 to 08/03/2013 are used in this research [29]. For this purpose, the statistical correlations of earthquakes in these two regions have been compared. Earthquakes with magnitude greater than 4 in Richter magnitude scale have been selected, and time series and the spatial series have been established by using time intervals and physical distances between the earthquakes, respectively.

Figure (2) shows the crossing with positive slope related to the obtained date (both original and shuffled data) for Iran and California earthquakes as can be estimated by Eq. (4).

The plots reveal the positive correlation of Iran and California earthquakes over time.

The level crossing for both original and shuffled data related to Iran and California earthquake have been depicted in Figure (2). From the figure, the earthquakes correlation is also evident in this case; however, the correlation is lower than that related to Iran earthquakes.

After examining the original and shuffled date for earthquakes occurred in Iran and California, now we will lie in an effort to compare earthquakes occurred in these two regions. Earthquakes in these two regions with similar area have been investigated at the same time.

For the original data related to earthquakes occurred in Iran and California, the crossing with positive slope has been estimated using Eq. (4),

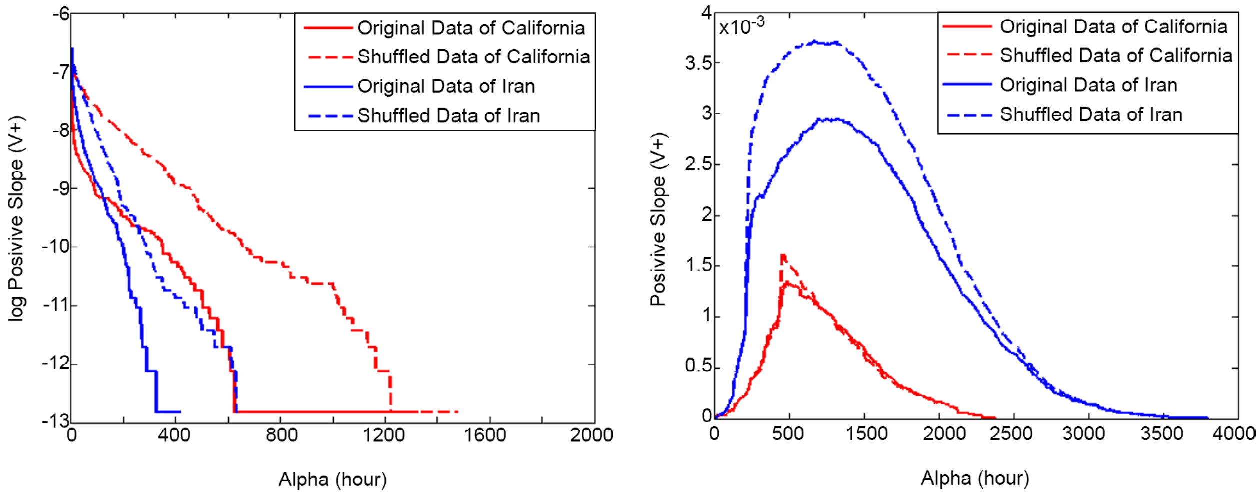

Figure 2. Plot of v_{α}^{+} for original and shuffled date related to Iran and California earthquakes in time series.

which are plotted in Figure (2). As it can be seen from the figure, the earthquake with smaller magnitude in Iran is more probable than California and the earthquake with large magnitude in California is more probable than Iran.

The estimated crossing with positive slope from Eq. (4), the spatial series for the original data related to earthquakes occurred in Iran and California has been depicted in Figure (3). It is clear from the figure that the earthquakes occurred in Iran show a higher correlation. This means that the earthquakes affect and stimulate each other. In other word, the percentage of induced earthquakes in Iran is more than California.

The final number of crossings with positive slope have been calculated from Eq. (5) and plotted in Figure (3).

The slope of the plot for California is higher than that of Iran. This means that earthquake occurrence in California is more probable than Iran and in terms of time, the possibility increases with an increase in earthquake magnitude. In addition, it is expected to occur a larger earthquake in California compared with Iran at far times.

The final number of crossings with positive slope estimated from Eq. (5) in the spatial series have been calculated and plotted in Figure (4) for earthquakes in Iran and California.

The slope of the plot is almost identical for Iran and California. This means that earthquake occurrences in California have a similar behavior in terms of spatial position.

