Effects of Foundation Geometry on the Natural Periods of Cylindrical Tank-Liquid-Soil Systems

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ABSTRACT: *Liquid storage tanks are essential structures in water, oil and gas industries, and their seismic safety is of great importance. On the other hand, modifying the dynamic characteristics of tank systems can be very useful for improving their seismic behavior. In this paper a study has been performed on the effect of the geometry of the tank foundation on the modal properties of the tank-liquid-soil system, in which both fluid-structure and soil-structure interactions have been considered. For this purpose a set of cylindrical steel tanks with various height over radius (H/R) and thickness over radius (t/R) ratios have been considered. The tank foundations have been assumed to have two main different geometries, namely square and circular in plan with different thicknesses, as well as various dimensions and/or diameters. Various conditions have been considered for the subsoil varying from very soft to very stiff based on the value of shear wave velocity (v_s). The first three modes of the tank system have been taken into account for modal characteristics calculations. The numerical results show that the natural periods of the system are quite sensitive to the foundation geometry. This sensitivity is much higher in the case of circular foundations, especially for lower H/R ratios and lower v_s values. By choosing appropriate values for foundation dimensions, it is possible to make the period values a few times longer. Therefore, using a specific foundation geometry can be a good tool for modifying the period of the whole tank-liquid-foundation system in earthquake prone regions to make it far from the dominant frequency of the site.*

Keywords: Cylindrical steel tanks; Tank-liquid-soil system; Foundation geometry; Natural period; Seismic Safety

1. Introduction

Liquid storage tanks are essential structures in water as well as oil and gas industries. On the other hand several cases of damage to tanks have been observed in past earthquakes. Regarding the importance of these systems, specially their seismic safety for avoiding the adverse consequences such as fires and explosions and environment pollution, better understanding of their seismic behavior still seems necessary. There are several factors, which affect the dynamic characteristics of a liquid storage tank. Some of these factors are:

- Wall thickness over radius ratio, (*t*/*R*), which shows the relative flexibility or rigidity of the tank wall;
- Height over radius ratio, (*H*/*R*), and filled height over tank height ratio, (H_f/H), which are both effective on sloshing phenomenon and the required free-board;
- Type of connection of the tank wall to the foundation, which affects the stability of the tank wall in its lower portions;
- Sub-soil conditions, specially the shear wave velocity (v_s), which is an index of the soil dynamic behavior;
- Shape of the tank foundation in plan, and finally
- Dimensions of the foundation.

Many of these factors have been studied by several researchers since early 1930's. It seems that Hopkins and Jacobsen [11] have been the first researchers who have worked on water pressure in tanks. Jacobsen [16] has also studied the hydrodynamic pressure in tanks. Several studies have been performed on tanks, including hydrodynamic pressure by Housner [12] as well as vibration tests and analysis by Housner and Haroun [14] and [15], and design by Haroun and Housner [10] for tanks subjected to earthquake forces. Epstein [6] has also worked on the seismic design of liquid storage tanks. Aslam et al [1] have studied the sloshing phenomenon in both annular and cylindrical tanks. Haroun [7] has also worked both theoretically and experimentally with special attention to flexible tanks. Haroun and Ellaithy [9] have studied the rocking motion of flexible tanks during earthquake. The rocking response of the tanks has also been studied by Veletsos and Tang [26]. Barton and Parker [2] have studied the effect of anchorage conditions on the seismic response of tanks.

In recent years the soil-structure interaction effect has been one of the most attractive subjects for many researchers. Velestos and Tang [27] have studied comprehensively the effect of soil-structure interaction on the tank seismic response. James and Raba [17] have studied the behavior of steel tanks from various aspects, including soil-structure interaction. Liquid-structure interaction as well as sloshing phenomenon have been also matters of interest for several researchers in recent years. Lay [20] has studied the modeling of axisymmetric tanks by taking into account the liquid-structure interaction. The sloshing phenomenon has been studied by Veletsos and Shivakumar [25] in the case of rigid tanks. Large amplitude sloshing has been also studied recently by Chen and his collegues [3] for tanks subjected to sever earthquakes. Soil-structure interaction has been taken into consideration again in a recent work by Malhotra [21] for unanchored tanks.

Most of the aforementioned studies have been performed for the anchored tanks. Nevertheless, some research have been also conducted for unanchored tanks, specially in recent years. In addition to studies of Barton [2] and Malhotra [21], some other researchers such as Haroun [8], Lau [18], Manos [22], and Peek [24] have been also worked on the behavior of unanchored tanks subjected to lateral or seismic loads. The use of unanchored tanks has not been recommended for seismic areas as the separation of tank walls and bottom from the foundation usually leads to heavy damages to the system in addition to the loss of content and environment pollution.

More recently the soil-structure interaction has been a matter of interest for some researchers. Course [4] has studied the energy dissipation due to this phenomenon for several systems including LNG tanks. Zou and Kong [29] have suggested a simplified method for seismic analysis of cylindrical tanks, in which the geometric parameters of tanks have been taken into consideration.

Although the soil-structure effect has been studied by several researchers, part of which were mentioned above, these studies have been mainly concentrated on soil modeling techniques, and less attention has been paid to the geometry of the tank foundation, namely its shape and dimensions, perhaps because it is generally thought that there is usually some limitation on the foundation design, which leads to almost particular shape and dimensions in each case.

