Random-Vibration-Based Response Spectrum Method for Multi-Support Structural Systems

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ABSTRACT: *A random vibration approach for the response analysis of linear multi-support structural systems, previously developed by the authors, is briefly reviewed. It serves as the basis for the formulation of a response spectrum method for the seismic response analysis of multiply-supported structures. The response of a two-span beam subjected to spatially-varying earthquake ground motion is analyzed. The influence of uniform and varying local soil conditions, wave passage and incoherence effects on the response of the beam is studied. The significance of each effect is assessed and the relative importance of the pseudo-static and the dynamic response is examined. For comparison purposes, the results also include the cases of statistically independent and perfectly correlated ground motion input.*

Keywords: Random vibration; Multi-support excitation; Stochastic response; Response spectrum; Soil response

1. INTRODUCTION

A classical representation in seismic response analysis is that of a single-or multi-degree-of-freedom linear oscillator subjected to an earthquake ground motion at its support, assuming that at all support points of a structure the ground motion excitation is the same. While this assumption may be appropriate for residential and high-rise buildings where the distances between supports are relatively small, it may not be realistic for structural systems such as bridges and parts of lifelines extending over long distances. Empirical data obtained from dense seismographic arrays has shown that significant variability exists in ground motion parameters over distances of the same order of magnitude as those between the multiple supports of spatial extended structures. The operational integrity and serviceability of lifeline systems after an earthquake are essential to the safety of the built environment and for the efficient implementation of disaster mitigation strategies. Experience shows, however, that lifeline systems may be severely damaged during earthquake (Iwasaki et al 1972; Narita 1976; EERC 1989).

Early studies on the response of extended structures assumed spatial variation of the support motions due to a traveling signal (Masri 1976; Werner et al 1979a,b; 1980). Some research was also carried out to extend the notion of the one-degree response spectrum to the response analysis of multi-support systems. Nelson and Weidlinger (1979) introduced the Interference Response spectrum for the analysis of long segmented pipelines, and Loh et al (1982) developed a dynamic response ratio for characterizing multi-support excitations which was expanded later by Abrahamson and Bolt (1985). More recently, response spectrum and time-history methods have been used for evaluation of the response to multi-support input (Kang and Weiland 1988; Yamamura and Tanaka 1990).

Random vibration and random fields theory have also been used for stochastic response analysis of multi-support systems (Abdel-Ghafar and Rubin 1982; Ukon and Vanmarcke 1989; Zerva et al 1986, 1988; Harichandran and Wang 1988, 1990; Zerva 1990; Nadim et al 1991). Since for practical use in earthquake engineering a method of analysis based on the response spectrum is desirable, attempts have been made to develop random-vibration-based response spectrum methods for multi-support systems. Berrah and Kausel (1990) suggested a modified response spectrum method for the dynamic response based on a correction factor relating the mean values of a maximum modal response for partially and perfectly correlated ground motion. A combination rule for the peak response, in terms of peak ground displacements, mean displacement response spectra, and a set of support and modal correlation coefficients, has been developed by Der Kiureghian and Neuenhofer (1991).

In this paper, a response spectrum method for multi-support systems is presented based on a random vibration methodology (Heredia-Zavoni and Vanmarcke 1993) which reduces the seismic response analysis of linear multi-support multi-degree-of-freedom (MS-MDOF) structural systems to that of a series of one-degree modal oscillators. A significant advantage of the methodology is that, given a stochastic model for the ground motion spatial variation, response analyses of the one-degree modal oscillators can be carried out by specifying only the location of the structural supports.

A brief outline of the random vibration approach for the analysis of multi-support systems is presented first. Next, we formulate a random-vibration-based response spectrum method relating response maxima to standard deviations by means of peak factors. The response of a two-span continuous beam is analyzed and results are presented for several parametric studies aimed at assessing the influence of local soil conditions, wave passage and incoherence effects. The findings are summarized in the conclusions.

