The Effect of Fuzzy Uncertainties on Performance Level and Performance Evaluation of Steel Moment Frames

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Received: 19/09/2010
Accepted: 11/08/2012

ABSTRACT

This paper aims to ponder the effect of fuzzy uncertainties on performance evaluation of steel moment frame structures. Since the performance evaluation of a structure depends on its seismic demand and capacity spectra, any uncertainties in these two spectra causes uncertainty in performance level and performance point. Among many sources of uncertainties in structural dynamic analysis, in this paper, the modulus of elasticity, gravity load on the structure, dynamic properties of structure and soil properties have been considered and treated as fuzzy variables. To investigate the effect of these uncertainties, first, a nonlinear static pushover analysis program was written in MATLAB medium. Then, fuzzy inference model was used for determination of the design spectrum for different kinds of soils and seismic zones. The Effects of fuzzy uncertainties on capacity curve and capacity spectrum have been investigated on a typical example based on a new fuzzy concept in construction of the capacity spectrum of structures. Finally, performance point and performance level of structure has been determined as a fuzzy output.

Keywords:
Static pushover analysis; Performance point; Performance level; Fuzzy uncertainties, Fuzzy inference model, Adaptive pushover analysis

1. Introduction

Development of performance-based design method opens new views in earthquake engineering. Heavy damages to structures designed according to the current design provisions indicate that current provisions are not completely reliable. One source of error in determination of seismic capacity and demand of structures is the inherent uncertainties of input parameters used for the evaluation of performance of structures. In some researches, the uncertainties have been pointed out, but no methodology has yet been proposed for consideration of these uncertainties. The goal of this research is to propose such a methodology for investigating the effect of consideration of some of these uncertainties in determination of seismic performance of structures. In this paper, fuzzy model is used for description of these uncertainties.

Most of researches in application of fuzzy models in civil engineering and mechanics of structures are in the fields of linear analysis of simple structures, linear finite element, structural reliability analysis, safety assessment of structures, modal analysis of structures, and control of vibration of structure during earthquake excitement. Here are some samples:

Oettli [1] suggested a linear programming method for evaluation of the solution set of a linear system with inaccurate coefficients. Later, Aberth [2] proposed a relatively simple solution algorithm by generalizing the linear programming method. However, both of proposed algorithms are applicable only to small size systems.

Dong and Shah [3] proposed a vertex method for computing functions of fuzzy variables based on the α-cut concept and interval analysis. A vertex solution or a combinatorial approach for giving the exact bounds of the solution set of linear interval equations has been studied by Neumaier [4], Hansen [5], Jansson [6] and Qiu et al [7].

Moens et al [8] used fuzzy variables through modal analysis of structures to obtain shape mode vectors and frequencies of structures in the form of fuzzy vectors and fuzzy variables. Then, they compared the results with Mont Carlo's simulation method.

Nahhas [9] used *UBC* provision criteria with application of fuzzy inference method, to propose a new method for producing design response spectrum.

Skalna et al [10] expanded the solution methods of the system of linear equations in the interval variables domain to solution of systems of fuzzy equations in structural mechanics. They employed the concept of interval vectors and interval matrices in their solution algorithm.

Buckey and Qy [11] proposed a method of solving fuzzy equations based on α-cut method.

Since mid 90*s*, intensive researches have been carried out in the field of application of fuzzy methods in various branches of civil engineering. Samples of these diverse publications are: structural reliability analysis through fuzzy number approach [12], structural design based on nonlinear fuzzy analysis [13-14], damage assessment of structures and its numerical simulation [15], safety assessment of maritime structures [16], and application of fuzzy probabilistic method for analysis under dynamic loads [17].

Although many researches have been carried out in the field of fuzzy analysis and design of structures, few of them have been fulfilled in the field of fuzzy seismic analysis of structures and fuzzy performance-based design of structures. In this paper, it is intended to use fuzzy concepts in determination of some structural characteristics like seismic capacity, seismic demand, and performance evaluation of structures.

2. Performance Evaluation of Structures

Performance evaluation of structures is done in different methods. In this paper, the evaluation is adopted identical to determination of performance point of structures in accordance to recommendations of *FEMA* 440 [18] for nonlinear static procedure. In this procedure, to find a performance point, two models are required: 1) a model for obtaining lateral load bearing capacity of structure, and 2) a model for determining seismic demand of structure.

Capacity curve, which is obtained from pushover analysis, is the basis of nonlinear static procedures. It is generated by subjecting a detailed structural model to one or more lateral load patterns (vectors) and then increasing the magnitude of the total load to generate a nonlinear inelastic force-deformation relationship for the structure at a global level. The load vector is usually an approximate representation of the relative accelerations associated with the first mode of vibration for the structure.

As shown in Figure (1), after determination of the capacity curve that is in force-displacement coordinate, this curve has to be converted via Eqs. (1) and (2) to a spectral acceleration-spectral displacement ordinates to produce the capacity spectrum.

$$S_a = \frac{V}{\alpha_1 \times W} \tag{1}$$

$$S_d = \frac{\delta_{roof}}{\Phi_{roof} \times \Gamma_1} \tag{2}$$

where, S_a is spectral acceleration of structure and S_d is spectral displacement of structure, V is base

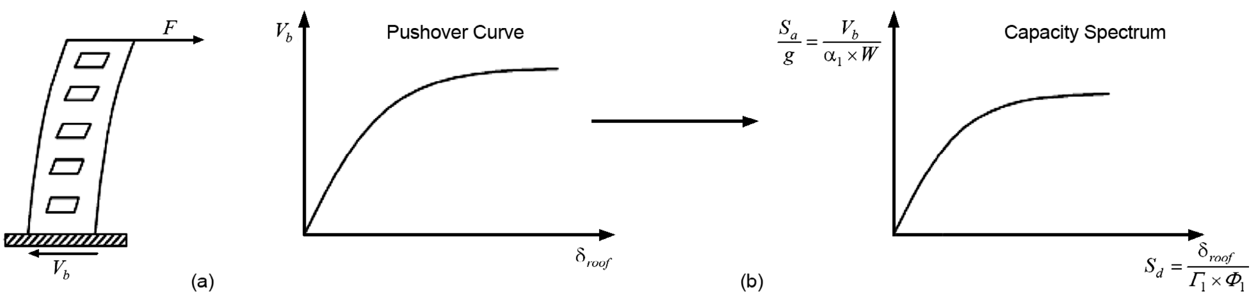

Figure 1. Capacity spectrum method: (a) development of pushover curve, (b) conversion of pushover curve to capacity spectrum.

shear, W is weight, δ_{roof} is displacement of control node in roof level, Φ_{roof} is amplitude of first mode in roof level, α_1 and Γ_1 are mass modal factor and modal participation factor for the first vibration mode and are defined as follows:

$$\alpha_1 = \frac{\left[\sum_{i=1}^{N}(m_i\phi_i)\right]^2}{\left[\sum_{i=1}^{N} m_i\right]\left[\sum_{i=1}^{N}(m_i\phi_{i1}^2)\right]}$$

$$\Gamma_1 = \left[\frac{\sum_{i=1}^{N}(m_i\phi_{i1})}{\sum_{i=1}^{N}(m_i\phi_{i1}^2)}\right]$$

where, m_j is the lumped mass at the j^{th} floor level, ϕ_{j1} is the j^{th} component of the fundamental mode shape ϕ_1, N is the number of floors.

For determining seismic demand of structure, elastic response (or design) spectrum must be used. Determination of seismic demand of structure is done in following steps:

1. The elastic response (or design) spectrum, must be converted from the standard S_a - T_n, format to the S_a - S_d format, where S_a is pseudo acceleration, T_n is natural period of structure and S_d is the deformation spectrum ordinate, see Figure (2).

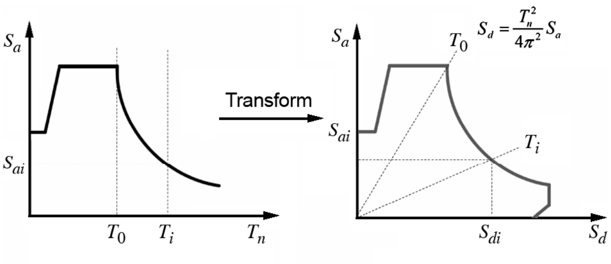

Figure 2. Conversion of elastic response spectrum from standard format to S_a - S_d format [18].

2. Since in the process of nonlinear analysis the natural period and the effective damping of the structure, β_{eff} do change due to birth of plastic hinges in the structure, the response spectrum does change accordingly, and this change has to be somehow considered in evaluating the performance of structures. To that end, for a hypothetical performance point, the effective damping, β_{eff} is calculated and as shown in Figure (3), the initial response spectrum is adjusted according to spectral reduction factor *M*, obtained from section 6.2.3 of *FEMA*-440. This gives the modified acceleration-displacement response spectrum (*MADRS*).

