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A Poisson Process Hidden Markov Cellular Automata Model in Earthquake Genesis and Conflict Analysis: A Physical Approach

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ABSTRACT

All large-scale events, like natural disasters and conflict mechanisms follow certain endogenous and exogenous interactions, which are still unclear to geoscientists by normal surface observations. It has been established that complex scale analysis provides a direct insight into the precursory framework for occurrence of endogenous and exogenous events related to large-scale dynamics. The present study is based on the mathematical derivation of a theoretical framework for the process in Earth system dynamics using Poisson process and Markov analysis to identify the endogenous and exogenous stress distribution and their redistributions beneath the sub-surface earth. The transition probability matrix derived for the optimal state sequences of the Markov model, which is relevant in analysis of shock wave of a complex system. This study validates the concept of exogenous events using sand pile cellular automata system, an approach to study complex behavior of multi-component system. In this paper, Poisson hidden Markov model implemented on a continuous space of sand pile behavior shows that seismogenesis and conflicts occur due to accumulating stress, representing disequilibria in energy and interactions between active agents (faults and heterogeneities) in which the stress may find its release through the onset of a tremor. Earthquake occurrence in the critical state or a critical shock due to the contributor states or conflict in societal analysis is not adequately known. Subsequently, the influence of each contributor tries to come back into stasis (meta-stable state or stable equilibrium state), the spatial system position of earth dynamics or conflicts through a series of tremors (aftershocks) or post earthquake responses.

Keywords:
Endogenous and Exogenous interactions; Earthquake genesis; Conflict; Avalanche; Emergence; Poisson; Markov; Transition probability matrix

1. Introduction

The process of complex system analysis to validate processes occurring in the system Earth characterized by complex spatio-temporal dynamics [1], has become a challenging topic of research to get deep insights into the earthquake generating processes by deploying various tools of multi-disciplinary science and technology available with the world [2]. It is always hard to establish clear-cut assessments of the causal roles that individual drivers play in conflict oriented situations that may result in a pre-determined change in the system. A systematic study for earthquake physics involves analysis of occurrence of events that evolves through multi-scale structures in a dynamical network of relations and mutual interrelations. Specifying every state and transition in the state space of even a moderately complex Markov process with hundreds of states is an infeasible task. The natural process of earthquake occurrence involves non-linear time delayed process that requires redistribution of stress in earthquake occurrence. However, the models of geophysics fail to propose a clear-cut model that might effectively interpret the chain of causality process interacting in the geodynamic process, and to ensure an easy formulation of the predictability of the system behavior. The present study is a way forward to assess the system by drawing resemblances with natural occurring phenomena and societal viewpoints, processes at work in the Earth's crust, where stresses build up slowly to be released in sudden earthquakes as per the Reids Elastic Rebound theory. The earth system mechanisms can be understood in the likelihood of an egg kept in a pressure cooker in boiling hot water. The surface of the outer shell of the egg is highly fragile and cracks start showing up in meta-stable conditions as the temperature of water rises. An hourglass calibrated alongside to analyze the time sequence of processes that eventually lead to catastrophic events like earthquakes serves as a time scale measure of the global coupling effects that take place in earth system. The shell breaking and the internal fluid of egg are two components that interact among themselves in non-equilibrium states. In order to address the issue of understanding the earthquake mechanism, we need to assess some dynamic mechanisms that help analyze critical state of a system in non-equilibrium states with many interdependent and diverse components that interact locally resembling the earthquake initiating process [3]. At some point, these pressures release their accumulated energy with catastrophic effect, creating shock waves. It has been earlier observed by Cederman [4] that the dynamic analysis of shock waves can provide the analogies between earthquakes and conflicts. Earthquakes and conflicts are complex systems, exhibiting emergent features associated with critical states. A quantitative signature of precursory (endogenous) behavior helps recognize and then reduce growing conflict. Many studies have tried to interpret physical interpretation of earthquake genesis mechanism with precursor activity [2, 5] to shed light on sharp and sudden emerging behavior associated with critical states in geodynamic system. In order to understand the complex dynamics involved in the exogenous and endogenous behavior of the dynamic system, there is a need to establish the interplay dynamics between these effects. These components can be realized as sand grains self-organizing to form a sand pile [6]. Knopoff [7] was the first one who analyzed the earthquake sequence as a Markov chain for the first stochastic model for earthquake occurrence. The proposed work deals with the study of emergent properties can themselves interact with each other in the rupture stress accumulation dynamics through sand deposition as per the methodology described by Winslow [8]. This study provides an analytical validation by deriving properties of a sand pile model to respond to the questions raised by Meier [9] to identify the exogenous and endogenous behaviors in earthquake physics and conflict analysis. In our study, a Poisson process reflects the exogenous process involved in stress dynamics, and Markov process reflects the endogenous interactions for an earthquake system. The fixed energy sand pile responses to endogenous and exogenous processes quantified as the fluctuation-dissipation theorem of energy states in statistical mechanics. The sudden drop of sand grains through pile can be representative of the stress drop that occurs prior to an earthquake where each sand grain site plays a definitive role based on density of the pile. Sudden drop from the sand pile is an emergent phenomena [10] having interactive forces in earthquakes and conflict system [11] analysis. The paper provides a mathematical basis for the endogenous and exogenous process and provides a step-by-step approach to study associated dynamics in a large-scale system through a computational framework. The paper is structured as follows:

