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# Slope Stability and Bearing Capacity of Footings on Top of Slopes Under Repeated Dynamic Loads

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## ABSTRACT

Although bearing capacity of footings on top of slopes under monotonic loadings have been investigated widely so far, safety and bearing capacity of these structures against repeated dynamic loads have not been considered well enough. In this paper, lower bound dynamic shakedown theorem in its numerical form has been employed to firstly obtain bearing capacity of strip footings on top of slopes, and secondly to determine safety factor of slopes by strength reduction method subjected to repeated dynamic loads. Following a try and error procedure, factor of safety against cyclic loads were determined based on the strength reduction concept. Results indicate that bearing capacity of footings is affected by dynamic properties of both slope and the loads so that minimum bearing capacity obtained when resonant occurs. Furthermore, results show that the safety factor of slopes is extremely affected by dynamic properties of both slope and dynamic load as well. The minimum factor of safety was found to arise at resonance. Besides, findings suggest that unlike Pseudo-Static method that does not differentiate between embankments (with wide crest) and slopes, here factor of safety of slope and embankment are not the same due to their difference in dominant natural period.

### Keywords:

Slope stability;  
Shakedown; Dynamic load; Bearing capacity; Factor of safety; Strength reduction method

## 1. Introduction

Stability of slopes against seismic loads has always been a great cause of concern in seismic regions. Pseudo-static, dynamic and mixture of pseudo-static and dynamic methods are usually utilized to evaluate the behaviour of slopes against earthquake loading. It is typical of all the aforementioned approach that slope stability is evaluated under one-time imposition of seismic. However, in reality, slopes can be subjected to many static and dynamic loads, in particular earthquakes loads, with different features during their lifetime. In other words,

repetition of loads is not taken into account by prevalent methods. Structures under repeated loads might perform completely different than against single time-imposed loads. Consequently, to have an insight into the real behaviour of structures, they should be analysed against the repetition of loads of any kind.

In general, mechanical systems respond to repeated loads in three different ways as:

- 1) If the intensity of the applied load is small enough, the whole body reflects back to its initial position

and behaves solely in an elastic manner.

- 2) Under repetitive large-intensity loads, plastic strain accumulation may result to the collapse of the system due to the excessive deformation (ratcheting).
- 3) A range of load intensity in-between the first two groups can be imagined under which the loaded body initially develop plastic strains, but plastic strain increment gradually tends to diminish as load repetitions proceed so that the system behaves elastically after sufficient load cycles. This kind of phenomenon is recognized as shakedown or adaptation.

In order to investigate the behaviour of slopes against repeated dynamic loading, two different approaches may be followed. The first way is to conduct a load-displacement nonlinear dynamic analysis, which in addition to being time consuming, is not trustable due to uncertainties associated with load and material properties. The second approach is to take advantage of shakedown limit theorems that directly attain a load domain under which, slopes cease to develop further permanent deformation after a limited number of load cycles. Clearly, the second approach is more affordable in terms of simplicity and implementation. Nevertheless, this method does not give any account on deformations and in particular permanent deformation prior to shakedown limit. In spite of this fact, shakedown limit theorems may be used to investigate the design failure criteria. In this respect, their result are comparable to methods such as limit equilibrium or limit analysis.

Shakedown limit theorems, similar to collapse limit theorems, have been developed in the forms of lower and upper bound theorems. Ceradini [1] developed the lower bound dynamic shakedown theorem and Corradi and Maier [2] and Koiter [3] introduced the upper bound dynamic shakedown theorem.

The first numerical shakedown solutions by composition of finite element method and mathematical programming, was developed by Maier [4]. Although shakedown approach has been used extensively in a variety of engineering fields, the first serious application of shakedown theorems was conducted by Sharp and Booker [5] to find the shakedown solution of road pavements under repeated

wheel loading. Most of the works on the application of shakedown theorems to geotechnical structures have been devoted to pavement design under traffic loads. Hossain and Yu [6] and Yu and Hossain [7], extended the method of Bottero et al [8], which was used previously to find the limit loads of shallow footings, to shakedown problems. This method consists of finite element elastic analysis, finite element stress analysis and linear programming.