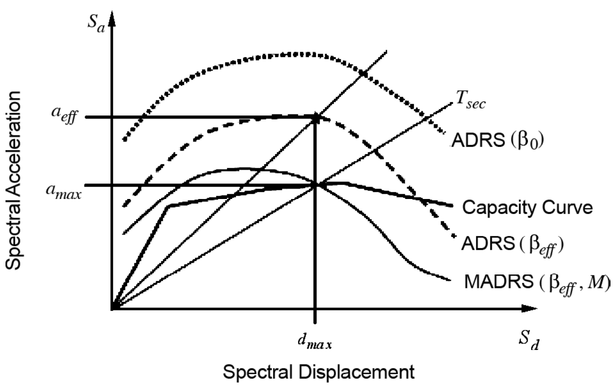

Figure 3. MADRS spectrum [18].

3. The intersection of capacity spectrum and the *MADRS* gives a performance point. This point has to be the same as the assumed one. If not, the process has to be repeated.
4. A different way to obtain the performance point is to use the locus of performance point as in Figure (4). To that end, several demand spectra are drawn according to various ductility values. Since there is a relation between pseudo displacements and pseudo accelerations, the intersection of the radial secant period, *Tsec*, with the *MADRS*, see Figure (4), can be obtained as a possible performance point on each demand

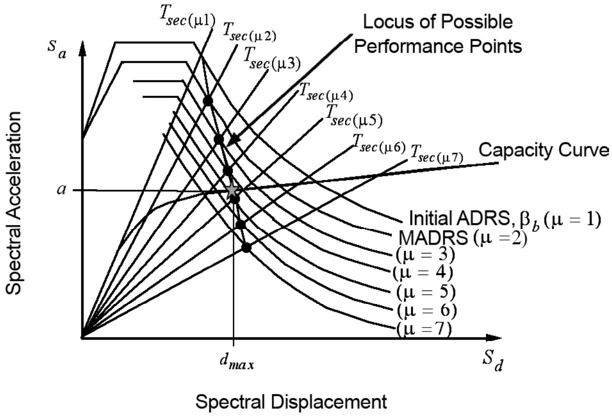

Figure 4. Locus of possible performance points using MADRS [18].

spectrum. If these points are connected to each other, they will establish the locus of performance point of the structure. Therefore, the intersection of capacity spectrum with the locus of the performance point will give the pseudo displacement corresponding to performance point.

3. Sources of Uncertainties

Although there are many sources of uncertainties in obtaining a performance point, the three more important of them are addressed in this paper. The first source is the abstraction process. The second one is the result extraction. The third is the uncertainty in material properties. With today's computational capabilities, minimizing the uncertainties in the abstraction process and consideration of uncertainties in the analysis is quite possible, and results of such analysis are much more valuable and reliable than analysis without consideration of uncertainties. In addition, the assessment of behavior of structure will be realistic when uncertainties are considered.

3.1. Abstraction Process

Abstraction process is the process of imposing simplifying assumptions during construction of a model. One of the major simplifying processes in determination of performance point is the method of analysis.

The result of pushover analysis, which is an abstracted process for dynamic analysis, obviously conceives some uncertainties. The major resource of uncertainties is the fact that dynamic properties of structure are assumed invariable during pushover analysis while they are variable.

3.2. Method of Result Extraction

The major source of uncertainties in result extraction is the set of assumptions made for extraction of a design spectrum. These assumptions are essentially made on the soil type and seismic zone determination. The soil types in seismic design codes are step functions of shear velocity in soil with 30 *m* depth. Similarly, different seismic zones with different ground accelerations are separated from each other by a boundary. Therefore, there are step functions for ground accelerations in two neighbor zones. These step functions produce uncertainty in result extraction. There will be more discussion on this subject in the coming sections.

3.3. Uncertainties in Structural Design Assumptions

Assumptions on material properties such as modulus of elasticity and material yield stress in steel structures are the basis of analysis of structures and calculation of displacements and stresses in structural members. The uncertainties in material properties will end up with uncertainties in the results. The uncertainties in gravity loads are also important parameters that comprise some sources of uncertainties in structural design. The effects of these sources of uncertainties are considered in the coming sections.

4. Uncertainties in Pushover Analysis

The lateral load bearing capacity of structure is usually done by pushover analysis. The source of uncertainty of pushover analysis was briefly described in the previous section. To prevent the effect of this source of uncertainty, in this research the adaptive pushover analysis that considers the variation of lateral load is used.

To that end, a nonlinear static analysis program has been written in *MATLAB* environment in which the structure is first analyzed under gravity loads. Then, based on displacement control concept of Argyris [19], and considering a constant displacement step for the control node, the lateral load of the structure is increased. Briefly, the technique determines the load in each incremental step based on the displacements obtained in the preceding analysis step. The program benefits from general concept of controlling a single displacement component that was proposed by Pian and Tong [20].

In the conventional pushover analysis methods, the lateral load is calculated based on initial dynamic properties of the structure and is kept constant during the nonlinear analysis. However, because the dynamic properties of structure do change during pushover analysis, in the present algorithm at the beginning of each incremental step, a modal analysis is performed according to the state of structure, plastic hinge distribution, etc. Then, based on the current dynamic properties, the distribution of lateral load is modified, and the pushover analysis is continued. In other words, an adaptive lateral load pattern is used for pushover analysis.

Since the dynamic properties of structure do change during the pushover analysis, the conversion of capacity curve to capacity spectrum with the conventional methods will carry uncertainties. On the other hand, if the variation of dynamic properties during the analysis is considered, the resulting curve will not be a smooth curve, rather it will be a jagged one. Consequently, determination of performance point will become very difficult. To solve this difficulty, in this research, the effective dynamic parameters were treated as fuzzy variables. As a result, performance point and performance level of the structure became a fuzzy output variable. This method is explained in the following sections.

5. Uncertainties in Seismic Design Spectrum

As mentioned in previous sections, the seismic demand spectrum is constructed based on the seismic design spectrum given by seismic design provisions and is modified according to effective damping of the structure.

The seismic design spectrums provided by seismic design provisions are based on crisp classification of soils. This kind of classification is a source of uncertainty in determination of performance point because of uncertainty in classification of soils that is explained in the following paragraph.

The classification of soil profile from Soft Soil to Hard Rock is based on average shear wave velocity (V_S) for upper 30 meters (100 feet) of soil profile. A crisp value of V_s classifies the class of soil profile. For example, a soil profile with V_s between the crisp boundaries of 180 *m/s* to 360 *m/s* is classified as “Stiff Soil” whereas soil with V_s between 360 *m/s* and 760 *m/s* is classified as “Soft Rock”.

Consider two types of soils with V_s values close to the boundary of two classified soils, e.g. V_{S1} = 350 *m/s*. and V_{S2} = 370 *m/s*. Since the soil properties of two samples are not as much different as the classification of soils in the seismic code, it is obvious that the crisp boundaries for the classification of soils are not good decisions. Therefore, to avoid step change (in boundaries) for soil types, the soil type can be specified in terms of fuzzy sets.

6. Structure of Fuzzy Inference Model

Fuzzy logic systems address the imprecision of the input and output variables by defining fuzzy numbers and fuzzy sets that can be expressed in linguistic variables (e.g. small, medium and large).

In this paper, the fuzzy inference model that has been developed for our problem of fuzzy decision-making is briefly introduced. Basically, a fuzzy inference system is composed of five functional blocks, as depicted hereunder, see Figure (5). The various parts of this fuzzy inference model will be described in the coming sections.

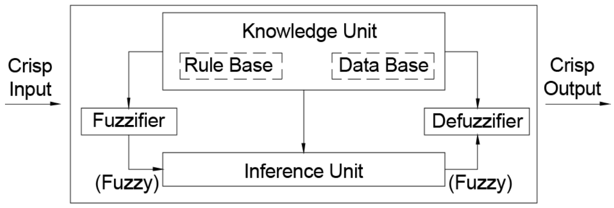

Figure 5. Fundamental blocks of fuzzy inference system.

6.1. Fuzzification

The fuzzification comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. The membership function is used to associate a grade to each linguistic term.

6.2. Fuzzy Database

Database contains information on the membership functions of the fuzzy sets used in fuzzy rules, the domains of the variables, and kinds of normalization.

6.3. Fuzzy Rule Base

Fuzzy rules are linguistic IF-THEN- constructions that have the general form “IF *A* THEN *B*” where *A* and *B* are (collections of) propositions containing linguistic variables. *A* is called the premise, and *B* is the consequence of the rule. In effect, the use of linguistic variables and fuzzy IF-THEN rules exploit the tolerance for imprecision and uncertainty. For example, a rule for the problem under consideration will look as follows:

IF “Soil Profile Type” is “Soft Rock” AND “S_s” is “0.5”, THEN “F_a” is “1.2”

IF “Soil Profile Type” is “Soft Rock” AND “S_1” is “0.5”, THEN “F_v” is “1.6”

In this case, the premise is a combined one. The AND operator is used to combine two conditional statements into one. The consequence is the value for F_a or F_v.