In Section 2, we evaluate the mathematical models associated with endogenous and exogenous processes for sand pile process where, for section 2.1 and 2.2, a deterministic prediction of emergent features using point probability that can occur from a single source and multi sources in a sand pile in continuous time interval for an exogenous process, respectively. In section 3 and 3.1, analysis of the different Markov states for endogenous changes in the sand pile for transition probability matrix of the recurrent energy state of the sand grain is derived, respectively. In section 4, we calculate transition probability matrix and computational model analysis. Section 5 concludes the paper on future work about using more real life constraints to the sand pile architecture.

2. Mathematical Model

An evaluation for the physical processes to earthquake genesis for a single fault based on past fault behavior can be the same as the correlation between earthquake magnitude and parameters of fault trace based on dislocation theory. $X(t)$ is a Markov process, having the Markov or memory less property: given the value $X(t)$ of the system at some time t, its future evolution depends only on the current state and not on the knowledge of history. In case of initial distribution period as t_0 increases in calculating the probability distribution over the state space at an arbitrary instant $t_0 < ... < t_{n+1}$: $\Pr(X(t_{n+1}) = x_{n+1} | X(t_n) = x_n, \ldots, X(t_0) = x_0) = \Pr$ $(X(t_{n+1}) = X_{n+1} | X(t_n) = x_n)$. The main objective with respect to a Markovian model is to calculate the steady state probability distribution that is the probability distribution of the random variable $X(t)$ over the state space *S*, as the system achieves a regular pattern of behavior. Each state of the process is a node in a graph; the arcs represent possible transitions between states, and are labeled by their respective rates (parameters of the exponential distributions determining the transition durations) *n* states has a one step $n \times n$ transition matrix *P* with elements p_{ij}. From the q_{ij}, we have derived the exit rates, q_i, and the transition probabilities, p_{ij}.

The generalized semi-Markov models [12] take into account time and space dependencies of large earthquakes. The study shows the graph of occurrence of events having a time series of conflict events (frequency) of earthquake occurrence over a particular period of time. The study reveals the event occurrence in the macro-scale for transition threshold of event occurrence. The transient state distribution of a stochastic transition probability matrix $P\{X(t), t \in R\}, pi \sim j (\sim j$ represents a set of states), is the probability that the process is in a state in p_j at time *t*, given that it was in state *i* at time 0 for mathematical validation of Poisson process and Markov model. Sand grains drop at the source for a sand pile in a continuous probability distributions, drop of the sand grain from sand pile by avalanche decay tends to follow a simple Poisson point process for a single source and multi source events of the sand pile. Any sand-pile deformation process shows that time delay before a transition from *i* to *j* is exponentially distributed with a parameter q_{ij} for instantaneous transition rate. This transition in the sand pile responds in the form of shock waves [13] that occurs with the variance of stationary active site energy density originating from zero. Inference for large-scale system can be explained as transition in the sand pile responds in the form of shock waves sites as density increases at points from zero. The exogenous process exhibited by the system can be similar to a sand pile system. This shock wave can be analogous to an earthquake occurrence in the critical state and can be explained from the Poisson source having λ rate of drop [14], see Figure (1).