In seismic regions, slopes are subjected to a variety of earthquake loadings with different characteristics during their lifetime. In this regards, shakedown theory can be utilized to investigate the safety of slopes against seismic loads. Arvin et al [9] and Askari et al [10] extended the method of Hossain and Yu [6] to dynamic lower bound shakedown analysis and evaluated the safety of embankment and slopes under repeated seismic loads. Both dynamic characteristics of slope ( $T_s$ : dominant period of slope) and dynamic load ( $T_m$ : medium period of dynamic load) were taken into account by them. They represented the variation of dynamic shakedown factor versus  $T_s / T_m$ . Their study indicated that although slopes might be stable under major earthquakes, they may fail due to the repetition of minor seismic loads.

Strength reduction method has gained popularity in recent years due to its ability to represent the crowded results of a numerical analysis in the form of a single value, namely factor of safety. Yet, results of shakedown analysis have not been illustrated as a factor of safety obtained through strength reduction approach. If shakedown results can be determined as a factor of safety analogous to common slope stability analysis method, making comparisons between them will become more efficient.

In this paper, strength reduction method is employed to determine the safety factor of slopes against dynamic repeated loads.

## 2. Lower Bound Shakedown Theorem

As stated previously, using the shakedown limit theorems, a load domain as a portion of the general load domain imposed on the structure might be determined for which the structure behaves in accordance with the shakedown definition. According to the lower bound dynamic shakedown theorem, the obtained shakedown limit is certainly lower than

the true shakedown limit. Lower bound dynamic shakedown theorem states: If a fictitious response  $u_i^*(x, t)$ ,  $\varepsilon_{ij}^*(x, t)$ ,  $\sigma_{ij}^*(x, t)$  (displacement, strain and stress respectively) and a time independent residual stress field  $\sigma_{ij}^r(x)$  can be found such that:

$$f(\sigma_{ij}^*(x, t) + \sigma_{ij}^r(x)) \leq 0 \quad (1)$$

then, shakedown will happen at real response [1]

### 3. Numerical Method

In order to implement the shakedown theorem, it should be converted to the form of a mathematical programming. To do so, in this paper, method of Arvin et al [9], which is the extension of Hossain and Yu [6] and Yu and Hossain [7] method to dynamic problems, has been employed. Just like its static counterpart, this method merges together the finite element and linear programming.

To implement finite element method, the body is discretised spatially to triangular finite elements. Stresses, both residual stresses and elastic stresses, are assumed to vary linearly across the elements. To do so, six nodes and three node triangular elements were considered for elastic stress-strain analysis and residual stress analysis respectively. According to lower bound shakedown theorem, the elastic stress field resulting from external dynamic loads must be obtained through an elastic dynamic analysis assuming arbitrary initial conditions. Therefore, determination of elasto-dynamic solution (stress, strain and displacement fields) is the first step toward shakedown method implementation. In this study, the following equation was employed to do so.

$$M\ddot{u} + C\dot{u} + Ku = P(t) \quad (2)$$

where  $M$ ,  $C$  and  $K$  are mass, damping and stiffness matrix respectively,  $u$  represents displacement and  $P(t)$  is the applied dynamic load in the Eq. (2). In this study, dynamic loads are assumed sin load with certain periods and intensities. Damping in Eq. (2) is considered to be of classic type. That is, damping is a linear combination of mass and stiffness matrix as follows.

$$C = \eta M + \xi K \quad (3)$$

In Eq. (3),  $\eta$  and  $\xi$  are constant coefficient

obtained from first and second dominant natural period of the structure. Implicit integration method of Newmark is employed in the present study to solve Eq. (2).

Since  $\sigma_{ij}^*(x, t)$  is the elastic response of the structure under general loading, shakedown limit as the portion of the general loads, lets show it with a coefficient  $\lambda$ , will occur when the elastic response of the body is equal to  $\lambda\sigma_{ij}^*(x, t)$ . Therefore, lower bound shakedown load domain is determined searching for the maximum value of  $\lambda$  through an optimization process.