The relative changes between Iran and California earthquakes for time series of q between 0 and 3

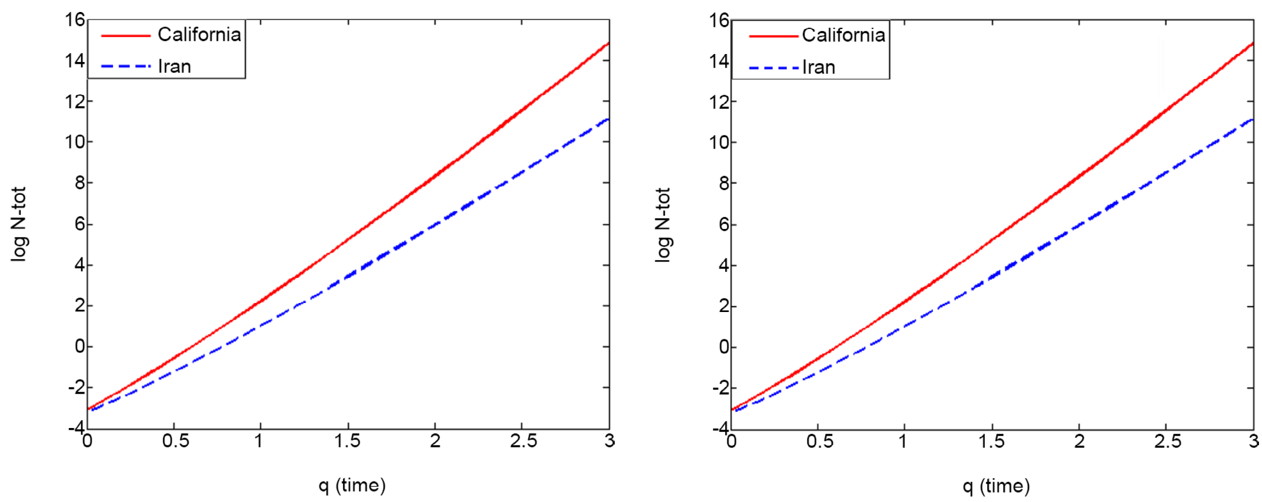

Figure 3. Plot of N_{tot}^{+} for Iran and California earthquakes in time series. q is a dimensionless parameter.

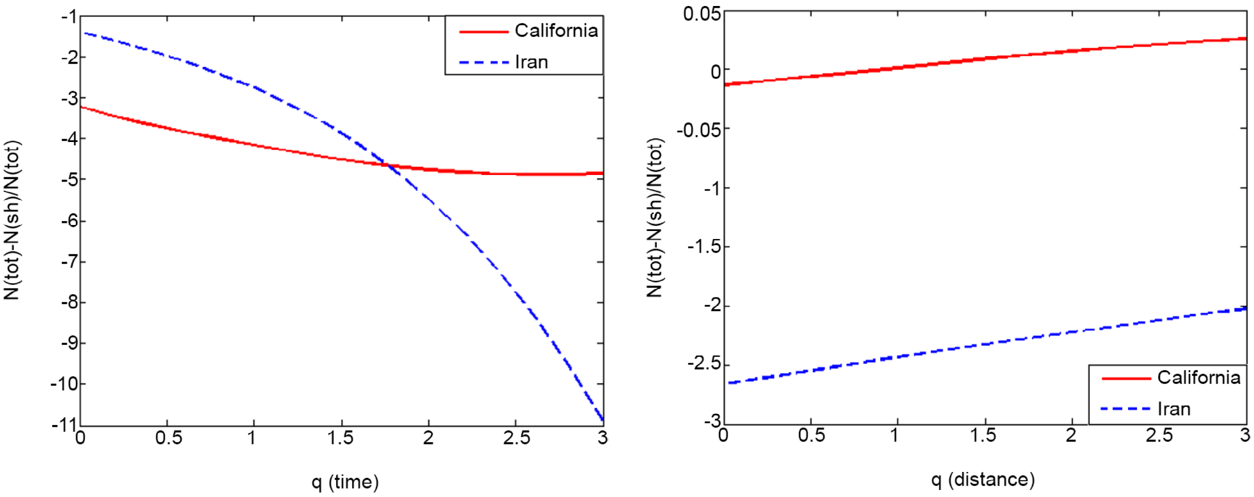

Figure 4. Comparing of $\left|N_{sh}^{+} - N_{tot}^{+}\right| / N_{tot}^{+}$ for Iran and California earthquakes. Temporal (Left panel) and spatial (Right panel). q is a dimensionless parameter, Ylabel is not suitable

have been calculated using Eq. (6), plotted in Figure (4).

The plot suggests that Iran is more active and sensitive in smaller earthquakes, but when earthquakes go larger than a limit, California shows more activity. In other words, the memory of Iran is higher than California for small earthquakes.

The crossing values for original and shuffled data as well as the relative differences of the time series for earthquakes in California and Iran have been presented in Table (1). The relative difference between original and shuffled data for earthquakes in California is more than Iran. This means that after an event in California we must wait longer for the next event.

The relative difference in time series for California earthquakes is more than Iran. In other words, earthquakes in California have a better memory in terms of time compared to Iran, i.e., the waiting time for occurrence of next earthquake in California is longer than that in Iran.

Figure (4) displays the relative changes between earthquakes occurred in Iran and California calculated using Eq. (6) for the spatial series of q between 0 and 3.

Table 1. The value of $N_{tot}^{+}(q=0), N_{sh}^{+}(q=0)$ and Hurst exponent for Iran and California earthquakes in temporal series.