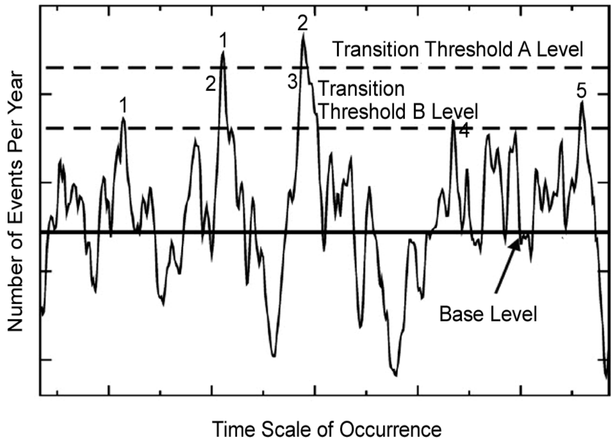

Figure 1. Statistical validation of precursory signature using arbitrary thresholds.

The study identifies few time-changing components in a sand-pile architecture. We have assumed a model for all events, P_p the probability of a point process in Δt time for sand grain drop, and *P* the associated event occurrence of avalanche occurrence probability in ΔT time, where *N* is the number of sand grain drop in ΔT time. Expected number of all earthquakes for the intervals involved would be some factor $N_0 = R_0 \Delta T P_p(M, \Delta M)$. A Poisson random process for a random sand grain drop can be characterized by an exponential distribution function that drops off sharply above an e-folding size scale [15].

$$N_\lambda = R \Delta T \{\lambda^k / k!\ e^{-\lambda}\} \quad \text{where} \quad k = M / \Delta M \qquad (1)$$

The factor $k!$ in the denominator is the number of ways in which the same multiple event can occur resulting in avalanche. By knowing the probability that sand grain drop will occur in an interval of time of width Δt for an avalanche is $r\Delta T$, we evaluate the distribution of the drop condition in some larger interval ΔT.

In the first interval, the sand grain fall occurrence is *p*. In the second interval of continuous probability distribution, it can be given by $(1-p)\times p$, where $1-p$ states does not occur in the first interval but is occurring in the second interval Δt of time ΔT.

In the third, it is $(1-p)\times(1-p)p$ and so forth. Thus, the event occurrence probability P dependence on the p point process is a partial sum of a geometric series and the total is the difference between two infinite sums.

$$\begin{aligned} p &= [p+(1-p)\,p+(1-p)^2\,p+(1-p)^m\,p- \\ &(1-p)^{m+1}p-(1-p)^{m+2}\,p] \text{ where } m=\Delta T/\Delta t. \\ &= p[1+(1-p)+(1-p)^2+...-(1-p)^m- \\ &(1-p)^{m+1}-(1-p)^{m+2}\,p....] \text{ where } m=\Delta T/\Delta t. \\ &=[1-(1-p)^{\Delta T/\Delta t}] \end{aligned} \quad (2)$$

This shows that the distribution of sand grains in the time interval ΔT is a geometric progression for point process events occurring in Δt time. Point process probability (q) for sand grain does not fall in the infinitesimal time period Δt and Q that the sand pile has no change in ΔT time interval.

$$Q = p^{\Delta T/\Delta t} \quad (3)$$

2.1. Continuous Probability for a Single Event in a Sand Pile

The lattice energy state for avalanche for the sand pile in the stationary state for a single event will occur in a time interval, dt, is $dP = Q\,r_s\,dt$ where $Q = 1-p$ is the probability that the lattice energy of the grain may vary from the energy at the state Q to 1 to occur in time ΔT. λ is the Poisson distribution that a single event occurs in the given time interval and can range from 0 to 1.