Goal function of this optimization process is  $\lambda$  and constraints of two main types, namely equality and inequality constraints, based on the shakedown theorem. Equality constraints consists of satisfaction of equilibrium equations both inside and on the boundaries of body and equations associated with stress discontinuity on the interfaces of adjacent elements. In addition, yield conditions is required to be met everywhere in the elements via inequality constraints. As shakedown theorem states, yield conditions must be fulfilled for the combination of elasto-dynamic stresses and residual stresses as well as for residual stresses alone. The Mohr-Coulomb yield surface has an original shape. However, it is possible to convert it to a linear formulation by piece-wise linearization technique. The optimization process to search for the shakedown factor  $\lambda$  can be summarized by the following equation:

$$\begin{aligned} & \text{Maximize } \lambda \\ & \text{Subjectd to} \\ & A_1 X = b_1 \\ & A_2 X \leq b_2 \end{aligned} \quad (4)$$

In Eq. (1),  $A_1$  and  $b_1$  are related to the equality constraints as discussed earlier. Similarly,  $A_2$  and  $b_2$  are linked to the inequality constraints (yield constraints). The vector  $X$  consists of all the residual stresses and  $\lambda$ . Since all functions involved in Eq. (4) are considered linear, Linear Programming (*LP*) as a powerful tool can be utilized to optimize the solution. The most important advantage of *LP* is that the output goals are global extremums. Formulations to constitute equality and inequality constraints of lower bound dynamic shakedown problems are available in detail in Arvin et al [9] and are not repeated in this paper.

#### 4. Strength Reduction Method for Repeated Loads

Results of slope stability analysis are usually represented in the form of factor of safety (FS). Limit equilibrium-based methods such as method of Bishop, method of Janbu and so on, define the factor of safety as a ratio of shear strength to existing shear stress along a pre-assumed slip surface. Hence, for a  $c$ - $\sigma$  soil,  $FS$  is calculated by the following equation:

$$FS = \frac{\tau_f}{\tau} = \frac{c + \sigma' \tan \phi'}{\tau} \quad (5)$$

where  $c$  and  $\phi'$  are the cohesion and internal friction angle of the soil respectively and  $\sigma'$  is effective stress normal to the slip plane. Although simple problems can merely be analysed using limit equilibrium method, slopes with complex loading and material properties have to be evaluated by more rigorous approaches such as finite element or finite difference methods. However, if numerical load-displacement approaches, such as finite element method is employed for the slope stability analysis, factor of safety and slip surface cannot be recognized straightforwardly. Therefore, a different approach other than the method described for limit equilibrium must be developed to find the  $FS$  and slip surface. Strength reduction method is an appropriate well-known tool to achieve this goal. In strength reduction method, soil strength parameters, namely  $c$  and  $\phi'$  are changed so that a failure mechanism is formed along which shear strength and existing shear stress are approximately identical. In this regard and as Figure (1) indicates, cohesion and internal friction angle at failure are introduced as Eq. (6).

$$\tau = \frac{c}{FS} + \sigma' \frac{\tan \phi'}{FS} = \bar{c} + \sigma' \tan \bar{\phi}' \quad (6)$$

Then, the factor of safety is calculated by Eq. (7):

$$FS = \frac{c}{\bar{c}} = \frac{\tan \phi'}{\tan \bar{\phi}'} \quad (7)$$

In order to directly compare the results of shakedown analysis to common slope stability evaluation approaches such as pseudo-static method, shakedown results are obtained employing

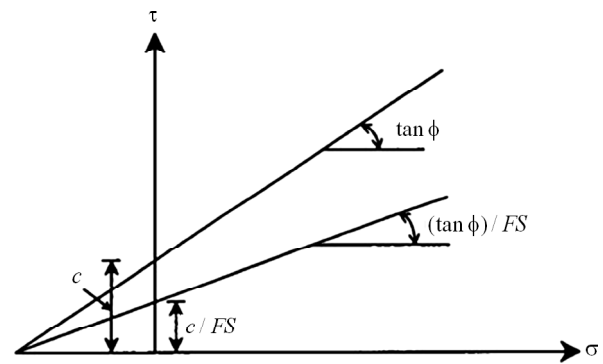


Figure 1. Mohr-Coulomb failure criterion before and after reduction of strength parameters.

the following step-by-step strength reduction procedure:

Step 1. Perform shakedown analysis to determine shakedown coefficient  $\lambda$ .

Step 2. If  $\lambda$  is approximately (with tolerable error) equal to unity, strength parameters employed in step 1 are regarded as  $\bar{c}$  and  $\bar{\phi}'$  and  $FS$  is obtained by Eq. (5).