6.4. Inference Mechanism

The core section of a fuzzy system is the inferring system, which combines the facts obtained from the fuzzification with the rule base and conducts the fuzzy reasoning process. This is called a fuzzy inference mechanism.

Several types of fuzzy inferring systems have been proposed in the literature. They differ in the types of fuzzy reasoning and fuzzy IF-THEN rules employed. In this paper, we use and present Mamdani inference mechanisms in fuzzy rule-based systems.

For satisfaction of two requirements, *A* and *B* in a resultant *C*, Mamdani uses the following architecture.

$$A \ AND \ B \rightarrow C = (A \cap B) \cap C \quad (3)$$

Mamdani inference mechanisms consist of three stages. In first stage, the weighting factor (firing strength) of each rule is computed. The weighting factor of each rule, which is expressed as α_i, $i=1, 2$, is determined by evaluating the membership expressions in the antecedent of the rule.

For example, in a defined fuzzy inferring system, suppose existing situation is covered by two rules as follows:

Rule 1: IF input 1 is A_{11} and input 2 is A_{12} THEN output is C_1.

Rule 2: IF input 1 is A_{21} and input 2 is A_{22} THEN output is C_2.

Then weighting factor (firing strength, α_i) of each rule is computed by:

$$\begin{aligned} \alpha_1 &= A_{11}(x_0) \cap A_{12}(y_0) = \min\{A_{11}(x_0), A_{12}(y_0)\} \\ \alpha_2 &= A_{21}(x_0) \cap A_{22}(y_0) = \min\{A_{21}(x_0), A_{22}(y_0)\} \end{aligned} \quad (4)$$

These are shown in first and second row of Figure (6), in which x_0 and y_0 are members of fuzzy sets and $A_{ij}(x_0)$ shows its membership in A_{ij}.

In second stage, the implication of each rule output membership function is computed by:

$$\begin{aligned} C_1'(z) &= \alpha_1 \cap C_1(z) = \min(\alpha_1 \cap C_1(z)) \\ C_2'(z) &= \alpha_2 \cap C_2(z) = \min(\alpha_2 \cap C_2(z)) \end{aligned} \quad (5)$$

In the third stage, the overall system output is computed to derive a consequent by combining the individual rule outputs by the max operator. This is shown in third column of Figure (6).

$$C(z) = C_1'(z) \cup C_2'(z) = \max\{C_1'(z), C_2'(z)\} \quad (6)$$

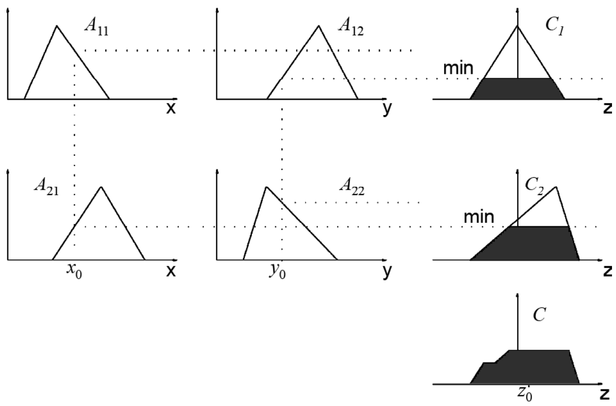

Figure 6. Graphical representation of mamdani inference mechanisms.

Graphical representation of Mamdani inference mechanisms is shown in Figure (6).

6.5. Defuzzification

For the application under consideration, we need the crisp values of output parameters. Defuzzification is introduced as mapping from a fuzzy subset (obtained after applying the inference mechanism) to the crisp point.

There are several defuzzification methods [21]. The most popular ones are: a) Centre of gravity method (*COG*), b) Centre of singleton method (*COS*), and c) Maximum methods. In this paper, the *COG* method is used for defuzzification.

7. Construction of Fuzzy Inference Model for Seismic Design Spectrum

In this section, the fuzzy inference model that has been developed for our problem of fuzzy decision-making will be explained. This explanation starts with identifying the input and output parameters of the problem.

The input parameters are "V_S and *Z* factor" in *UBC*97 and "V_s, S_s and S_1" in *IBC*. The two coefficients C_a and C_v are output of *UBC*97 method, and coefficients F_a and F_v are outputs of *IBC* method.

The fuzzy inference model developed in this paper will yield fuzzy output values. They will be converted to crisp output values using *COG* defuzzification method. In the following, the various parts of this fuzzy inference model will be described.

7.1. Fuzzification of Soil Profile

Soil profile type that is determined based on average shear wave velocity (V_S) is one of the input fuzzy parameters for *UBC* and *IBC*. Fuzzification of soil profile type is based on the assumption that 100% truth will be associated with a value or interval of values of V_s that is in between the lower and upper range for the soil type. Here in this research, a trapezoid membership function for each soil profile, as shown in Figure (7) is adopted. It is evident that any other membership functions may be adopted according to expert opinions. In fuzzy logic terminology, the input fuzzy parameter, "Soil Profile Type", has some associated fuzzy variables such as "Soft Soil", "Stiff Soil", etc., that serve as adjective to the noun "Soil Profile". The complete membership function is shown in Figure (7) where all soil profiles are shown together.

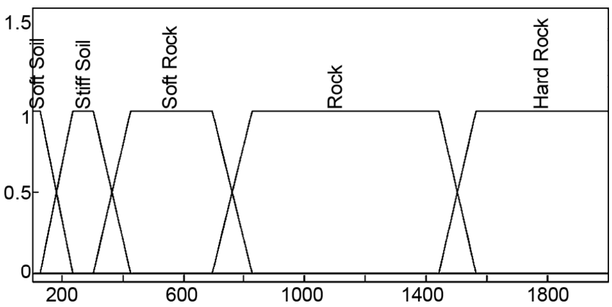

Figure 7. Fuzzy sets for soil profile.

7.2. Fuzzification of Seismic Zone Factor (UBC 97)

The seismic zone factor is another input parameter with associated fuzzy variables. The seismic zone classification is based on the value of the seismic zone factor *Z* that is specified for each zone. The truth value of (100% membership) has been associated with the value of *Z* of each zone. For each of the five seismic zones, a triangular or trapezoid membership function, as shown in Figure (8), has been assumed.

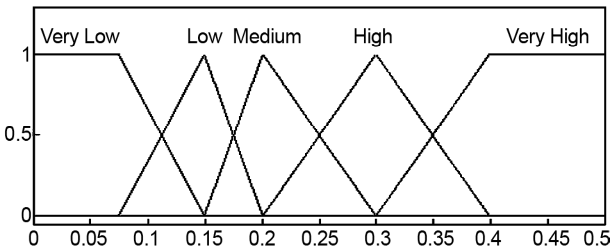

Figure 8. Fuzzy sets for seismic zone factor for UBC.

7.3. Fuzzification of S_s and S_1 (IBC)

S_s and S_1 are other input parameters with associated fuzzy variables in *IBC*. The seismic zone classification is based on the value of the seismic zone factors S_s and S_1. Here, the code specifies a value of S_s and S_1 for each zone. The truth value of (100% membership) has been associated with the value of S_s and S_1 specified for each zone. A triangular membership function has been assumed for each zone as shown in Figures (9) and (10).

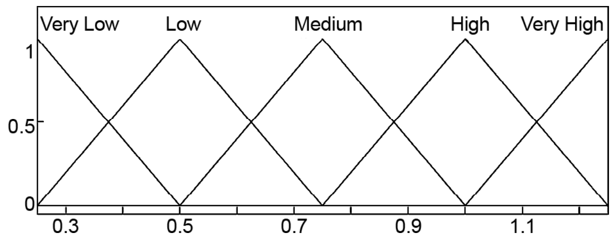

Figure 9. Fuzzy sets for S_S.

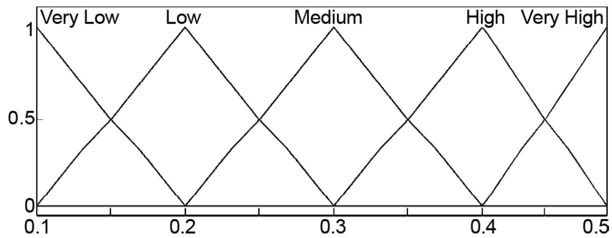

Figure 10. Fuzzy sets for S_1.

7.4. Fuzzification of Output Parameters for UBC

The output parameters in *UBC*97 are the seismic coefficients C_a and C_v. For both coefficients, the membership functions are shown in Figures (11) and (12). It is important to note that the names for the values of the linguistic variables have been selected the same as their numeric values in the Tables (1) and (2) [9].