$$Q = p^{\Delta T/\Delta t}$$

$$p = r_s\,\Delta T$$

$$\Delta Q = q(\Delta t+\Delta T)/\Delta t - q(\Delta t+\Delta T.)$$

$$= -q\Delta T/\Delta t(1-q)$$

$$= -q\Delta T/\Delta t.p$$

$$\Delta Q = -Q r_s\,\Delta t$$

$$dQ = -Q r_s\,dt$$

The system behavior says that the avalanche will collapse completely or either collapses with a rate $r_s\,dt$ in the time period *t*.

$$\int_1^Q 1/Q\,dQ = -\int_0^t r_s\,dt$$

$$\ln Q = r_s\,t$$

$$Q = e_s^{-rt} \quad (4)$$

$$P = 1-e_s^{-rt}$$

$$dP = e^{-rt}\,r_s\,dt = Q.r_s.\,dt$$

2.2. Multiple Events from a Single Source in the Sand Pile

The emergence phenomenon is a bottom-up process for avalanche decay. The avalanche decay initializes at Δt time interval and proceeds towards a stable state as it reaches ΔT for a stationary sand pile. The single event decay, see Eq. (4), may vary with probability distribution for a simple Poisson "counting" process [16] where multiple events may occur from the same source. The probability of observing *k* events in an interval of time *k* dimensions in which the time of observation of each *k* events is represented by an axis. The P_k is easily calculated in nD, a space with arbitrary number of dimensions on the sand pile system. For P_k, the intervals for the avalanche phenomena can be taken as independent random variables drawn from an exponentially distributed population, population with the density function $f(x) = le^{-1x}$ for some fixed constant. Each time the probability that the event happens at time t_k is the probability that it does not happen for an infinitesimal time interval Δt. We end up integrating higher powers of the same function over and over again and can deduce a general expression for P_k.

$$P_k = \int_0^{\Delta T} \int_{\Delta t}^{\Delta T} \left(\int_{\Delta t}^{\Delta T} (Qr\,dt)\,Qr\,dt \right) Qr\,dt$$

For a single event occurrence from Eq. (4):

$$P_k = \int_0^{\Delta T} \int_{\Delta t}^{\Delta T} \left(\int_{\Delta t}^{\Delta T} (dP)\,dP \right) dP$$

P' is the probability of single event observed within interval for change in the sand grain and P_k is the probability of observing *k* events.

$$P_1(t) = \int_{\Delta t}^{\Delta T} dP = P(\Delta T) - P(\Delta t) =$$
$$-|P(\Delta t) - P(\Delta T)) = -P'(t)$$

$$-P'(t) = P(\Delta t) - P(\Delta T)$$

$$dP' = dP$$

$$P_2(t) = \int P'\, dP' = [P'^2/2]_{\Delta T}^{t}$$

$$P_k(t) = -[P'(t)/k!]_{\Delta T}^{t} = [P(\Delta T) - P(t)]^k / k!$$

Initial system of probability $= [1 - e^{-r\Delta T}]^k / k!$

$= Pk/k!$

$$\sum P = \sum_{k=0}^{\infty} P^k = \sum P^k / k!$$

It is easily seen that the probability distribution for multiple observations is the Poisson distribution. The expected number of occurrences for *k* events by the time *t* is given by the integral.

$$E(k) = \sum_{k=1}^{\infty} kP^k/k! = e^{-\lambda t}\sum_{k=1}^{\infty} k(\lambda t)^k/k! =$$
$$e^{-\lambda t}.\lambda t\sum_{k=1}^{\infty}\lambda t^{k-1}/(k-1)! = \lambda t \tag{5}$$
$$= p\sum_{k=1}^{\infty} P^{k-1}/(k-1)! = P.\sum P^k/k!$$

$$E(k) = P.e^P = 1 - e^{-r\Delta T} \qquad 0 \le E(k) \le e \tag{6}$$

This single grain of sand in the previous state results in the present avalanche, which can be explained by a Markov process. It is a challenge to identify the precursory states and the after-shock distribution based on the precursor states and the imminent occurrence of the critical state that precludes the avalanche to anticipate the possibility of a collapse.