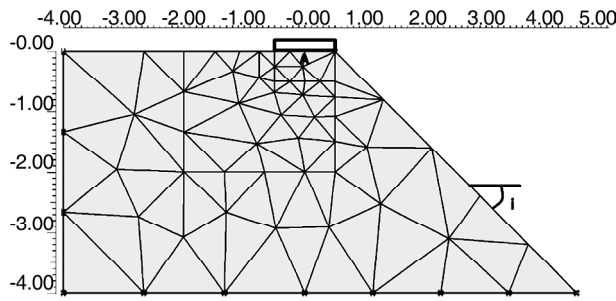
Step 3. If  $\lambda > 1$ , both  $c$  and  $\phi'$  are reduced with the same scale. Likewise, in case  $\lambda < 1$ , both  $c$  and  $\phi'$  are increased at the same rate. Then return to step 1 considering the altered values of  $c$  and  $\phi'$ .

Following the procedure described above, a single factor of safety is obtained that shows whether or not the slope is safe regarding the shakedown criteria.

#### 5. Problem Definition

Two different problems are considered and analysed through the present study. Firstly, shakedown method is employed to find the bearing capacity of a strip footing resting on top a slope under repeated vertical dynamic loads. Secondly, factor of safety of slopes are determined under repeatedly applied dynamic loads via strength reduction method.

For the first part of the investigation, a vertical dynamic load is imposed on a rigid smooth strip footing resting on top of a slope. The system is discretized by triangular elements using the mesh generation ability of Plaxis software (Figure 2). Load is applied on the node A in the middle of the foundation as depicted on Figure (1). Vertical movement at the base of the foundation are restricted to be identical. For numerical convenience,



**Figure 2.** Mohr-Coulomb failure criterion before and after reduction of strength parameters.

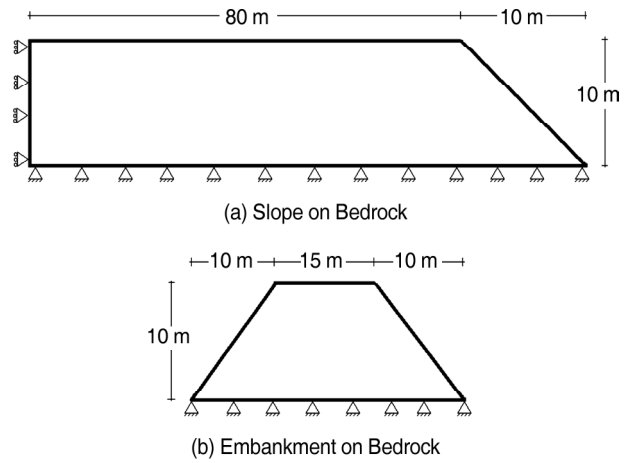
displacements of nodes at the base of the footing are tied to that of node A.

For the present study, vertical dynamic loading is considered as a number of successive half-sine loads, applied on the centre of footing. The peak value of load is equal to one. The maximum possible vertical intensity of dynamic load bearable by the footing is then calculated based on shakedown approach and via numerical method described previously.

As the second part of study, namely application of the strength reduction method, an embankment and a slope, resting on bedrock have been considered as model studies. Mesh generation module of Plaxis software was utilized for space discretization. Figure (3) depicts the geometrical characteristics and boundary conditions of the model studies.

Dynamic response of structures depends on both dynamic properties of structure and loads. Here, mean period of load ( $T_m$ ) and dominant natural period of structure ( $T_s$ ) are considered as the dynamic properties of load and slope respectively. In this regard, parameter  $T_s / T_m$  is the representative of the dynamic properties of both load and slope. To have different values for  $T_s$ , either geometrical characteristics or material properties must be changed. Doing so, resulting  $T_s$  have been presented in Table (1).

Sin loads with 0.15 g intensity and different periods ( $T_m$ ) are considered as imposed dynamic loads.