7.5. Fuzzification of Output Parameters for IBC

The output parameters in *IBC* are the seismic coefficients F_a and F_v. The membership functions for these coefficients are shown in Figures (13) and (14). Similar to C_a and C_v, the names for the values of the linguistic variables have been selected the same as their numeric values in the Tables (3) and (4).

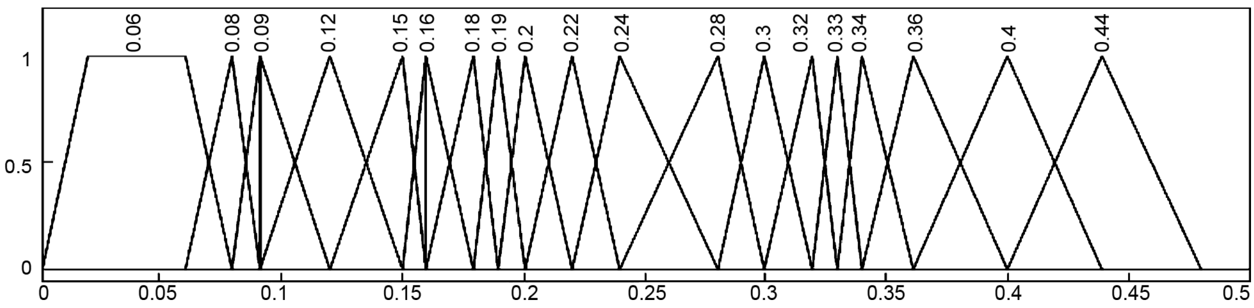

Figure 11. Fuzzy sets for C_a.

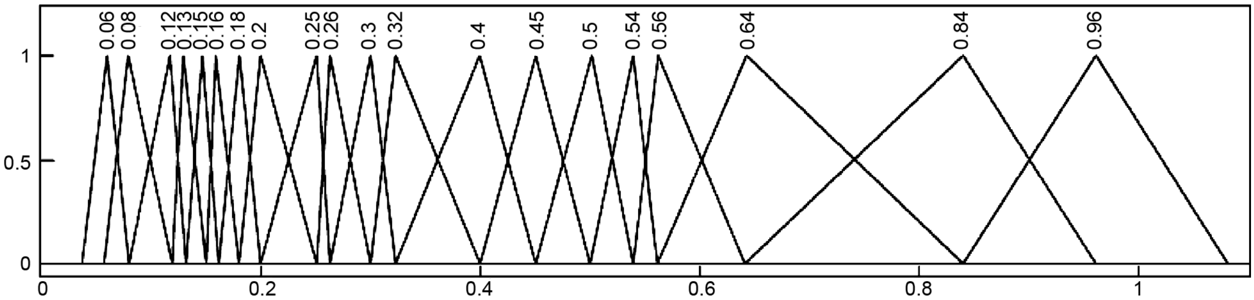

Figure 12. Fuzzy sets for C_v.

Table 1. Seismic coefficients C_a.

Soil Profile Type	Seismic Zone Factor				
	Z = 0.075	Z = 0.15	Z = 0.2	Z = 0.3	Z = 0.4
Soft Soil	0.06	0.12	0.16	0.24	0.32
Stiff Soil	0.08	0.15	0.2	0.3	0.4
Soft Rock	0.09	0.18	0.24	0.33	0.4
Rock	0.12	0.22	0.28	0.36	0.44
Hard Rock	0.19	0.3	0.34	0.36	0.36

Table 2. Seismic coefficients C_v.

Soil Profile Type	Seismic Zone Factor				
	Z = 0.075	Z = 0.15	Z = 0.2	Z = 0.3	Z = 0.4
Soft Soil	0.06	0.08	0.12	0.16	0.32
Stiff Soil	0.08	0.15	0.2	0.3	0.4
Soft Rock	0.13	0.25	0.32	0.45	0.56
Rock	0.18	0.32	0.4	0.54	0.64
Hard Rock	0.26	0.5	0.64	0.84	0.96

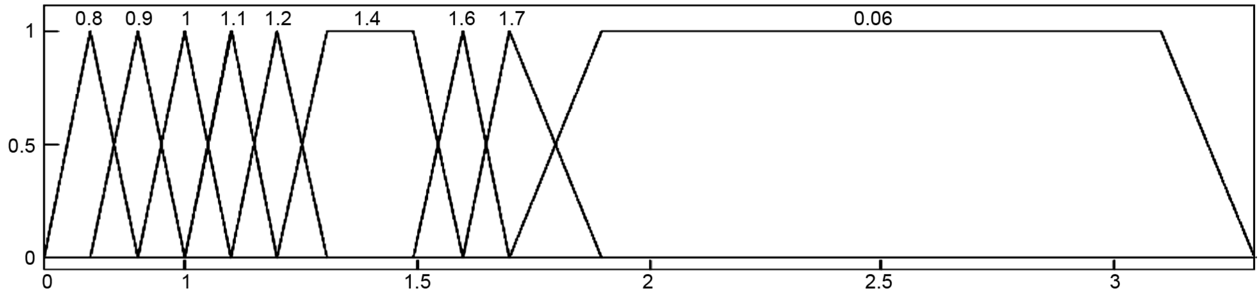

Figure 13. Fuzzy sets for F_a.

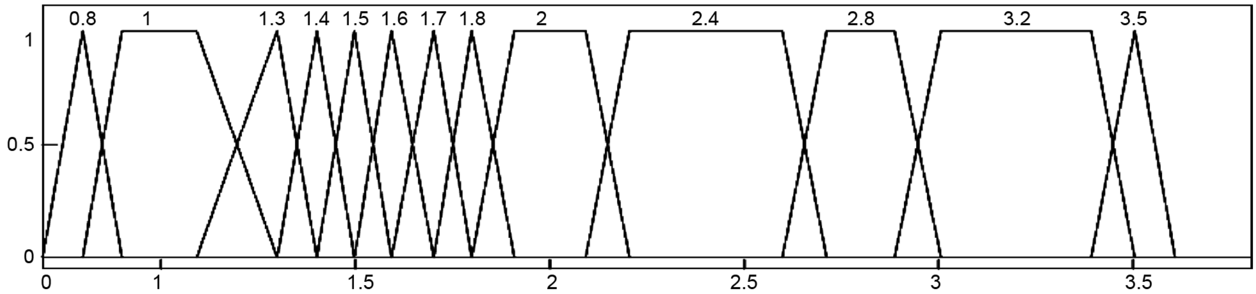

Figure 14. Fuzzy sets for F_v.

Table 3. Seismic coefficients F_a.

Soil Profile Type	Seismic Zone Factor				
	S_S < 0.25	S_S = 0.5	S_S = 0.75	S_S = 1	S_S = 1.25
Soft Soil	0.8	0.8	0.8	0.8	0.8
Stiff Soil	1	1	1	1	1
Soft Rock	1.2	1.2	1.1	1	1
Rock	1.6	1.4	1.2	1.1	1
Hard Rock	2.5	1.7	1.2	0.9	

Table 4. Seismic coefficients F_v.

Soil Profile Type	Seismic Zone Factor				
	S_1 < 0.1	S_1 = 0.2	S_1 = 0.3	S_1 = 0.4	S_1 = 0.5
Soft Soil	0.8	0.8	0.8	0.8	0.8
Stiff Soil	1	1	1	1	1
Soft Rock	1.7	1.6	1.5	1.4	1.3
Rock	2.4	2	1.8	1.6	1.5
Hard Rock	3.5	3.2	2.8	2.4	

7.6. Rule Base

When we use *UBC*, there are two input domains, namely, the soil profile type and the seismic zone, each with five fuzzy sets. The numbers of rules needed in this fuzzy system are 25 for each output. All the 25 rules for each of the output variables, namely, C_a and C_v are based on the values given in Tables (1) and (2). This ensures that the expert opinion embedded in the code through these tables will be used in deriving the results based on fuzzy inference. When using *IBC*, there are three input domains, namely, the soil profile type, and S_S and S_1, each with five fuzzy sets. The number of rules needed in this fuzzy system is 24 for each output. All the 24 rules for F_a and F_v are based on the values given in Tables (3) and (4) to ensure satisfaction of code provisions in deriving the results based on fuzzy inference. In these cases, the AND operator is used to combine two conditional statements into one. For example, a rule for the problem under consideration will look as follows:

IF “Soil Profile Type” is “Soft Rock” AND “S_s” is “0.5”, THEN “F_a” is “1.2”

IF “Soil Profile Type” is “Soft Rock” AND “S_1” is “0.5”, THEN “F_v” is “1.6”

7.7. Inference Mechanism

In this research, Mamdani inference method has been used for the inference mechanism because of its simplicity, and because it is most commonly used. The “MIN” inference rule is used where the inputs are combined logically with “AND” operator. This inference rule determines the minimum of the two antecedents as the share of every rule in the output. Then the Max operator is used to find the envelope of the outputs. The inference mechanism is thus based on what is usually referred to a “MAX-MIN” inference, i.e. the MAX composition and MIN inference.