3. Markov Model for Earthquake Stress Drop Analysis

A sand pile endogenous behavior has been described by a Markovian dynamics. The sand pile has a few endogenous elements interacting among themselves having each element dependent on the rate *r* function of the transition probability matrix represents the probability that the state is *j* at the time $t+1$, given that the state was *I* at the time *t*, and the probability can be written as: $p_{ij} = Prob[X_{t+1=j}|X_{t=i}]$ for a stationary sand pile. The $P(X_{n+1} = x_{n+1}|X_n = x_n)$ describe the one-step transition probabilities of a DTMC, that is then probabilities that the DTMC moves from state x_n to state x_{n+1} in a single transition. These values can be organized in a stochastic transition probability matrix *P*, the elements p_{ij} of which are defined as: $p_{ij} = P(X_{n+1=j}|X_{n=i})$ under the conditions that for all $i, j \in S, 0 \le p_{ij} \le 1$ and $p_{ij} = 1$. Equilibrium (point or trajectory) for a system that is at least locally stable, if the state ever gets "sufficiency close" to this equilibrium. The "sufficiently close" region is called the basin of attraction of the equilibrium. Emergent phenomenon is a bottom-up process that occurs with local interactions of sand grain distributed this equilibrium for interdependent spatial structures.

$$f(A_l|\lambda_l, N_l) =$$
$$\int_{\Phi_{\lambda l, Nl}} f(A_l|\lambda_l, N_l, \Phi)\,\rho(\Phi|\lambda_l, N_l)\,d\Phi \tag{7}$$

$$f(A_l|\lambda_l, N_l, \Phi) = \prod_{O\in A_l} f(O|\lambda_l, N_l, \Phi) \tag{8}$$

Φ_{λ_l, N_l} Hidden Markov Models triple the number of regional space parameters of the likelihood function $\rho(\Phi|\lambda_l, N_l)$.

Attractor conditions for the energy states for the growth of the sand pile where p_{ij} is the decay probability and q_{ij} is the growth probability of sand pile in normal condition. In a condition of self-organized criticality, a system exhibits a property of "homeostasis" [17] as the system returns back to the original state of energy under the effect of local stress. Sand grain can transit from a highly integrated state to a highly segregated state in response to a small local disturbance, which affects the activity of the system. The phase transition occurs in between set of states, $S = (s_1;\ s_2;\ s_3)$ for the sand grain system. The process is irreducible: all states in *S* can be reached from each other by following a path of transitions. The process starts in one of these states and moves successively from one state to another. If the chain is currently in state s_i, then it moves to state s_j at the next step with a probability denoted by f_k for the sand pile, and this probability does not depend upon which states the chain was in before the current state. In the present study, a continuous time Markov model is assumed having *N*, number of occupancy states, *r* is the probability of the active sand grains to sustain shift of occupational energy of the sand grain at f state transition probability of the sand grain that cannot sustain the shift in energy state, g is associated gain function to compute the fraction of numbers of sand grains in the states r. For a sand grain in critical state condition where every sand grain has a change of energy state, we can establish that the system continues to be in the active state with the shift of the occupational active state energy or undergoes transition to fall into a lower state of energy depending on the state requirements. From the law of conservation of energy for any occupational state, it is evident that for any state $r_k + f_k = 1$. In the sand pile grain lattice energy states can be categorized as either active (1), semi-active (2) or dormant (3) states, which act as the attractor states for the sand grains. States diagram, see Figure (2), is labeled 1 through 3 having three levels of occupational states. In a given interval of time, a fraction of the states, r, will produce sand grain decay while the remainder f will undergo transition to the next higher vacant state of energy. The exceptions are the initial and final states. Fraction of states r will produce grain drop and remaining f goes to the next higher states. We evaluate the transition matrix to evaluate the change of gain between two upper sequential states of the sand grain that is the sub-critical state after which the sand grain drops. Change in the sand grains after self-organization can be given as:

$$\Delta N_1 = rN_1 + rN_2 + N_3 + rN_1 - (1-r)N_1$$

$$\Delta N_2 = (1-r)N_1 - rN_2 - (1-r)N_2$$

$$\Delta N_3 = (1-r)N_2 - N_3$$

$$\Delta N = N' - N$$

$$N' = N + \Delta N$$

$$\Delta N = DN$$

$$N' = N + DN = (I + D)N = T(N)$$

For the energy state in k[th] level, $N'_k = N_k - rN_k + f_{k-1}N_{k-1}$.