In this paper a study has been performed on the effect of the geometry of the tank foundation on the modal properties of the tank-liquid-soil system for the case of anchored cylindrical steel tanks, in which both liquid-structure and soil-structure interactions have been taken into account. For this purpose a set of cylindrical steel tanks with various height over radius (H/R) and thickness over radius (t/R) ratios with two main different foundation geometries, namely square and circular in plan with different thicknesses have been considered. Several soil conditions have been employed varying from very soft (v_s = 110*m/s*) to very stiff (v_s = 900*m/s*). Both tank and foundation have been modeled in each case by finite elements using *SAP*-90 and *MATS* softwares, respectively. *MATS* program uses the Winkler model for soil springs and has the capability to omit the springs in tension to model the soil behavior realistically [5]. This kind of soil springs have been also used by Lau for studying the nonlinear behavior of tanks [19]. The first three modes of the tank system have been considered for modal characteristics calculations. As numerical results, the variation of the natural periods of the tank systems, both without the effect of soil-structure interaction and with that effect and the effect of foundation geometry, with respect to H/R as well as v_s values have been shown graphically. The results show the high sensitivity of the natural periods of the tank-liquid-soil systems to the foundation geometry, specially in the case of circular foundations and low values of v_s.

2. Study of the Foundation Geometry Effects

To study the effects of foundation geometry on the natural periods of the tank system a set of cylindrical steel tanks with various H/R and t/R ratios have been considered, with the general specifications as follows:

- Modulus of elasticity = 2.03E11kN/m^2
- Specific Gravity = 7.85$tonf/m^3$
- Poison Ratio = 0.3
- Water Specific Gravity = 1.00$tonf/m^3$

Four types of foundation as shown in Figure (1) have been employed for each tank. These include a square one having sides greater than the tank diameter (type I), which is the usual geometry for tank foundations, a square one having sides equal to the tank diameter (type II), a round one having a diameter greater than the tank diameter (type III), and finally a round one having a diameter equal to the tank diameter (type IV). Different values have been also considered for the thickness of foundations.

To consider the soil conditions, four different values have been assigned to the shear wave velocity, which

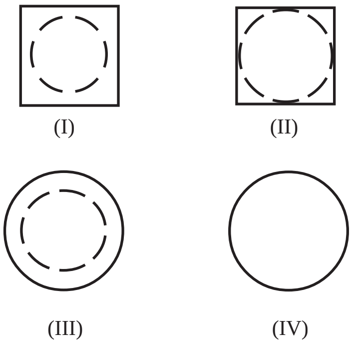

Figure 1. Various types of foundation used in the study.

are 110*m/s*, indicating very soft soil, then 160*m/s*, 250*m/s*, 450*m/s*, and 600*m/s*, for considering a wide variety of medium soils and finally 900*m/s*, which shows very stiff soil. The initial values for shear modulus of various types of soils have been calculated based on the shear wave velocity by using the following well-known equation

$$G = \rho . v_s^2 \tag{1}$$

in which ρ is the soil specific mass [5]. Then these values have been introduced to the *MATS* computer program for calculating the soil spring coefficients, to encompass the soil structure interaction.

To take into account the liquid-structure interaction and sloshing phenomenon, the method suggested by other researchers have been employed, which leads to consideration of an impulsive portion of the liquid in the lower part of the tank and some convective portions of liquid in the upper part. The added mass for considering the incorporation of impulsive portion of impounded water has been calculated using the formula proposed by Epstein [4] similar to the work of Barton and Parker [2], which is

$$M_{imp.} = \left[\frac{H}{\sqrt{3}R} tang\, h\left(\frac{\sqrt{3}R}{H} \right) \right] M \tag{2}$$

where M is the total mass of impounded water in the tank.

For the calculation of the modal characteristics the first three modes of the tank system have been considered. For this purpose the modal frequencies of the convective portion of the impounded water have been obtained by

$$f_{jc} = \frac{1}{2\pi} \sqrt{\frac{g}{R} \lambda_j \, tangh\left(\lambda_j . \frac{H}{R} \right)} \tag{3}$$

and the corresponding modal masses by

$$m_{jc} = \left[\frac{2}{\lambda_j^2 - 1} . \frac{1}{\lambda_j \frac{H}{R}} tangh\left(\lambda_j . \frac{H}{R} \right) \right] . M \tag{4}$$

and finally the corresponding heights for the calculation of overturning moments by

$$h_{jc} = \left[1 - \frac{1}{\lambda_j \frac{H}{R}} tang\, h\left(\frac{\lambda_j}{2} . \frac{H}{R} \right) \right] . H \tag{5}$$

In these formula $\lambda_j (\ j = 1 \rightarrow 3)$ are the roots of the first order Bessel function for the liquid vibration, and their values are

$$\lambda_1 = 1.8412 \quad \lambda_2 = 5.3314 \quad \lambda_3 = 8.5363 \tag{6}$$

Then by using the design spectra the related accelerations can be obtained, by which the maximum shear and overturning moments can be calculated as

$$V_c = \left[\sum_{j=1} \left(m_{jc} . A_{jc} \right)^2 \right]^{\frac{1}{2}} \tag{7}$$

$$M_c = \left[\sum_{j=1} \left(m_{jc} . h_{jc} . A_{jc} \right)^2 \right]^{\frac{1}{2}} \tag{8}$$

where A_{jc} is the spectral acceleration for mode j given by appropriate response or design spectra.