7.8. Defuzzification

In our fuzzy inference model, Centroid method will be used for defuzzification of fuzzy outputs. In this method, the crisp value of the output parameter is the value corresponding to the centre of gravity of the fuzzy sets of the output parameter.

$$Z_{COG} = \frac{\int_C \mu_A(z)\, z\, dz}{\int_C \mu_A(z)\, dz} \tag{7}$$

8. Example

In this example, a six-storey one-bay frame is considered. This example has been provided to show the methodology of finding the performance point via fuzzy inferring model in accordance to provisions of *UBC*97 and *IBC*2006. The frame specifications are as follows:

Beams and columns sections are European steel cross sections. The aforementioned program has been used for pushover analysis. Other properties are as follows:

1. Length of each bay is 5 *m*, and height of each storey is 3.2 *m*;
2. Dead load is considered 6.5 kN/m^2, and live load is considered 2.0 kN/m^2 in each storey;
3. Dead load and live load on the roof are considered 6.0 kN/m^2 and 1.50 kN/m^2 respectively;
4. Column sections are *IPB*240 for storey 1 to storey 4, and *IPB*220 for storey 5 to storey 6;
5. Beam sections are *IPE*300 for all floors;
6. Relative damping is considered 5% for design;
7. Yield stress for steel is considered 240 *MPa*.

There are other parameters that are dealt with as fuzzy variables as follows:

a) Soil type;
b) Seismic zone factor;
c) dynamic properties of the structure including mass modal factor, modal participation factor, and amplitude of first mode in roof level;
d) Modulus of elasticity in vicinity of 2.1×105 *MPa*;
e) Weight of mass on the structure (0.2 *L.L* + *D.L.*) in vicinity of crisp values stated in 2 and 3 above. From here forward, this is briefly called weight.

This frame is shown in Figure (15). In this example, the lateral load pattern is considered to be proportional in magnitude to the pattern that is derived from the combination of first three natural mode shapes of the structure. As explained before, at the beginning of each step, first, a modal analysis is performed to determine dynamic properties of structure at that state of structure and extract the

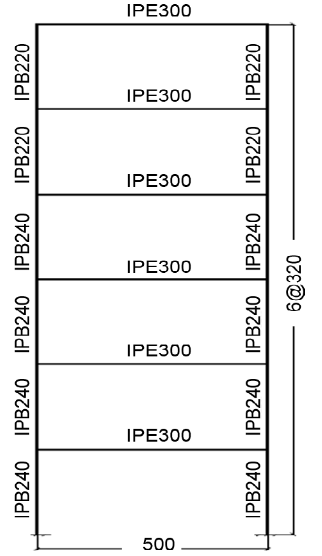

Figure 15. Structure of the example.

lateral load pattern. It is clear that because of change in the dynamic properties of the structure, including modal participation factor, mass modal participation factor and modal shape vectors of structure, the capacity spectrum that is derived from the capacity curve of structure will not be a smooth curve. The resulting capacity spectrum for this structure has been shown in Figure (16).

For determination of performance point, the capacity spectrum curve should be intersected to the demand spectrum curve that is produced from design response spectrum. Obviously, jagged curve that is derived for capacity spectrum is not appropriate for this task. Figures (17) show the variation of mass modal participation factor (α), modal participation factor (Γ), and amplitude of first mode

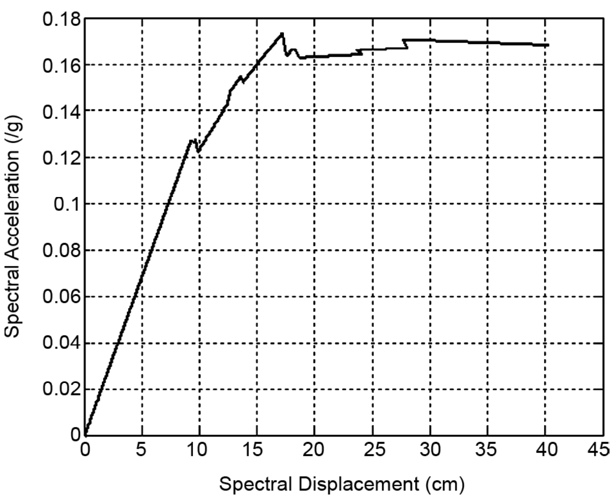

Figure 16. Jagged capacity curve obtained from conversion of pushover analysis to capacity spectrum.

in roof level (Φ_{roof}) respectively. These curves show that during pushover analysis, the dynamic characteristics of the structure do not remain constant, and they vary by formation of hinges in the structure.

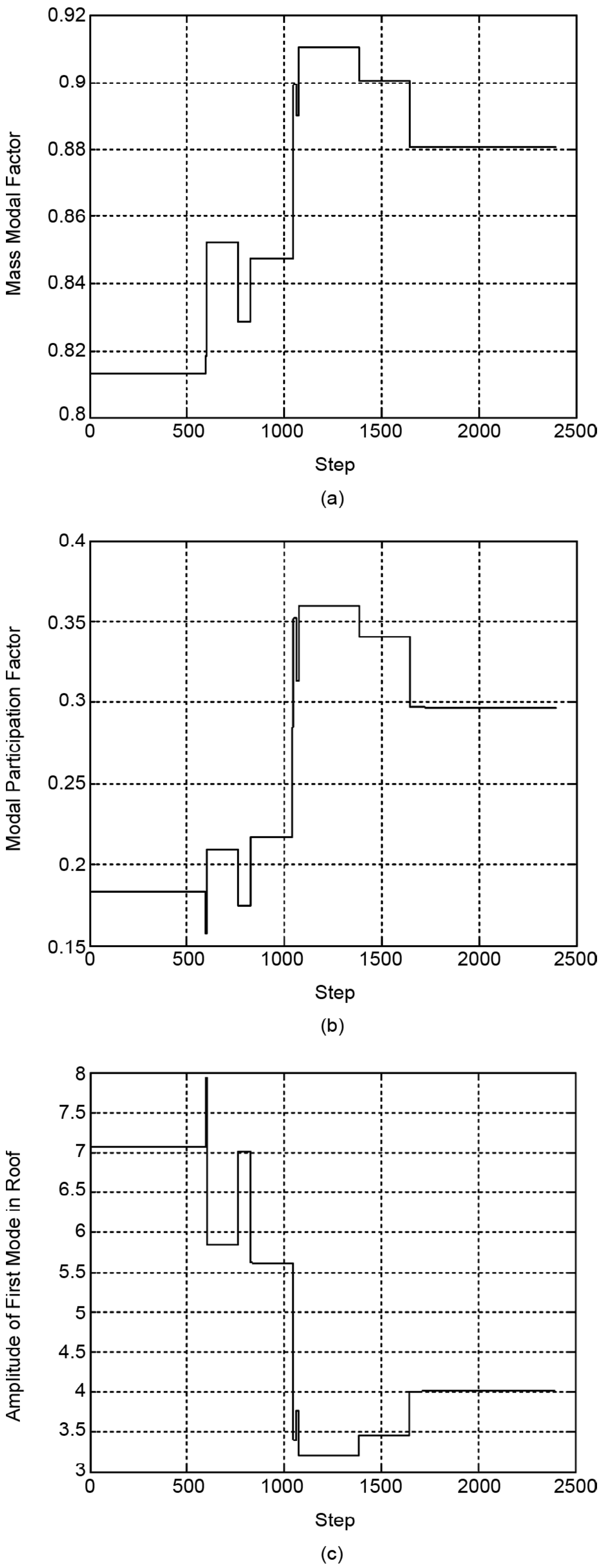

Figure 17. Variation of dynamic properties of the example during pushover analysis: (a) mass modal factor (α), (b) modal participation factor (PF), and (c) amplitude of first mode in roof level (λ_{roof}).

8.1. Introducing Uncertain Parameters as Fuzzy Variables

It is obvious that in the adaptive pushover analysis, the lateral load pattern is not constant and varies in accordance to the situation of structure and birth and fade of plastic hinges.