For instance, the condition of a system in state 1

$$\Delta N_k = N'_k - N_k = -r_k N_k + f_{k-1}N_{k-1}$$

$$N_k = f_{k-1} / r_k N_{k-1}$$

Energy level of state $E_k = r_k N_k = r_k \, f_{k-1} / r_k . N_k$ (9)

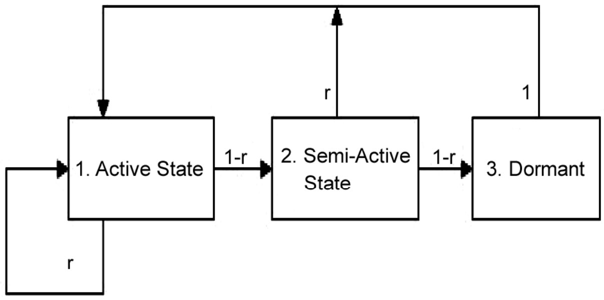

Figure 2. A simple Markov process for three level stress drop.

Energy level of the 2[nd] state defines the gain function of the previous state. Discrete gain functions for the previous occupational states have been evaluated.

$$g_k = N_k + 1/N_k \tag{10}$$

$$r_k = 1/1 + g_k \tag{11}$$

$$f_k = g_k r_k \tag{12}$$

3.1. Transition Matrix for a Continuous Time Markov Model

$$T[n] = \begin{matrix} r & r & 1 \\ 1-r & 0 & 0 \\ 0 & 1-r & 0 \end{matrix}$$

Energy state [18] of sand piles with height restrictions exhibit a phase transition due to gain of energy at the sequential states at the active phase sites as the particle steady state density z is increased beyond a critical value. A work on statistical mechanics by [19] has been used to explain the mathematical framework for the gain function between two sequential states of a Markov process. For a critical threshold λ as the spreading rate of sand pile with activity at the critical level λ_c of the active site, the activity density varies as $(\lambda\text{-}\lambda_c)p$. The survival probability of the sand pile $P(t)$ varies as $(\Delta t)^q$ while mean number of events grows as $(\Delta t/\Delta T)^q$ is the condition that the system will not respond to avalanche breakdown. In this system, the level gain will be dependent on $P(t)/(\lambda/\lambda_c)^p$. The system decays with the ratio of $\Delta t/\Delta T$, so knowing the gain function we can estimate the change of states from f_k to r_k.

4. Results and Analysis

Any set of forecast values given as $f = (f_1, f_2, ...,$

f_m), and that the set of observed values is: $x = (x_1, x_2, \ldots, x_n)$ characterizing the joint distribution of forecasts and observations: $p(f_i, x)$ for a continuum state model needs a joint distribution that includes observations as well as their mutual association. Defining the matrix p_{ij} as the joint relative frequency of forecast f_i and observation x_i, the dimension of this matrix is the product of m and n less one, since the sum of all entries is constrained to be unity. In the deterministic forecasting illustration the matrix dimension has been set to three which is minimal. Assuming $r = 0.3$ for the state transition matrix:

$$T(n) = A = \begin{array}{ccc} N_1 & N_2 & N_3 \\ 0.3 & 0.3 & 1 \\ 0.7 & 0 & 0 \\ 0 & 0.7 & 0 \end{array} \tag{13}$$

The non-continuous and continuous hidden Markov model transition rate comparison r is used as a measure of the dissipation in the semi-active state behavior. Current research indicates a correlation between the size of avalanche growth and energy occupation levels for observational matrix B [20]. In order to simplify disaster mechanism, we only consider three medium and large, or S, M and L for avalanches. The probabilistic relation- ship between energy states and avalanche sizes could be given by the possibility of observation of given region is $B = b_j(k)$ for the observed time t area Vk of the possibility of thousands of events Vk:

$$b_j(k) = P(ot = VK \mid s_t = q_i) \tag{14}$$

$$B = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \end{array} \begin{array}{|ccc|} S & M & L \\ 0.3 & 0.4 & 0.5 \\ 0.2 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.3 \end{array}$$