2. RANDOM VIBRATION ANALYSIS OF MS-MDOF SYSTEMS

Heredia-Zavoni and Vanmarcke (1993) developed a random vibration methodology that reduces the stationary seismic response analysis of linear multi-support MDOF systems to that of modal oscillators accounting fully for the multi-support input and the space-time correlation structure of the ground motion. Consider a MS-MDOF system with n response degrees of freedom and m prescribed support ground motions. Let $Z(t)$ and $V(t)$ denote two response quantities of interest and let $Z_d(t)$ and $V_d(t)$ denote their corresponding dynamic components. Three multi-support (MS) system coefficients, $\alpha_{kli}, \theta_{kli}$ and ϕ_{kli}, can be defined in terms of the effective modal participation factors $\mathbf{c}_i^T = \{c_{1i},...,c_{mi}\}$ and, $\mathbf{c}_i'^T = \{c'_{1i},..,c'_{mi}\}$, $i = 1,2,...,n$, associated with $Z(t)$ and $V(t)$, respectively, and of the cross-modal factors, A_{ij}, A_{ij} and B_{ij} which depend on modal frequencies and dampings,

$$\alpha_{kli} = c_{ki}c'_{li} + \frac{1}{2}\sum_{j=1,\,j\neq i}^{n}\left(c_{ki}c'_{lj} + c_{kj}c'_{li}\right)A_{ij},$$
$$\theta_{kli} = \frac{2}{\omega_i}\sum_{j=1}^{n}\left(c_{ki}c'_{lj} - c_{kj}c'_{li}\right)A_{ij}, \qquad (1)$$
$$\phi_{kli} = \frac{2}{\omega_i^3}\sum_{j=1}^{n}\left(c_{ki}c'_{lj} - c_{kj}c'_{li}\right)B_{ij}.$$

The stationary dynamic response covariance $\mathrm{Cov}[Z_d(t),V_d(t)]$ can be expressed approximately in terms of the MS system coefficients $\alpha_{kli}, \theta_{kli}$ and ϕ_{kli} which depend only on the structural properties, and of three spectral parameters, $\Gamma_{o,kli}, \Lambda_{1,kli}, \Lambda_{3,kli}$, associated with the response of one-degree systems,

$$\mathrm{Cov}\left[Z_d(t),V_d(t)\right] = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\left\{\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\Lambda_{1,kli} - \phi_{kli}\Lambda_{3,kli}\right\}\sigma[Y_{ki}]\sigma[Y_{li}], \qquad (2)$$

where $\sigma^2[Y_{ki}] = \int_{-\infty}^{\infty}|H_i(\omega)|^2 S_{\ddot{U}_k\ddot{U}_k}(\omega)\,d\omega$ is the variance of the stationary response Y_{ki} of a modal oscillator with frequency and damping ω_i, ξ_i, corresponding to the *i-th* normal mode, subjected to the random ground acceleration $\ddot{U}_k$, $k = 1,2,...,m$; $H_i(\omega) = [\,\omega_i^2 - \omega^2 + 2i\xi_i\omega_i\omega\,]^{-1}$ is a modal transfer function and $S_{\ddot{U}_k\ddot{U}_k}$ is the ground acceleration spectral density function.

The spectral parameter $\Gamma_{0,kli}$ is equal to the cross-correlation coefficient between the responses Y_{ki} and Y_{li} of the modal oscillator to the ground accelerations $\ddot{U}_k$ and $\ddot{U}_l$,

$$\Gamma_{0,kli} = \frac{1}{\sigma[Y_{ki}]\sigma[Y_{li}]}\int_{-\infty}^{\infty}\omega\,|H_i(\omega)|^2 R_{\ddot{U}_k\ddot{U}_l}(\omega)\,d\omega,$$

in which $R_{\ddot{U}_k\ddot{U}_l}(\omega)$ is the ground acceleration cospectrum. The spectral parameters $\Lambda_{1,kli}, \Lambda_{3,kli}$ can be expressed as follows,

$$\Lambda_{1,kli} = \frac{1}{\sigma[Y_{ki}]\sigma[Y_{li}]}\int_{-\infty}^{\infty}\omega\,|H_i(\omega)|^2 Q_{\ddot{U}_k\ddot{U}_l}(\omega)\,d\omega = -\rho_{\dot{Y}_{ki}Y_{li}}\Omega_{2,ki}, \qquad (3)$$

$$\Lambda_{3,kli} = \frac{1}{\sigma[Y_{ki}]\sigma[Y_{li}]}\int_{-\infty}^{\infty}\omega^3|H_i(\omega)|^2 Q_{\ddot{U}_k\ddot{U}_l}(\omega)\,d\omega = \rho_{\ddot{Y}_{ki}\dot{Y}_{li}}\Omega_{4,ki}^2\Omega_{2,li},$$