	N_{tot}^{+}	N_{sh}^{+}	$\left\|N_{sh}^{+} - N_{tot}^{+}\right\| / N_{tot}^{+}$	H
Iran	0.0405	0.0859	1.1232	0.82 ±0,04
California	0.0456	0.2149	3.7169	0.79 ±0,04

The plot suggests that California is more active and sensitive than Iran whether in small or large earthquakes.

Table (2) presents the values of crossing with positive slope for original and shuffled data as well as the relative differences of spatial series of $q=0$ for earthquakes in Iran and California.

Table 2. The value of $N_{tot}^{+}(q=0), N_{sh}^{+}(q=0)$ and Hurst exponent for Iran and California earthquakes in spatial series.

	N_{tot}^{+}	N_{sh}^{+}	$\left\|N_{sh}^{+} - N_{tot}^{+}\right\| / N_{tot}^{+}$	H
Iran	4.0054	5.0091	0.2506	0.66 ±0,04
California	0.9536	0.9387	0.0156	0.68 ±0,04

The relative difference in the spatial series related to Iran earthquakes is more than California, i.e., there is a higher (more) spatial correlation between Iran earthquakes. In other words, after earthquake occurrence, the fault energy is evacuated and the expectance that this fault or their neighbor ones lead to an earthquake is reduced.

Spatial-time distribution for Iran and California earthquakes is depicted in Figure (5).

Then the crossing of the graph at different radii has been calculated for the two regions and plotted in Figure (6).

In this plot, it is shown that the crossing with positive slope in the spatial with time distribution is in Iran probability of an earthquake in near time and place is more than California. This result could be

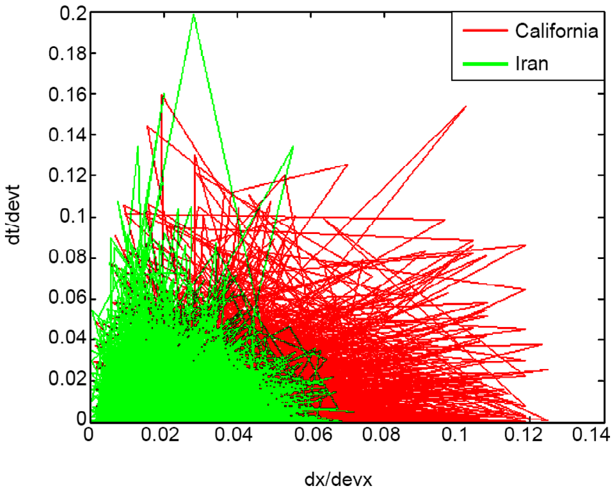

Figure 5. Spatial-time frequency plot for Iran and California earthquakes.

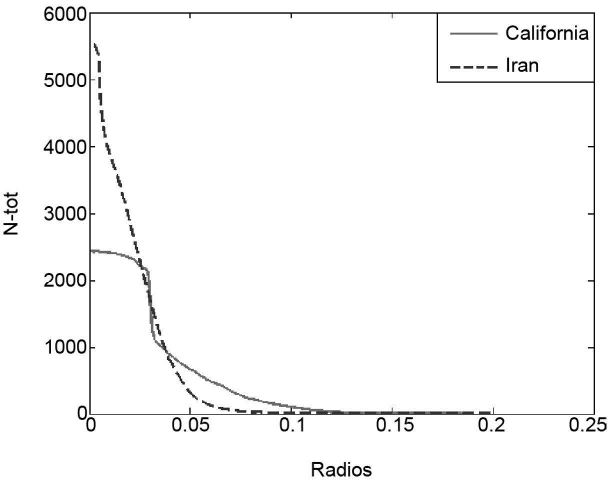

Figure 6. Plot of N^{+}_{tot} for Iran and California earthquakes in spatial-time distribution.

due to the differences of the faults in two regions. There are a large number of short length faults in Iran, but there are a few of large length faults in California (the length of the San Andreas Fault is about 1,300 km). Therefore, with occurrence of an earthquake in California, the earth energy is released and no other fault will cause an earthquake. Hence, it takes more time for the next earthquake to gather energy, or the earthquake take places in more distant places.

4. Conclusion

In this paper, the concept of level crossing analysis has been applied to Iran and California earthquakes. In level crossing method, no scaling feature is explicitly required and this is the main advantage of this method in estimating the statistical information of the series. Level crossing is based on stochastic processes that grasp the scale dependence of the time series. It is shown that the level crossing is able to detect the memory of series. This method with no required scaling feature is a powerful method in characterizing the time series. Considering all of the above discussions and results, we notice that Iran and California earthquakes are correlated over time and location. However, in California, the temporal correlation of earthquakes is lower than that of Iran. Most of California earthquakes are strike-slip, but most of the Iran earthquakes are trust. Thus, when an earthquake is occurred in California, the earth energy is released and the rest of the fault discharged stress; however, the fault behave independently in Iran. In other words, the percentage of induced earthquakes in Iran is more than California. Moreover, in Iran, the probability of an earthquake in near time or place or near time and place is more, but in California, the probability of an earthquake in far time and place or far time and place is more.

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