In most cases all of the first three modes are related to the lateral vibration of the tank as shown in Figure (2).

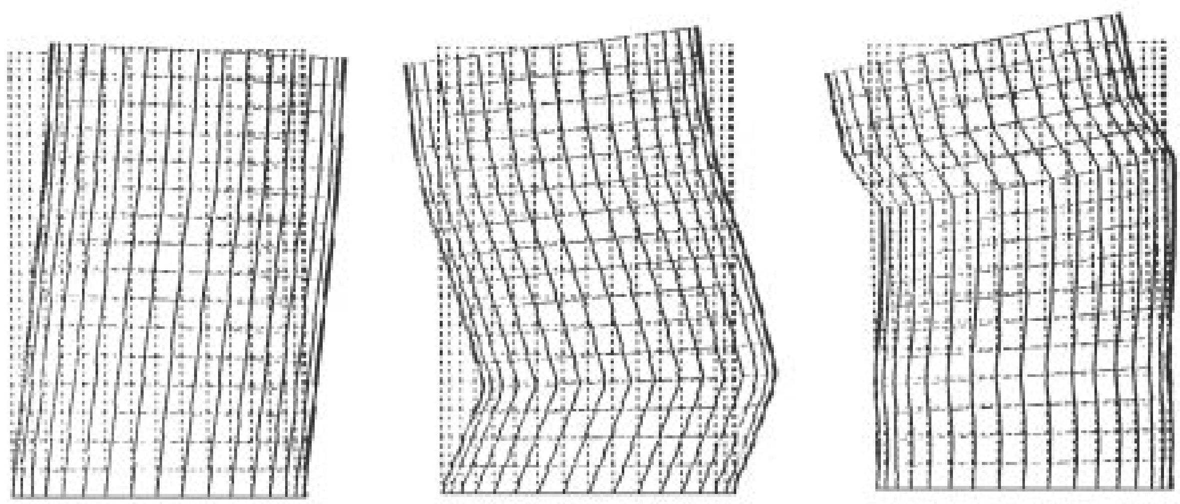

Figure 2. A sample of the first three mode shapes of cylindrical tanks under study in the case of rigid foundation.

Only for the case of very low rise tank the third mode may be related to the vertical vibration of the tank, depending on the *t/R* ratio. Furthermore, the cirumfrencial modes of the tank, which are of the $\cos n\theta$ type, are not usually excited by earthquake except for the first circumfrencial mode or n = 1. Therefore, the horizontal degrees of freedom in the tank wall has been constrained to each other for the calculation of first three modes. This may cause a little error in the case of very low *t/R* ratios, but generally has not any significant effect.

Modal characteristics of the tank-foundation-soil system have been obtained in the following procedure. At first, modal frequencies and modal shapes have been calculated for only the superstructure by assuming the foundation as a rigid base by using the *SAP*-90 program. Then by applying the forces and moments exerted on the foundation in each mode, its deformations have been computed by taking into account its flexibility as well as the soil deformability by using the *MATS* program. Successively, these deformations have been applied to the superstructure and its modal characteristics have been modified. By repeating this calculations in an iterative manner, as described by Mohajer [23], the final values of modal properties have been obtained by satisfactory precision.

For soil damping, which is mainly of the radiation type, the results of an investigation accomplished by Yamamoto and his colleagues have been used [28]. on this basis a value between 2% to 25% can be assumed for the tank-soil system depending on *H/R* ratio and v_s value. Similar values have been given for this purpose in reference [5] as well. The damping ratio decreases with increase in v_s value and for each value of v_s has its maximal in a particular value of *H/R* ratio. This particular value of *H/R* ratio, giving the maximum damping ratio, is usually around 0.65. For other values of *H/R* ratio, the damping ratio is much lower. Therefore, for simplicity, in this study an average value of 10% has been used for all cases.

3. Numerical Results

The ratio of two main values has been considered for most of the numerical calculations. These are modified and unmodified natural frequencies of the tank system. Modified natural frequency, denoted here by $\tilde{f}$, refers to the value for which both foundation flexibility and soil deformability have been considered. The unmodified natural frequency, shown by *f*, is then the value for which the foundation has been assumed as a rigid base. The variations of the $\tilde{f} / f$ ratio, called hereinafter the frequency ratio, with respect to various values of *H/R* as well as v_s, are shown in Figures (3) to (8) for *t/R* = 0.001.