The transition from one state to the next is a Markov process since the next state depends only on the current state and the fixed probabilities in (11). However, the actual states are "hidden" since we cannot directly observe the state of energy in the past. A computational model of 3×3 transition matrix for the simulated states [1, 3] of the Markov model chain based on parametric data approach fluctuates between different regimes where the data comes from three observable data and partition space of empirical transition matrix. The next step is given in a particular state to generate a sample from that state compute empirical discrete probability distribution of each regime with a precision 'dt'. This model can now be used to simulate the observed process dynamics, the results show that the sample paths as well as histograms match quite well. The general mechanism can be applied via a wide array of dataset for nonparametric simulations, though we cannot observe the state of the empirical measurement of the size of avalanche.

It is spectacular to mention that the erratic aftershock trend of the 2004-2005 Sumatra - Andaman tsunamigenic earthquakes created a wide-scale panic among the people because of a series of subsequent tremors and shocks in the region [2, 21]. Forecast density at time t is a weighted sum of the three exponential densities, with weights the conditional probabilities of being in states 1, 2, or 3 at time $t+1$. These are computable for a given model. People of the Andaman-Nicobar region of India anticipated whether possibilities of another big tsunamigenic earthquake in the vicinity of Andaman - Nicobar region exist! Mishra et al [21] described genesis of erratic aftershocks and continuous shaking of the region after the main shock was related to the strong heterogeneities with the causative faults and the subsequent stabilization of the sub-surface earth resulted in a series of tremors and aftershocks, manifesting the healing process of the sub-surface earth beneath Andaman-Nicobar region of India during the year 2004-2005. This observation vindicates the concept of seismogenesis and conflicts that occur due to accumulating stress (disequilibria in energy and relations) between active agents (faults and heterogeneities) in which the stress may find its release through sudden burst or emergence of new features as shock waves analogous to an earthquake occurrence in the critical state or a critical shock or conflict in societal analysis. The influence of each contributor tried to come back into stasis (meta-stable state or stable equilibrium state), the spatial system position of earth dynamics or conflicts through a series of tremors (aftershocks) in accordance to a Poisson process hidden Markov cellular automata model.

There is additional evidence that the initial state distribution, denoted as:

$$\pi, = \begin{array}{c} N_1 \\ N_2 \\ N_3 \end{array} \begin{array}{|c|} 0.5 \\ 0.3 \\ 0.2 \end{array}$$

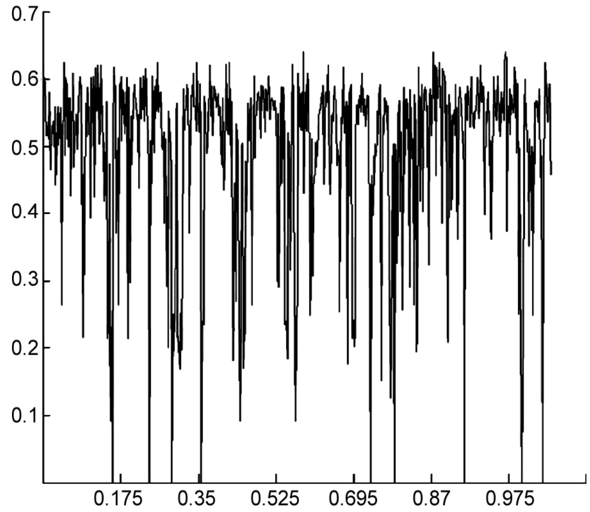

Figure 3a. Synthetic data set for transition probability.