The equation of equilibrium for entire process of analysis can be written as below:

$$[\underline{K} \quad \overline{K}]\times[\underline{d} \quad \overline{d}] = [\underline{F} \quad \overline{F}] \tag{8}$$

where, the interval $[\underline{F} \quad \overline{F}]$ shows the variation of lateral load pattern. Seismic provisions recommend two lateral load distribution patterns to be used for pushover analysis. In this paper, the subject has been dealt with a fuzzy approach. For this purpose, according to variation of Φ_{roof}, α and Γ during the pushover analysis, some fuzzy sets are defined for them. In this example, with regard to Figure (17), for 6 storey frame, the following fuzzy sets have been defined:

$$\tilde{\alpha} = \langle \alpha_l \ \alpha_o \ \alpha_r \rangle \tag{9}$$

$$\tilde{\Gamma} = \langle \Gamma_l \ \Gamma_o \ \Gamma_r \rangle \tag{10}$$

$$\tilde{\Phi}_{roof} = \langle \Phi_l \ \Phi_o \ \Phi_r \rangle \tag{11}$$

Membership functions of these fuzzy sets, are shown in Figure (18). It is noted that the range of variation of each fuzzy variable depends on the extent of nonlinearity of the structure behavior, which in turn is a function of formation of hinges in the structure. It is understood from Figure (17) that minimum and maximum of fuzzy variables $\tilde{\Phi}_{roof}$, $\tilde{\alpha}$, $\tilde{\Gamma}$ during pushover analysis are as below.

$$\tilde{\alpha} = [0.813 \quad 0.847 \quad 0.91] \tag{12}$$

$$\tilde{\Gamma} = [0.158 \quad 0.217 \quad 0.36] \tag{13}$$

$$\tilde{\Phi}_{roof} = [3.215 \quad 5.616 \quad 7.94] \tag{14}$$

It is important to note that the values of α_0, Φ_0, Γ_0, should be such determined that they get close to the average of the boundary values, and they occur simultaneously. In this example, it occurs approximately when the frame is pushed to have nine hinges.

In this example, in addition to those dynamic parameters, the mass and module of elasticity of the material, are considered as fuzzy variables. Membership functions of these two variables have been shown in Figure (19).

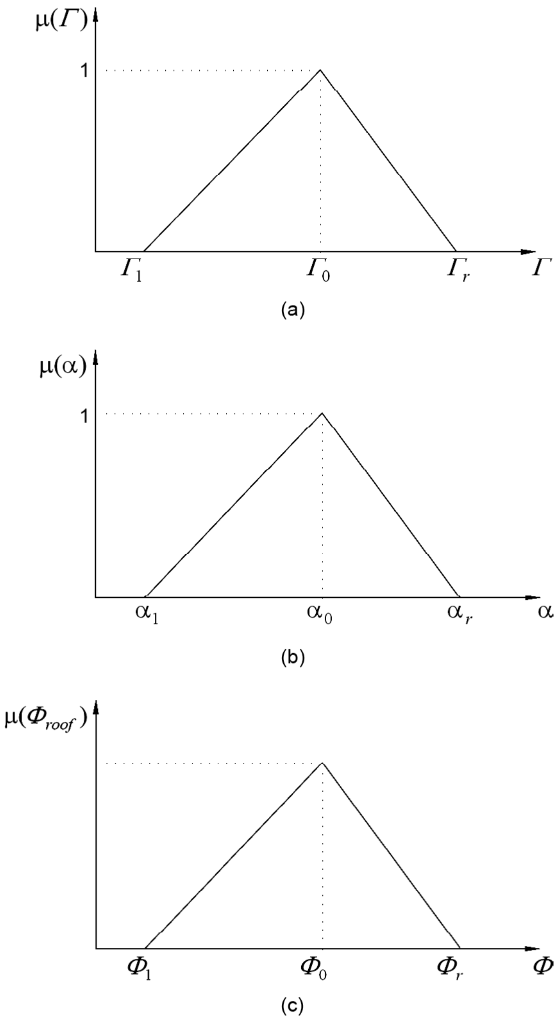

Figure 18. Membership functions of (a) modal participation factor, (b) mass modal factor and (c) amplitude of the first mode at the roof level in the first mode.

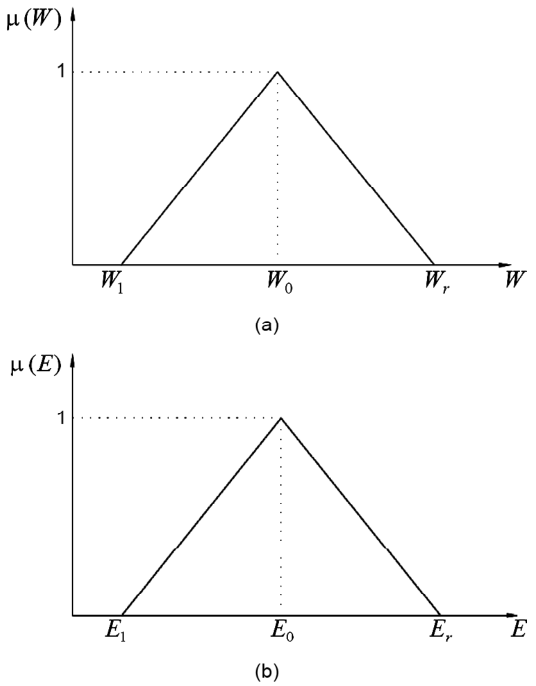

Figure 19. Membership function of (a) weight, and (b) module of elasticity.

8.2. Pushover Analysis Considering Fuzzy Variables

Fuzzy variables of weight and module of elasticity influence the pushover analysis. Membership functions of these two variables have been divided into five levels. These five levels correspond to:

$\mu(\tilde{w}), \mu(\tilde{E}) = 0, \quad \mu(\tilde{w}), \mu(\tilde{E}) = 0.25,$

$\mu(\tilde{w}), \mu(\tilde{E}) = 0.5, \quad \mu(\tilde{w}), \mu(\tilde{E}) = 0.75,$

$\mu(\tilde{w}), \mu(\tilde{E}) = 1$

It is clear that upper bound of capacity curve of structure corresponds to lower bound of weight/mass and upper bound of modules of elasticity of the material. Conversely, lower bound of capacity curve of structure corresponds to upper bound of weight and lower bound of module of elasticity. Therefore, in order to have a variety of capacity curves, diversity (at least 5) of memberships of E and W can be used. Figure (20) shows the capacity curves for $(0.9E, 1.1W)$, $(1E, 1W)$ and $(1.1E, 0.9W)\}$.

The conversion of capacity curve to capacity spectrum is performed according to Eqs. (1) and (2). Since in this example, parameters $\tilde{\Phi}_{roof}$, $\tilde{\alpha}$, $\tilde{\Gamma}$ and weight are fuzzy variables, this conversion is represented as fuzzy formulae as below:

$$\tilde{S}_a = \frac{V}{\tilde{\alpha} \times \tilde{W}} \tag{15}$$

$$\tilde{S}_d = \frac{\tilde{\delta}_{roof}}{\tilde{\Phi}_{roof} \times \tilde{\Gamma}} \tag{16}$$

Because the capacity curve is obtained in five levels, the conversion of the capacity curve to

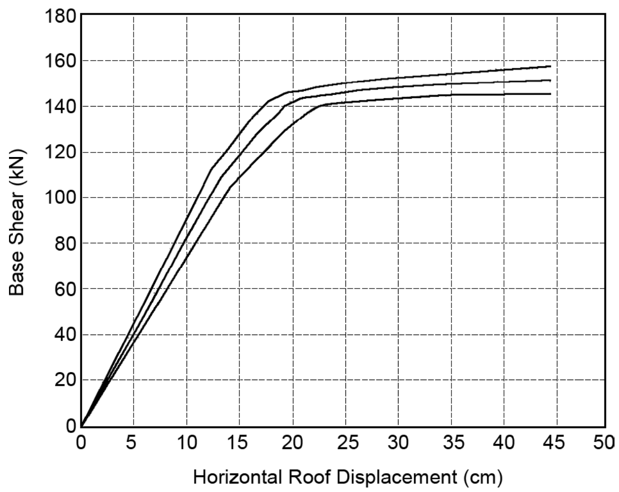

Figure 20. Capacity curve of structure in each level.

capacity spectrum is done in each of these five levels. Since parameters $\tilde{\Phi}_{roof}$, $\tilde{\alpha}$, $\tilde{\Gamma}$ and weight are fuzzy variables, therefore, in each level, the corresponding capacity curve is mapped to set of capacity spectrum curves. In each level, to obtain upper bound and lower bound of capacity spectrum, upper bound and lower bound of spectral acceleration, and spectral displacement must be calculated based on the following equations:

$$(\overline{S}_a)_\alpha = \frac{(\overline{V})_\alpha}{\underline{(\tilde{\alpha}\times\tilde{W})}}$$

$$(\underline{S}_d)_\alpha = \frac{(\underline{\delta})_{roof}}{\overline{(\tilde{\Phi}_{roof}\times\tilde{\Gamma})}} \tag{17}$$

$$(\underline{S}_a)_\alpha = \frac{(\underline{V})_\alpha}{\overline{(\tilde{\alpha}\times\tilde{W})}}$$

$$(\overline{S}_d)_\alpha = \frac{(\overline{\delta})_{roof}}{\underline{(\tilde{\Phi}_{roof}\times\tilde{\Gamma})}}$$

In denominator of Eqs. (17), product of two fuzzy variables must be obtained. According to interval mathematics, if two interval variables are independent, then products of these two interval variables are obtained as below:

$$\tilde{A} = [\underline{A}\ \ \overline{A}]\ \ ,\ \ \tilde{B} = [\underline{B}\ \ \overline{B}]$$

$$\tilde{A}\times\tilde{B} = [\min(\underline{A}\times\underline{B}, \underline{A}\times\overline{B}, \overline{A}\times\underline{B}, \overline{A}\times\overline{B})\ \ \max(\underline{A}\times\underline{B}, \underline{A}\times\overline{B}, \overline{A}\times\underline{B}, \overline{A}\times\overline{B})] \tag{18}$$

In Eq. (15), variables $\tilde{\alpha}$ and $\tilde{W}$, are independent. Therefore, the product of these two fuzzy variables can be obtained as below:

$$\tilde{\alpha}\times\tilde{W} = \langle W_l\times\alpha_l \quad W_r\times\alpha_r\rangle \tag{19}$$

where W_l and W_r, are the lower and upper bounds of weight variable.