where $\rho_{\dot{Y}_{ki}Y_{li}}$ is the cross-correlation coefficient between the response velocity $\dot{Y}_{ki}$ and the response Y_{li}; $\rho_{\ddot{Y}_{ki}\dot{Y}_{li}}$ is the cross-correlation coefficient between the response acceleration $\ddot{Y}_{ki}$ and the response velocity $\dot{Y}_{li}$; $\Omega_{2,ki} = \sigma[\dot{Y}_{ki}]/\sigma[Y_{ki}]$ and $\Omega_{4,ki}^2 = \sigma[\ddot{Y}_{ki}]/\sigma[Y_{ki}]$ are characteristic frequencies; and $Q_{\ddot{U}_k\ddot{U}_l}(\omega)$ is the ground acceleration quadrature spectrum.

The analysis of the MS-MDOF system is thus reduced to that of independent one-degree systems with natural frequencies and damping ratios corresponding to those of the normal modes. The spectral parameters $\Gamma_{0,kli}, \Lambda_{1,kli}, \Lambda_{3,kli}$ can be computed for a wide range of natural frequencies ω_i and for different damping ratios ξ_i given the location of the MDOF system supports.

One can think of Eq. (2) as a modal combination rule since it expresses the dynamic response covariance as a sum of modal contributions $C_i\left[Z_d(t),V_d(t)\right]$,

$$C_i\left[Z_d(t),V_d(t)\right] = \sum_{k=1}^{m}\sum_{l=1}^{m}\left\{\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\Lambda_{1,kli} - \phi_{kli}\Lambda_{3,kli}\right\}\sigma[Y_{ki}]\sigma[Y_{li}]. \qquad (4)$$

Eq. (2) can also be used for MDOF systems subjected to single support excitation; in this case m=1, $\Gamma_{0,i} = 1$ and $\Lambda_{1,i} = \Lambda_{3,i} = 0$ since the ground acceleration spectral density function $S_{\ddot{U}\ddot{U}}(\omega)$ is real. The dynamic response covariance $\mathrm{Cov}\left[Z_d(t),V_d(t)\right]$ in (2) becomes

$$\mathrm{Cov}\left[Z_d(t),V_d(t)\right] = \sum_{i=1}^{n}\alpha_i\sigma^2[Y_i] \qquad (5)$$

where

$$\alpha_i = c_ic'_i + \frac{1}{2}\sum_{j=1,\,j\neq i}^{n}\left(c_ic'_j + c_jc'_i\right)A_{ij},$$

and $\sigma^2[Y_i] = \int_{-\infty}^{\infty} |H_i(\omega)|^2 S_{\ddot{U}\ddot{U}}(\omega)d\omega$. Notice that the response covariance is expressed as a sum of mode-specific variance contributions; this is an extension of the modal combination rule derived by Vanmarcke (1976),

$$\sigma_{Z_d}^2 = \sum_{i=1}^{n}\left\{c_i^2 + \sum_{j=1, j\neq i}^{n} c_i c_j A_{ij}\right\}\sigma_i^2, \quad (6)$$

which is obtained by letting $Z_d(t) = V_d(t)$ in Eq. (5).

The total response covariance is obtained by adding to Eq. (2) the contributions from the pseudo-static response and its cross-correlation with the dynamic response,

$$\begin{aligned}\text{Cov}[Z(t),V(t)] = &\sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l' \rho[U_k,U_l]\sigma[U_k]\sigma[U_l] + \\ &\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\{a_k c_{li}' + a_k' c_{li}\}\rho[U_k,Y_{li}]\sigma[U_k]\sigma[Y_{li}] + \quad (7)\\ &\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\{\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}A_{1,kli} - \phi_{kli}A_{3,kli}\}\times \\ &\sigma[Y_{ki}]\sigma[Y_{li}],\end{aligned}$$

where $a^T = \{a_1,...,a_m\}$ is the vector of effective influence coefficients which depends on structural properties; $\rho[U_k,U_l]$ is the cross-correlation coefficient between the ground displacements U_k and U_l; $\rho[U_k,Y_{li}]$ is the cross-correlation coefficient between U_k and the modal response Y_{li} and $\sigma[U_k]$ is the standard deviation of the ground displacement U_k.