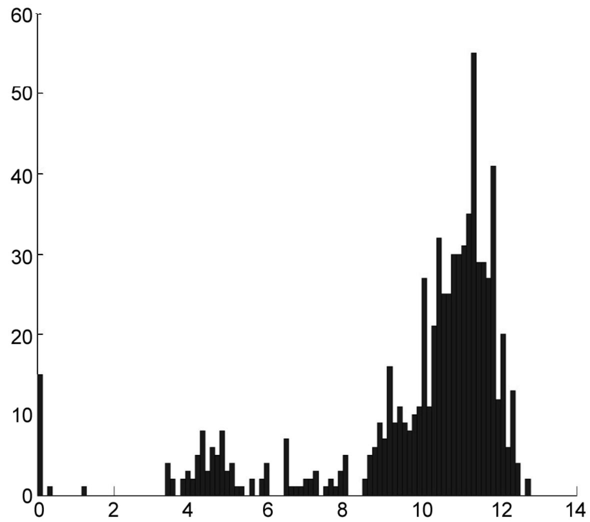

Figure 3b. Synthetic histogram output on event analysis.

The matrices π, *A*, and *B* are column stochastic, as the elements in the column have to add up to 1, which can then prove, by energy conservation rule, that the system is stable. An HMM is defined by *A*, *B* and π as the stochastic sand pile is denoted by σ = (*A*, *B*, π). A sample distribution by observing the series of avalanches *S*, *M*, *S*, *L*, where 0 represents *S*, 1 represents *M* and 2 represents *L*, as the observation sequence is *O* = (0, 1, 0, 2) is the most likely for state sequence of the Markov process which has state sequences as length four.

If we assume a state generic sequence of length four $X=(x_0, x_1, x_2, x_3)$ with corresponding observations $O=(O_0, O_1, O_2, O_3)$, then πx_0 is the probability of starting in state x_0. Moreover, $b x_0 (O_0)$ is the probability of initially observing O_0 and $a x_0$, x_1 is the probability of transiting from state x_0 to state x_1. Continuing, we see that the probability of the state sequence X is given by

$$P(X) = P(N_1 N_1 N_2 N_2) = \pi_{x0} b_{x0}(O_0) a_{x0,x1} b_{x1}(O_1) \times a_{x1,x2} b_{x2}(O_2) a_{x2,x3} b_{x3}(O_3) = 0.00001764$$

We can compute the probability of each of the possible state sequences of length four, assuming the fixed observation sequence.

$$\sum_{i=1}^{n=4} P_i(x) = 0.00001764 + 0.00001856 + 0.000014335 + 0.000003245$$

According to our study, the average stress, before and after a shock $(\sigma b + \sigma a)/2$ decreases with increasing $P(x)$ found for the nature of the shock; that is, the absolute stress level decreases with increasing magnitude. The distribution is then estimated by the empirical distribution function, or the histogram of the samples. Thus, using the sand pile criterion, an active relationship between magnitude and energy condition for stress variability with density energy condition is derived for sand pile concurrency with stress drop that emerges as a characteristic property of the system.

5. Conclusion

The study shows that exogenous and endogenous events interact and trigger spatially in large-scale disasters and conflicts. The spatial existence of the trigger basins is of vital concern in large-scale networks. We have proved the interior stress distribution and coupling effects for sources using sand pile model. It needs to be seen whether such observation and data can be validated for other complex systems as well for the observed state of extreme event occurrence, which all are products of both endogenous and exogenous effects. This study validates the concept of exogenous events using sand pile cellular automata system, an approach to study complex behavior of multi-component system.

The methodology has clear-cut explanation about the genesis of aftershocks and conflicting relationship with faults and heterogeneities that bring the system into a brittle failure. The special role in the given model is the study of "triggering" functions elaborated in [22]. At all stage of accumulation of elastic energy in the block of rocks, there is a probability of external influence on the block and dump of the elastic energy, which has been saved up in this block. However, while the saved up energy will not reach (achieve) a critical point, dump of energy is realized as earthquakes or power shocks in the critical state or a critical shock or conflict in societal analysis. The influence of each contributor tries to come back into stasis (meta-stable state or stable equilibrium state), the spatial system position of earth dynamics or conflicts through a series of tremors (aftershocks) or post earthquake responses. The present approach is very much valid for explanation of aftershock sequences of the 2004 - 2005 tsunamigenic earthquake sequences.

It is ongoing research to see the predicting potential of the sand pile model in unraveling the dynamics of conflict analysis model in future observations based on detailed emphasis on environment biased condition and real life constraints.

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