However, $\tilde{\Phi}_{roof}$, $\tilde{\Gamma}$ and $\tilde{\alpha}$, are dependent interval parameters, if these parameters are considered as independent parameters, the boundary of results and capacity spectrum curves will be falsely expanded. The product of $\tilde{\Phi}_{roof}\times\tilde{\Gamma}$ should be such obtained that fuzzy variables Φ_{roof} and Γ are attributed simultaneously. Therefore, the minimum and maximum of this product is obtained from a set of values that have been calculated at a certain level of formation of hinges.

$$\tilde{\Phi}_{roof}\times\tilde{\Gamma} = \langle 1.157 \quad 1.295\rangle \tag{20}$$

According to Eqs. (5) and (6), spectral displacement and spectral acceleration of the structure in each level can be rewritten as below:

$$\tilde{S}_a = \frac{V}{\langle W_l\times 0.8131 \quad W_r\times 0.9102\rangle} \tag{21}$$

$$\tilde{S}_d = \frac{\delta_{roof}}{\langle 1.157 \quad 1.295\rangle} \tag{22}$$

$$\left(\tilde{S}_d, \tilde{S}_a\right) = \left(\langle \underline{S}_d \quad \overline{S}_d\rangle, \langle \underline{S}_a \quad \overline{S}_a\rangle\right)$$

$$\underline{S}_d = \frac{\delta_{roof}}{1.295}\ ,\ \ \overline{S}_d = \frac{\delta_{roof}}{1.157}\ , \tag{23}$$

$$\underline{S}_a = \frac{V}{1.1w\times 0.9102}\ ,\ \ \overline{S}_a = \frac{V}{0.9w\times 0.8131}$$

In the above equations, the over-bar superscript and under-bar subscript denote the upper bound and lower bounds, respectively. Finally, capacity spectrum curve of structure is achieved in each level and illustrated in Figure (21). It is noted that situation of performance point and performance level of the frame is determined in each level.

As mentioned in previous section, in this example, performance point of structure is calculated with capacity spectrum method. This method requires the use of a seismic design spectrum. Seismic provisions propose several methods for construction of seismic design spectrum. In this example, for construction of this spectrum, both methods of *UBC*97 and *IBC*2006

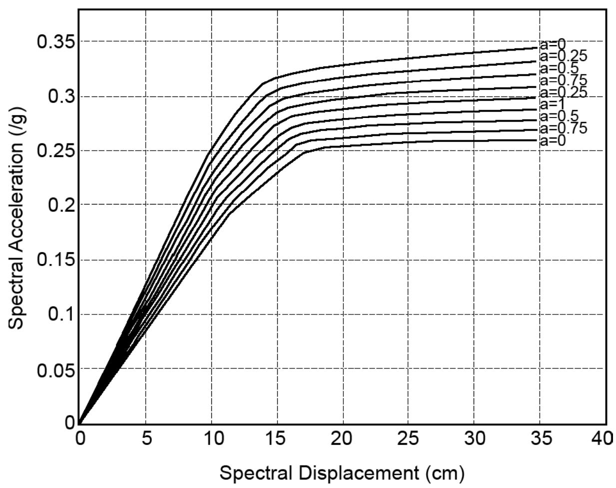

Figure 21. Capacity spectrum curve of structure in each level.

have been used. Then, results have been compared with each other.

8.2. Determination of Performance Point of Structure

In this section, effect of fuzziness of weight, module of elasticity and dynamic characteristics on performance point is investigated. The method of determination of performance point that is used in this research has been previously pointed out in section 3. For achieving seismic demand spectrum, the design spectrum has to be constructed. In this example, the design response spectrum has been built based on seismic data and the fuzzy inference model that was explained in section 6.

Let us assume that the frame under consideration is located in San Diego on a soil in which $V_S = 380$. This type of soil is classified as soft rock in *UBC*97. The seismic zone is $Z = 0.4$. For this example, the seismic demand spectrum will be plotted based on *UBC*97 and *IBC*2006.

For *UBC*97, we have $V_S = 380$ and $Z = 0.4$. The outcomes of these inputs from the *UBC*97 are $C_a = 0.4$ and $C_v = 0.56$. However, the outcomes of fuzzy inference model are: $C_a = 0.416$ and $C_v = 0.664$. The descending part of the design spectrum in *UBC*97 is built based on the following equation:

Smooth branch $= 2.5C_a$

Descending branch $= C_v/t$

This curve has been shown in Figure (22). In the *IBC*2006 for San Diego, we have $S_S = 1.25$, $V_1 = 0.457$ which results: $F_a = 1$ and $F_v = 1.4$. The outcomes of fuzzy inference model are: $F_a = 1$, and $F_v = 1.44$.

To construct the demand spectrum in *IBC*2006, we need to calculate S_{DS} and S_{D1}.

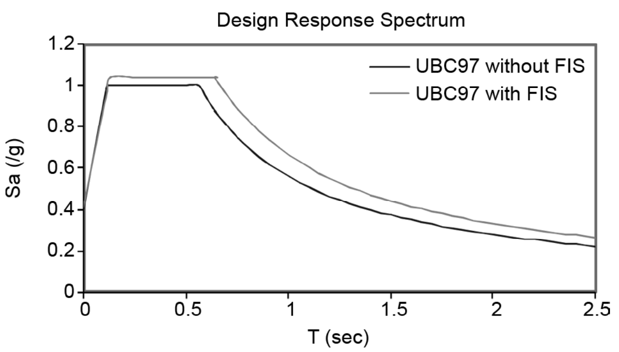

Figure 22. Design response spectrum of UBC97 with and without application of Fuzzy Inference System (FIS).

$$S_{DS} = (2/3)F_a \times (S_S), S_{D1} = (2/3)F_v \times (S_1)$$

The equation used for construction of descending part of the design spectrum in *IBC*2006 is as follows:

Smooth branch = S_{DS}

Descending branch = S_{D1}/t

This curve has been shown in Figure (23). In this example, for determination of seismic demand spectrum, design spectrum obtained from fuzzy inference model is used. This spectrum has been transformed from time-acceleration coordinates to spectral displacement-spectral acceleration coordinates (*ADRS* format). As explained in *FEMA*440 [18], determination of performance point is an iterative process. In this example, instead of one curve, a series of curves is obtained for capacity spectrum. Therefore, instead of one performance point, we will have a set of performance points. But, these performance points will have different membership functions or degrees of possibilities. In this example, using *UBC*97 for construction of design spectrum, the membership function of performance point will be obtained as in Figure (24).

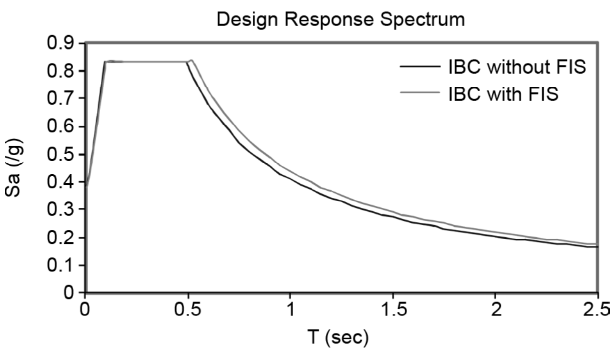

Figure 23. Design response spectrum of IBC2006 with and without application of Fuzzy Inference System (FIS).