In can be seen in Figure (3) that for low-rise tanks located on soft soils the frequency ratio is relatively low, which increases with increase in the height of the tank. Figures (4) and (5) show that the second and the third

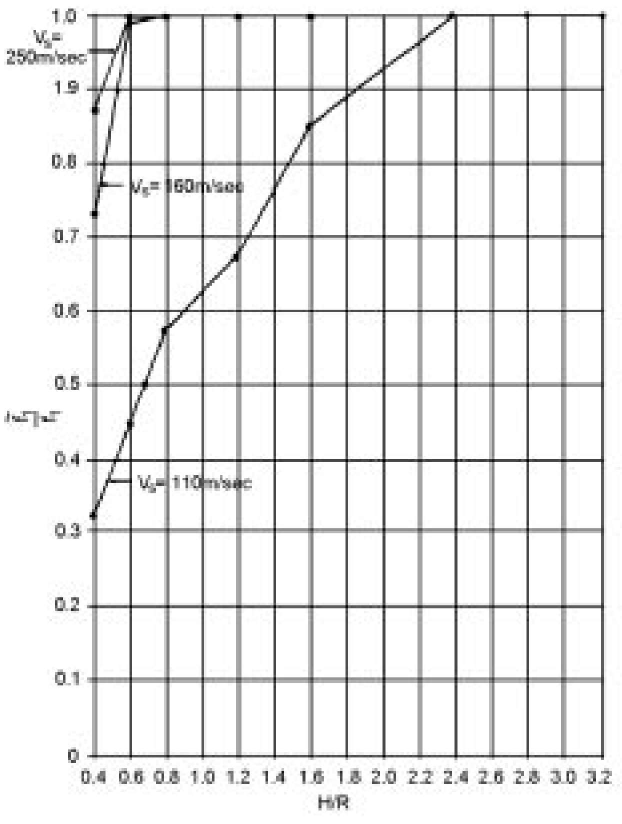

Figure 3. The variation of frequency ratio for the first mode of the tank systems with *t/R* = 0.001, having foundation of type (I) located on soft to medium soils.

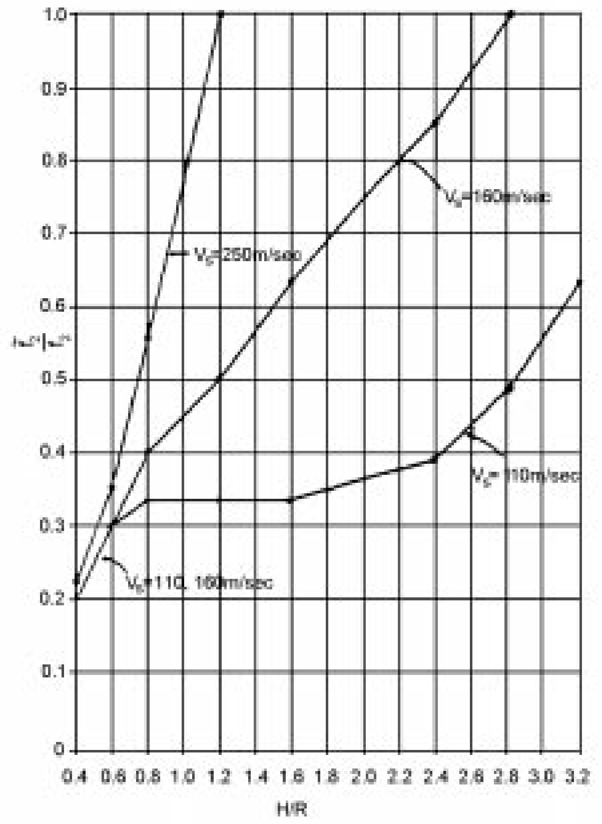

Figure 4. The variation of frequency ratio for the second mode of the tank systems with *t/R* = 0.001, having foundation of type (I) located on soft to medium soils.

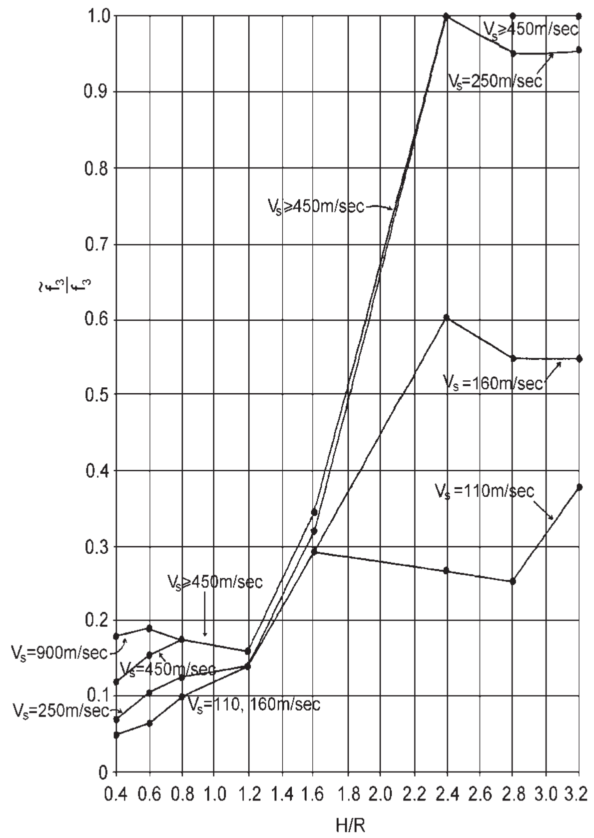

Figure 5. The variation of frequency ratio for the third mode of the tank systems with t/R = 0.001, having foundation of type (I) located on soft to stiff soils.