**Figure 3.** Model studies considered in the present study.

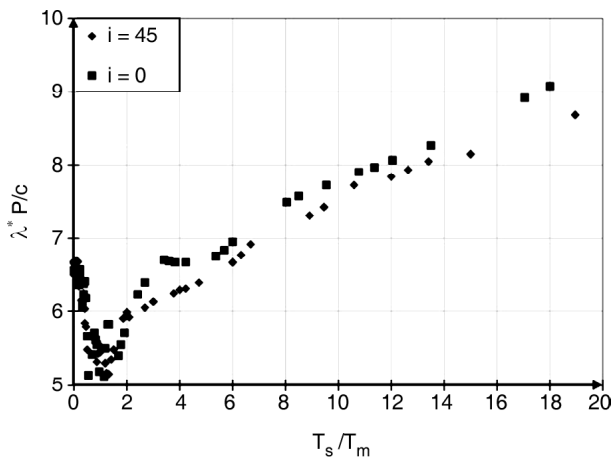
### 6. Results

Dynamic shakedown bearing capacity of a footing resting on the crest of a slope as defined in Figure (2) was investigated first. Results are presented for different values of  $T_s / T_m$ . In order to verify the effects of ground inclination on the results, dynamic shakedown bearing capacity of footings were calculated for level ground and slope inclination angle of  $i = 45^\circ$ . In addition,  $\phi = 10^\circ$ ,  $n = 2$ ,  $\gamma = 20 \text{ KN/m}^3$  and  $DR = 5\%$  are considered for the cases at hand where  $n$  is the number of successive half-sin load and  $DR$  is damping ratio. Then shakedown analysis were performed to find the maximum shakedown factor ( $\lambda$ ) that can be tolerated by the footing. Obviously,  $\lambda P$  is the dynamic bearing capacity of the footing, where  $P$  is the initial domain of the applied sin load.

Figure (4) shows the variation of  $\lambda P/c$  versus  $T_s / T_m$  for footing lain on the level ground and for  $i = 45^\circ$ . As shown in the Figure (4),  $\lambda P/c$  first decrease sharply as  $T_s/T_m$  grows, and then rises gradually as  $T_s / T_m$  increase.  $\lambda P/c$  reaches a low at about  $T_s / T_m = 1$ , where resonant occurs. Furthermore, Figure (4) suggests that dynamic shakedown bearing capacity of footings diminishes as level ground turns into a slope.

**Table 1.** Dominant period of model studies for different soil shear modulus.

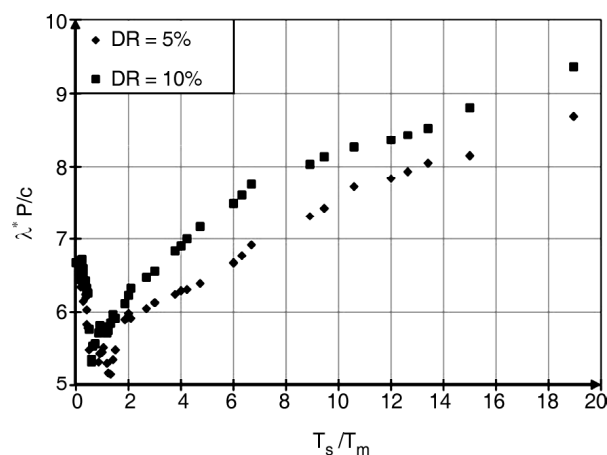
Model	G(MPa)							
	2	3	4	6	15	30	100	300
Slope	1.195	0.976	0.845	0.69	0.437	0.309	0.169	0.098
Embankment	1.126	0.92	0.796	0.65	0.411	0.291	0.159	0.092



**Figure 4.**  $\lambda^*P/c$  values vs  $T_s/T_m$  for a footing resting on the crest of a slopes with  $i = 45^\circ$  and  $i = 0$  and soil properties  $DR = 5\%$ ,  $\gamma = 20 \text{ KN/m}^3$  and  $\phi = 10^\circ$ .

Effects of damping on dynamic shakedown bearing capacity were also investigated for slope angle  $i = 45^\circ$ . As Figure (5) shows, shakedown bearing capacity for  $DR = 10\%$  is always higher than that of  $DR = 5\%$  regardless of the  $T_s/T_m$  value.