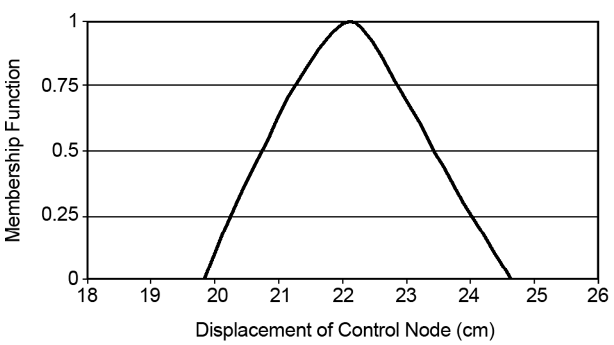

Figure 24. Membership function of performance point.

If fuzzy uncertainties are not considered, performance point is equal to 16.6 *cm*, while Figure (24) shows that the minimum displacement is about 19.8 *cm*.

Since the drift values are used as a general means for description of performance level, the attention will be paid to the drift ratio of the structure in different storeys. The boundaries of allowable drift ratio in each performance level for steel moment frames are 0.7%, 2.5% and 5% for *IO*, *LS* and *CP* respectively [22]. When the fuzzy inference model is used, depending on the membership function of the performance point, there will be different drift values. Figure (25) shows drift ratios for different storeys of this structure.

In this situation, comparison of storey drifts, with and without fuzzyfying the design parameters, is shown in Figure (26).

Similarly when *IBC*2006 is used for construction of design spectrum, membership function of performance point can be obtained as in Figure (27).

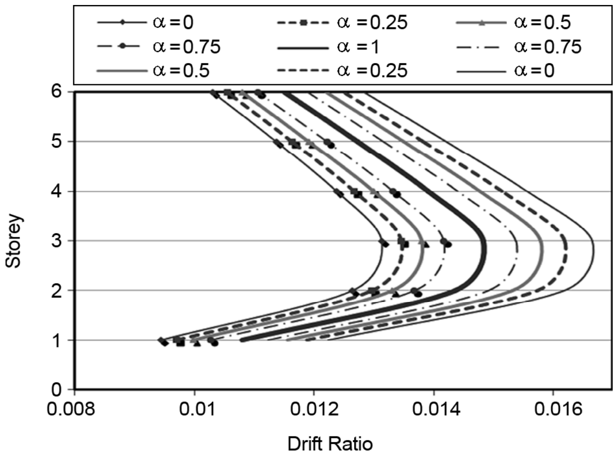

Figure 25. Drift ratio for storeys of structure in each level.

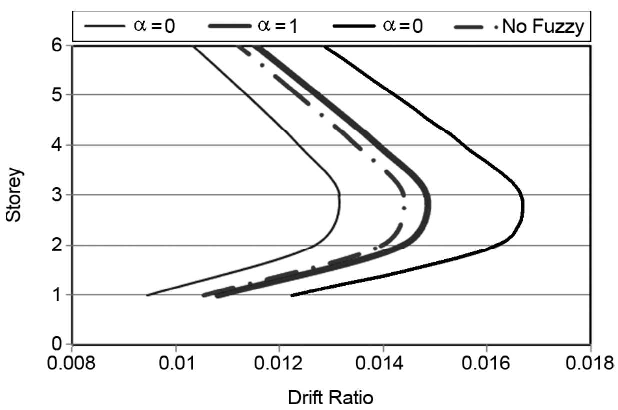

Figure 26. Comparison of storey drift with and without fuzzyfying design parameters.

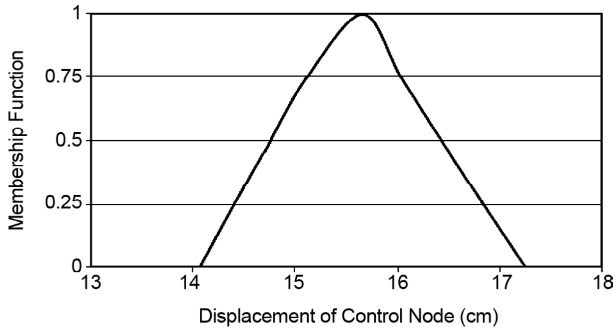

Figure 27. Membership function of performance point.

If fuzzy uncertainties are not considered, performance point is equal to 14.7 *cm*. However, with consideration of uncertainties, the minimum displacement is about 14.1 *cm*, and the maximum displacement is about 17.2 *cm*.

When the fuzzy inference model is used, depending on the membership function of the performance point, similar to previous section, there will be different drift values. Figure (28) shows drift ratios for different storeys of this structure when *IBC*2006 is the basis of analysis.

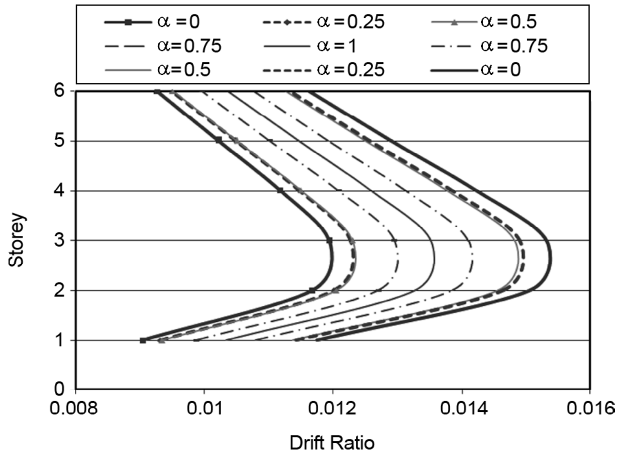

Figure 28. Drift ratio for storeys of structure in each level.

When *IBC* is the basis of earthquake design, storey drift with and without fuzzyfying design parameters, will be as shown in Figure (29).

Upper bound and lower bound of drift ratios of storeys correspond to upper bound and lower bound of performance point of structure. In this example, performance point and corresponding drift ratio have been calculated in five α levels. Since values of drift ratio in all storeys are less than %2.5, it is concluded that life safety performance level for this frame is expected.

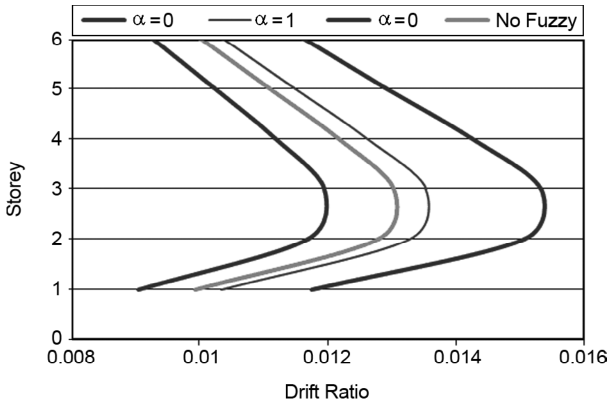

Figure 29. Storey drift ratios with and without fuzzyfying design parameters.

9. Summary and Conclusion

In this paper, special attention was paid to determination of structural response to earthquake, considering fuzzy uncertainties. The modulus of elasticity, gravity load on the structure and dynamic properties of structure were considered as fuzzy parameters.

The capacity spectrum was obtained from the capacity curve for weight and modal properties of structure including: 1) modal participation factors of first three modes, 2) mass modal factor, and 3) amplitude of first mode in the roof.

Since the dynamic properties of structure do change during formation of plastic hinges in the structure, in traditional method, the transformation of the capacity curve to capacity spectrum is accompanied by some errors/uncertainties. In this paper, to show the effect of uncertainties, the dynamic properties (Φ_{roof}, α and Γ), the weight, and soil properties were treated as fuzzy variables. Consequently, performance point and performance level of structure were produced as fuzzy outputs. Some results from this research are as follows:

- The result of determining performance point and performance level show that the crisp evaluation of performance point compared to fuzzy evaluation of the point may accompany some errors. Importance of this result is highlighted if we remember that in the process of rehabilitation of structures, a structure is such rehabilitated that it just meets the code specified criteria and does not have any hidden capacity. So, using the traditional way of performance evaluation that according to this study may be underestimates the drift, may be unsafe.
- Importance of consideration of uncertainties was highlighted with the typical example.
- It was observed that the effect of uncertainties in transformation of the capacity curve to capacity spectrum in *IBC*2006 method was less than that of *UBC*97. This is because in *IBC*2006, when S_S>1.25 and S_1>0.5, S_{DS} is (2/3)×S_S and S_{D1} is (2/3)×S_1. In fact, when S_S>1.25 and S_1>0.5, value of S_{DS} and S_{D1} are independent of type of soil. As a result, in this situation, uncertainty in determination of type of soil has no effect on design response spectrum. As a general conclusion, it is possible to write the codes in such a way that the effects of uncertainties are in a minimum.
- It is therefore suggested that some parts of provisions of codes of practice be revised accordingly. This study shows which parameters need more attention.
- The example shows that the effect of uncertainties of the design spectrum on performance point is more than uncertainties of dynamic properties of the structure.
- The fuzzification of soil type may be readily taken into practice by introduction of some graphs similar to Figure (7), or some formulae for averaging the soil dynamic parameters.

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