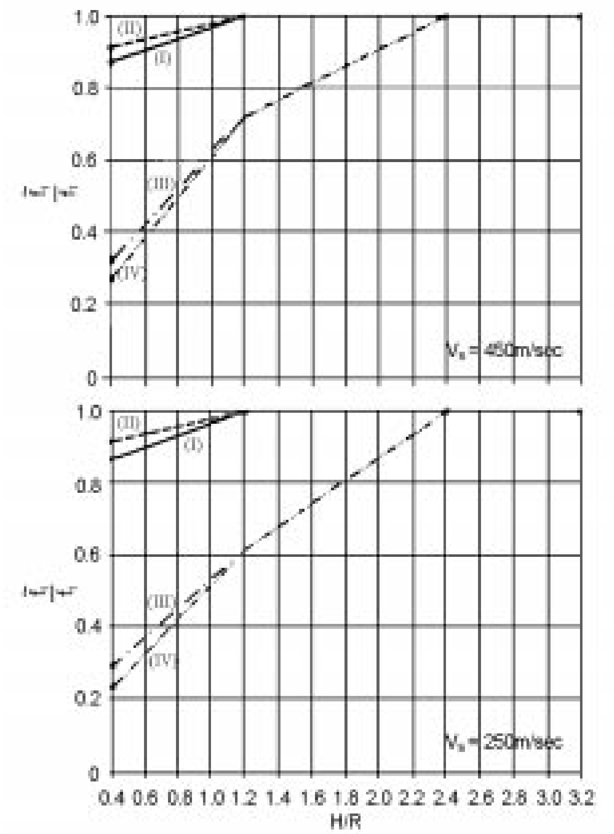

Figure 7. The effect of foundation geometry in plan on the frequency ratio of the first mode for tank systems with t/R = 0.001 located on medium soils.

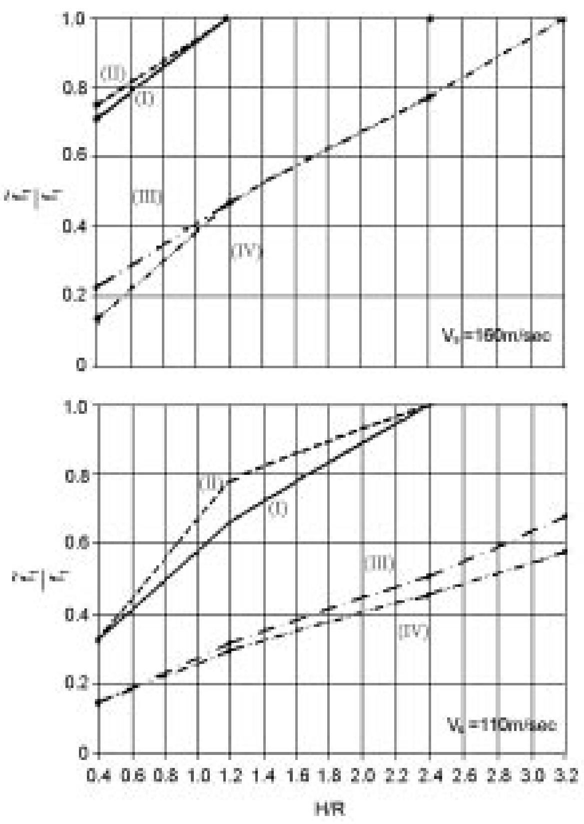

Figure 6. The effect of foundation geometry in plan on the frequency ratio of the first mode for tank systems with t/R = 0.001 located on soft soils.

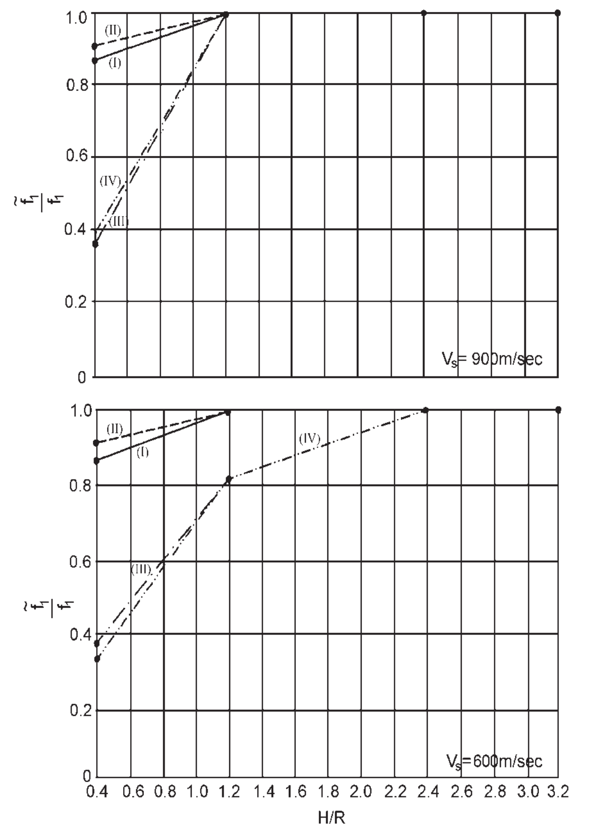

Figure 8. The effect of foundation geometry in plan on the frequency ratio of the first mode for tank systems with t/R = 0.001 located on stiff soils.