The theory can be used to test the conditions under which it is valid to assume that motions are either statistically independent or fully coherent. Depending on the degree of correlation between motions at the supports, the response of a structure should approach that to fully coherent or statistically independent ground motions. For a given medium and ground motion covariance structure, the response would tend towards that for statistically, independent ground motion if the supports were very far apart, or approach the response for fully coherent motions if the supports were very close together.

The response covariance in Eq. (7) can be rewritten as follows:

$$\begin{aligned}\text{Cov}[Z(t),V(t)] = &\sum_{k=1}^{m}\left\{a_k a_k' \sigma_{U_k}^{\ 2} + \sum_{l=1,l\neq k}^{m} a_k a_l' \rho_{U_k U_l}\sigma_{U_k}\sigma_{U_l}\right\} + \\ &\sum_{k=1}^{m}\sum_{j=1}^{n}\left\{\{a_k c_{kj}' + a_k' c_{kj}\}\rho_{U_k Y_{kj}}\sigma_{U_k}\sigma_{Y_{kj}} + \right. \\ &\left.\sum_{l=1,l\neq k}^{m}\{a_k c_{lj}' + a_k' c_{lj}\}\rho_{U_k Y_{lj}}\sigma_{U_k}\sigma_{Y_{lj}}\right\} + \quad (8)\\ &\sum_{k=1}^{m}\sum_{i=1}^{n}\left\{\alpha_{kki}\sigma_{Y_{ki}}^2 + \right. \\ &\left.\sum_{l=1,l\neq k}^{m}\{\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}A_{1,kli} - \phi_{kli}A_{3,kli}\}\sigma_{Y_{ki}}\sigma_{Y_{li}}\right\}.\end{aligned}$$

Notice that the terms associated with sums over the subscript $l, l \neq k$ account for the contributions from the cross-correlation between ground motions at different supports, while the others account for the contributions to the response from each support ground motion. Let $\rho^*_{U_k U_l}, \rho^*_{U_k Y_{lj}}, \Gamma^*_{0,kli}$ denote cross-correlation coefficients for fully coherent ground motion; $\rho^*_{U_k U_l}, \Gamma^*_{0,kli} = 1$ if uniform soil conditions are considered (in which case the ground motions are perfectly correlated). If full coherence is assumed, then $A_{1,kli}, A_{3,kli} = 0$, $\gamma_{kl}(\omega) = 1$ for any k, l, and Eq. (8) becomes:

$$\begin{aligned}\text{Cov}_{FC}[Z(t),V(t)] = &\sum_{k=1}^{m}\left\{a_k a_k' \sigma_{U_k}^{\ 2} + \sum_{l=1,l\neq k}^{m} a_k a_l' \rho^*_{U_k U_l}\sigma_{U_k}\sigma_{U_l}\right\} + \\ &\sum_{k=1}^{m}\sum_{j=1}^{n}\left\{\{a_k c_{kj}' + a_k' c_{kj}\}\rho_{U_k Y_{kj}}\sigma_{U_k}\sigma_{Y_{kj}} + \right. \\ &\left.\sum_{l=1,l\neq k}^{m}\{a_k c_{lj}' + a_k' c_{lj}\}\rho^*_{U_k Y_{lj}}\sigma_{U_k}\sigma_{Y_{lj}}\right\} + \quad (9)\\ &\sum_{k=1}^{m}\sum_{i=1}^{n}\left\{\alpha_{kki}\sigma_{Y_{ki}}^2 + \sum_{l=1,l\neq k}^{m}\alpha_{kli}\Gamma^*_{0,kli}\sigma_{Y_{ki}}\sigma_{Y_{li}}\right\}.\end{aligned}$$

If ground motions are assumed to be statistically independent, then $\gamma_{kl}(\omega) = 0$, for any $k \neq l$, and the response covariance is then given by,

$$\begin{aligned}\text{Cov}_{SI}[Z(t),V(t)] = &\sum_{k=1}^{m} a_k a_k' \sigma_{U_k}^{\ 2} + \quad (10)\\ &\sum_{k=1}^{m}\sum_{j=1}^{n}\{a_k c_{kj}' + a_k' c_{kj}\}\rho_{U_k Y_{kj}}\sigma_{U_k}\sigma_{Y_{kj}} + \\ &\sum_{k=1}^{m}\sum_{i=1}^{n}\alpha_{kki}\sigma_{Y_{ki}}^2.\end{aligned}$$