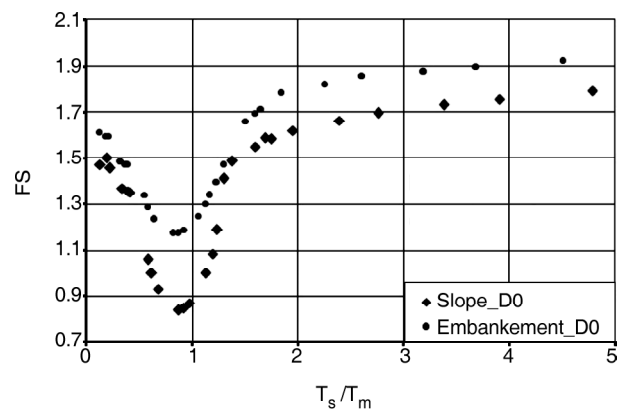
Moreover, shakedown factor of safety for the model studies described earlier (Figure 3) were calculated based on the strength reduction approach. To do this, firstly, slope and embankment were subjected to the dynamic sin load. Then, employing numerical lower bound shakedown analysis, their shakedown factor ( $\lambda$ ) were determined. Secondly, Soil strength parameters (cohesion  $c$ , internal friction angle  $\phi$ ) are changed (either reduced or increased) by a try and error procedure, as discussed previously, so that the slope reaches the critical conditions



**Figure 5.** Effect of damping ration on the shakedown dynamic bearing capacity of a footing resting on the crest of a slope ( $i = 45^\circ$ ,  $\gamma = 20 \text{ KN/m}^3$  and  $\phi = 10^\circ$ ).

with respect to dynamic shakedown criterion. Finally, factor of safety was determined by Eq. (7).

Factor of safety ( $FS$ ) against  $T_s/T_m$  for embankment and slope with  $\phi = 30^\circ$ ,  $\gamma H/c=5$  ( $\gamma$  is soil unit weight and  $H$  is the slope height), and damping ratio  $DR=0.05$ , are depicted in Figure (6). Results indicate that as  $T_s/T_m$  increases, first,  $FS$  value decreases and then increases. At high levels of  $T_s/T_m$ ,  $FS$  value tends to a constant value. The minimum value of  $FS$  comes about  $T_s/T_m = 1$ , when the slope and embankment undergo resonant. In addition, for all  $T_s/T_m$  shakedown factor of safety of embankment is larger than the slope of the same geometrical properties. This finding is in contradiction to pseudo-static method, which does not differentiate between embankment (with wide crest) and slope. Besides, contrary to Pseudo-Static method that gives rise to a single factor of safety regardless of the dynamic properties of structure and load, shakedown factor of safety is clearly connected to foregoing characters.



**Figure 6.** Shakedown factor of safety attained by strength reduction method against  $T_s/T_m$  for embankment and slope with  $\phi = 30^\circ$ ,  $\gamma H/c=5$  and  $DR = 0.05$ .

### 7. Conclusions

In the present study, lower bound shakedown dynamic theorem in its numerical form was employed to find bearing capacity of footing on top of slopes under dynamically repeated applied loads in addition to assessment of safety factor of slopes and embankments under repeated dynamic loads by strength reduction method.

A strip footing resting on the crest of a given slope was analysed under vertical repeated dynamic load and its maximum shakedown factor were

determined using numerical lower bound dynamic shakedown method. Furthermore, an embankment and a slope with specified geometrical and material properties under repeated dynamic loads were evaluated to obtain their factor of safety with respect to shakedown failure criterion. Numerical dynamic lower bound shakedown theorem was employed to find the shakedown coefficient ( $\lambda$ ). Then, by strength reduction method through a try and error procedure, soil strength properties on the verge of inadapation (a type of failure that occurs if load domain exceeds shakedown load domain) were calculated. Factor of safety then determined as the rate of initial and reduced soil strength properties. Results were presented for different slope and load dynamic properties ( $T_s / T_m$ ). Followings are the summary of the main findings:

- ❖ Dynamic shakedown bearing capacity of footings on top of slopes are extremely affected by  $T_s / T_m$  so that it first drops dramatically and then increases gradually as  $T_s / T_m$  grows. A low in bearing capacity is observed at around  $T_s / T_m = 1$  where resonance occurs.
  - ❖ Higher values of damping ratio give rise to higher shakedown dynamic bearing capacity of footings on top of slopes.
  - ❖ Unlike Pseudo-Static method, FS is dependent on slope and load dynamic properties ( $T_s / T_m$ ).
  - ❖ As  $T_s / T_m$  increases, first FS value decreases to a minimum and then start to increase. FS tends to a constant value at higher values of  $T_s / T_m$ .
  - ❖ Minimum value of FS occurs at approximately  $T_s / T_m = 1$ , where slope resonate under the imposed load.
  - ❖ FS is different for embankment and slope. Embankments have higher FS than slope. This is contrary to results of Pseudo-Static method for slope and embankment with large crest.
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