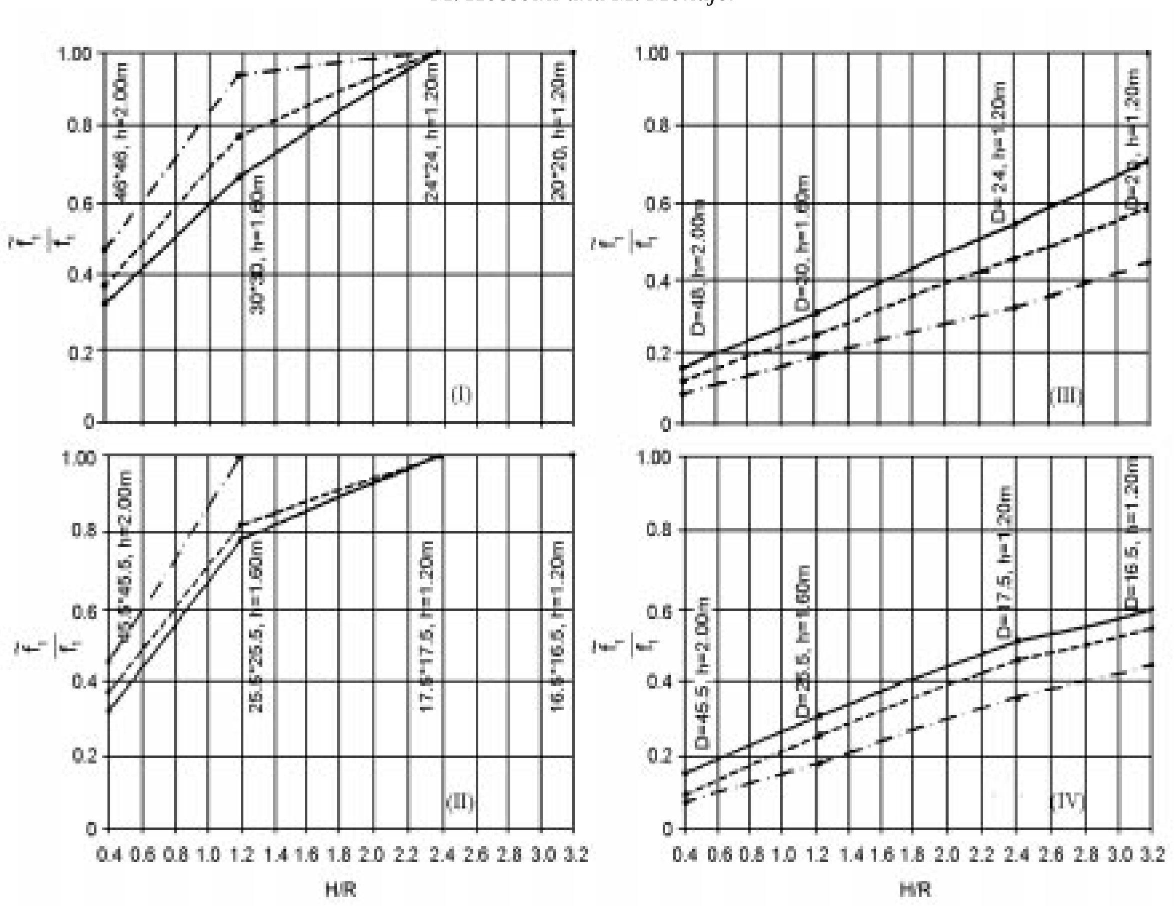

Figure 9. The effect of foundation thickness on the frequency ratio for various types of foundations located on soft soil.

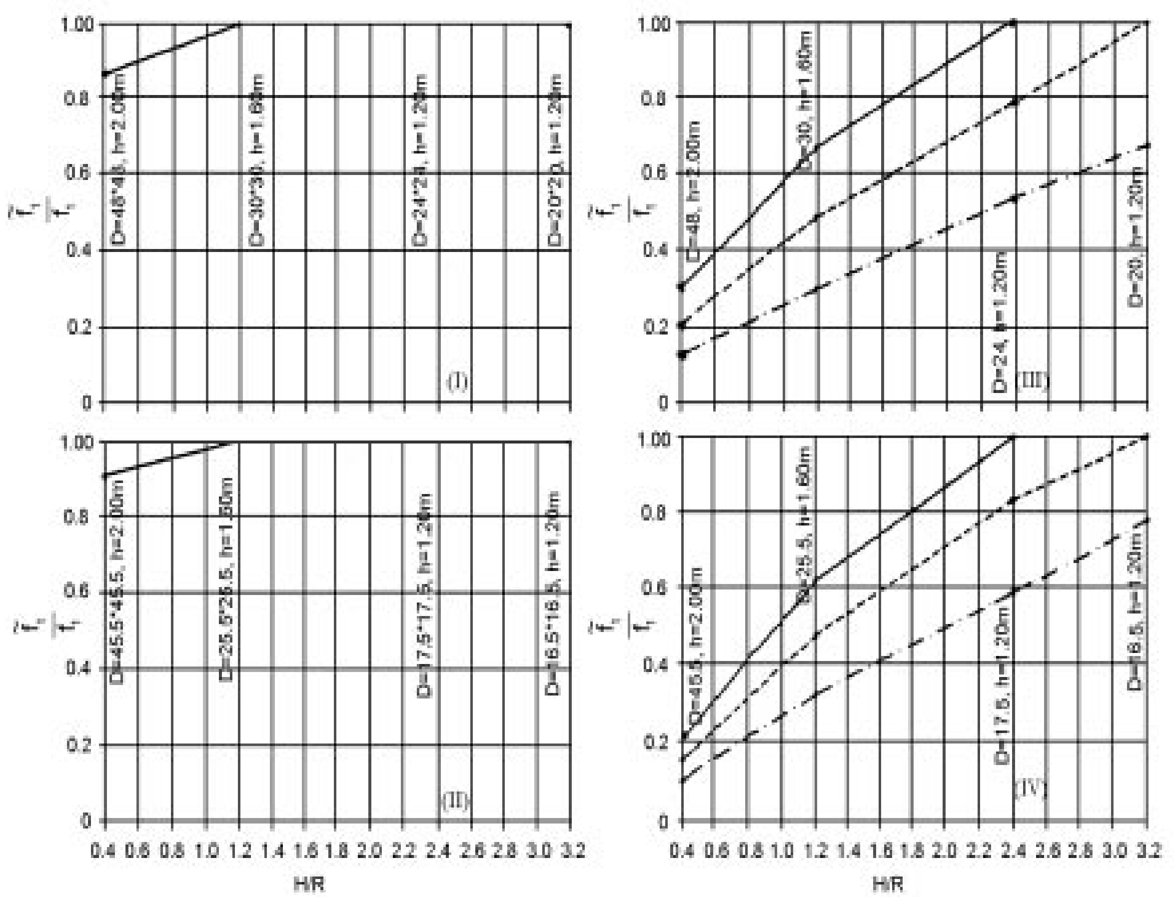

Figure 10. The effect of foundation thickness on the frequency ratio for various types of foundations locatd on medium soil.

modes are more sensitive to the foundation flexibility and soil deformability.

Figure (6) shows the effect of the foundation shape in plan on the first frequency of the tank systems, located on various types of soils. It can be seen in this figure that the round foundations can decrease extensively the modified frequency of the tank systems, specially for low-rise tanks, located on soft soils. Figures (7) and (8) show that the effect of foundation shape in plan decreases slightly by increase in v_s value.

Figures (9) and (10) show the effect of the thickness of the foundation, implying somehow its flexibility, on the frequency ratio of the tank systems for v_s = 110m/s and v_s = 250m/s, respectively. In these figures the heavy lines are for the thickness of foundation shown in the body of figures for different sizes of foundations, the dash lines are for the case in which the thickness of foundation is 75% of the initial value, and the third group of lines are for the case in which the thickness of foundation is 50% of the initial value. It can be seen in Figures (9) and (10) that the modified frequencies are much less than the unmodified corresponding ones again in the case of round foundations, while this decrease can only be seen for the low-rise tanks in square foundations cases. These figures also show that the modified frequency decrease with the decrease in of the foundation thickness. Finally, it can be seen that in the case of square foundations locating on medium to stiff soils the difference between modified and unmodified frequencies is almost negligible unless for very low values of *H/R* ratio.

4. Conclusions

Based on the numerical results it can be concluded that

- The natural periods of the tank systems are quite sensitive to the foundation geometry, namely its form and dimensions.
- Any decrease in the foundation size in plan or its thickness leads to an increase in the natural periods of the tank system.
- The period sensitivity is much higher in the case of circular foundations, especially for lower *H/R* ratios and lower v_s values, namely softer soils.
- This sensitivity can be seen in the natural periods of higher modes as well as the first mode of the tank system; sensitivity of higher modes is also more extensive in soft soils.

On this basis, it can be said that by choosing appropriate shape and proper values for foundation dimensions it is possible to make the period values even served 8 times longer. Therefore, use of foundation geometry can be a good tool for controlling the period of the whole tank-liquid-foundation system in earthquake prone regions to make it far from the dominant frequency of the site.

Finally, it should be noted that for the case of very soft soil it is likely to use the pile foundations. To extend the results of this study to the case of pile foundations more investigation should be performed. Furthermore, to check the accuracy of the range of frequency ratio variations, especially because of the frequency-dependance nature of soil stiffness and damping properties, some further research is required.

References

1. Aslam, M., Goodden, W.G., and Scalise, D.T. (1979). “Earthquake Sloshing in Annular and Cylindrical Tank”, *Journal of Engineering Mechanics Division, ASCE*, **105**, 371-389.

2. Barton, P.C. and Parker, J.V. (1987). “Finite Element Analysis of the Seismic Response Anchored and Unanchored Liquid Storage Tanks”, *Earthquake Engineering and Structural Dynamics*, **15**, 299-322.

3. Chen, W., Haroun, M.A., and Liu, F. (1996). “Large Amplitude Sloshing in Seismically Excited Tanks”, *Earthquake Engineering and Structural Dynamics,* **25**, 653-669.

4. Course, C.B. (2000). “Energy Dissipation in Soil-Structure Interaction”, *Proc. of the 12WCEE, New Zealand*, Paper No. 366.

5. Dowrick, J.D. (1987). Earthquake Resistant Design for Engineers and Architects, 2nd ed., *John Wiley & Sons*, Singapore.

6. Epstein, H.I. (1976). “Seismic Design of Liquid Storage Tanks”, *Journal of Structural Engineering Division, ASCE*, **102**, 1659-1673.

7. Haroun, M.A. (1983). “Vibration Studies and Test of Liquid Storage Tanks”, *Earthquake Engineering and Structural Dynamics*, **11**, 179-206.

8. Haroun, M.A. (1988). “Seismic Behavior of Unanchored Ground-Based Cylindrical Tanks”, *Proceedings 9th of the World Conference on Earthquake Engineering*, Japan, **3**, 643.

9. Haroun, M.A. and Ellaithy, H.M. (1985). “Model for Flexible Tanks Undergoing Rocking”, *Journal of Structural Engineering Division, ASCE*, **111**, 143-157.

10. Haroun, M.A. and Housner, G.W. (1981). “Seismic Design of Liquid Storage Tanks”, *Journal Tech. Council, ASCE, TC1*, 999-207.

11. Hopkins, L.M. and Jacobsen, L.S. (1934). “Water Pressure in a Tank Caused by Simulated Earthquake”, *Bell. Seismic. Sec. Am.*, **24**.

12. Housner, G.W. (1957). "Dynamic Pressures on Accelerated Fluid Containers", *Bull. Seism. Soc. Am.*,**7**(1), 15-37.

13. Housner, G.W. (1963). "The Dynamic Behavior of Water Tanks", *Bull Seism. Sec. Am.*, **53**(2), 389-387.

14. Housner, G.W. and Haroun, M.A. (1979). "Vibration Tests of Full Scale Liquid Storage Tanks", *Proc. and US National Conference on Earthquake Engineering*, Stanford, California, 137-145.

15. Housner, G.W. and Haroun, M.A. (1980). "Dynamic Analyses of Liquid Storage Tanks", *Proc. of the 7th WCEE*, Istanbul, Turkey, **8**, 431-438.

16. Jacobsen, L.S. (1949). "Impulsive Hydrodynamics of Fluid Inside a Cylindrical Tank and of a Fluid Surrounding a Cylindrical Piers", *Bull. Seism. Soc. Am.*, **39**.

17. James, Ray W. and Roba, Gray W. (1991). "Behavior of Welded Steel Water Storage Tank", *Journal of Structural Engineering Division, ASCE*, **117**(1).

18. Lau, D.T. (1992). "Dynamic Uplift Analysis of Unanchored Cylindrical Tanks", *Proceedings of the 10th World Conference on Earthquake Engineering*, Spain, **9**, 5011.

19. Lau, D.T. (1995). "Nonlinear Behaviour of Bottom Plate in Cylindrical Liquid Storage Tanks for Seismic Applications", *Canadian Journal of Civil Engineering*, **22**(1), 180-189.

20. Lay, Khai Seong (1993). "Seismic Coupled Modeling of Axisymmetric Tanks Containing Liquid", *Journal of Structural Engineering Division, ASCE*, **119**, 1747-1761.

21. Malhotra, Parveen K. (1997). "Seismic Response of Soil Supported Unanchored Liquid Storage Tanks", *Journal of Structural Engineering Division, ASCE*, **123**(4), 440-450.

22. Manos, G.C. (1992). "Study of the Vibratory Characteristics of Unanchored Cylindrical Liquid Storage Tanks Models", *Proceedings of the 10th World Conference on Earthquake Engineering*, Spain, **9**, 4969.

23. Mohajer, M. (1997). "Seismic Response of Cylindrical Liquid Storage Tanks Considering Soil-Tank-Liquid Interaction", (In persian) Master Thesis Submitted to Hydraulic Structures Group, Civil Engineering, Dept., Graduate School, South Branch of the Islamic Azad University, Tehran, Iran.

24. Peek, R. (1988). "Analysis of Unanchored Liquid Storage Tanks under Lateral Loads", *J. Earthquake Engineering Structure Dynamic*, **16**, 1087-1100.

25. Veletsos, A.S. and Shivakumar P. (1993). "Sloshing Response of Layered Liquids in Rigid Tanks", *Earthquake Engineering and Structural Dynamics*, **22**(9), 801-821.

26. Veletsos, A.S. and Tang, Y. (1987). "Rocking Response of Liquid Storage Tanks", *Journal of Structural Engineering Division, ASCE*, **113**(11).

27. Veletsos, A.S. and Tang, Y. (1990). "Soil-Structure Interaction Effects for Laterally Excited Liquid Storage Tanks", *Earthquake Engineering and Structural Dynamics*, **19**, 473-496.

28. Yamamoto, S., Kawano, K., Shimizu, N., Umebayashi, S., and Yamagata, M. (1985). "Radiation Damping of Cylindrical Liquid Storage Tnak Resting on Elastic Body", *Proc. of 8WCEE, Sun Fransisco*, **5**, 231-238.

29. Zou, D. and Kong, X. (2000). "A Simplified Seismic Response Analysis Method for Cylindrical Liquid Storage Tanks", *Proc. of the 12th WCEE, Auckland, New Zealand*, Paper No. 0486.