Therefore, as $\gamma_{kl}(\omega) \to 1$, i.e the ground motions approach full coherence, $\rho_{U_k U_l} \to \rho^*_{U_k U_l}$, $\rho_{U_k Y_{lj}} \to \rho^*_{U_k Y_{lj}}$, $\Gamma_{0,kli} \to \Gamma^*_{0,kli}$, $A_{1,kli}$ and $A_{3,kli} \to 0$, an $\text{Cov}[Z(t),V(t)] \to \text{Cov}_{FC}[Z(t),V(t)]$. Also, when $\gamma_{kl}(\omega) \to 0$, the ground motions become statistically independent, $\rho_{U_k U_l}$, $\rho_{U_k Y_{lj}}$ and $\Gamma_{0,kli}$ all approach zero, and $\text{Cov}[Z(t),V(t)] \to \text{Cov}_{SI}[Z(t),V(t)]$.

3. RESPONSE SPECTRUM METHOD

Equation (7) provides the basis for a random-vibration-based response spectrum method for MS-MDOF systems. The standard deviation of the response quantity $Z(t)$, σ_Z, can be related to the spectral value $Z_{s,p}$, representing a mean peak response, given the duration s of stationary response and a specified exceedence probability p. One may write,

$$Z_{s,p} = K_{s,p}[Z]\sigma_Z, \quad (11)$$

where $K_{s,p}[Z]$ is a peak factor. When $p = 0.5$ one predicts the median response spectral values. The evaluation of the peak factors involves the solution of the first-crossing problem and is necessarily approximate. Under the assumption of Gaussian ground motion input and high level-crossings by the absolute value of the response, the peak factor is given by,

$$K_{s,p}[Z] = \sqrt{2\ln\{2[\Omega_Z s / 2\pi](-\ln p)^{-1}\}},$$

where Ω_Z is a characteristic frequency that depends on the second spectral moment of $Z(t)$.

The standard deviations of the ground displacement and the modal oscillator response, $\sigma[U_k]$ and $\sigma[Y_{ki}]$, can likewise be related to the peak displacement $U_{k,max}$ and the response spectrum ordinate $S_{d,k}(\omega_i, \xi_i)$ representing the peak relative displacement response to the ground motion $\ddot{U}_k$, as follows,

$$U_{k,max} = K_{s,p}[U_k]\sigma[U_k],$$

$$S_{d,k}(\omega_i, \xi_i) = K_{s,p}[Y_{ki}]\sigma[Y_{ki}],$$

where $K_{s,p}[U_k], K_{s,p}[Y_{ki}]$ are the ground displacement and modal response peak factors. For simplicity of the notation, the subscripts s and p have been omitted from $U_{k,max}$ and $S_{d,k}(\omega_i, \xi_i)$.

Substituting for $\sigma_Z, \sigma[U_k]$, and $\sigma[Y_{ki}]$ in Eq. (7) and setting $Z(t) = V(t)$, one obtains a rule combining modal response spectra and peak ground displacements for predicting the spectral value $Z_{s,p}$

$$Z_{s,p} = \Bigg[\sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l\, \rho[U_k, U_l]\frac{K_{s,p}^2[Z]}{K_{s,p}[U_k]K_{s,p}[U_l]}U_{k,max}U_{l,max} + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\{a_k c'_{li} + a'_k c_{li}\}\times \rho[U_k, Y_{li}]\frac{K_{s,p}^2[Z]}{K_{s,p}[U_k]K_{s,p}[Y_{li}]}U_{k,max}S_{d,l}(\omega_i, \xi_i) + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\{\alpha_{kli}\Gamma_{0,kli} - \theta_{kli}\Lambda_{1,kli} - \phi_{kli}\Lambda_{3,kli}\}\times \frac{K_{s,p}^2[Z]}{K_{s,p}[Y_{ki}]K_{s,p}[Y_{li}]}S_{d,k}(\omega_i, \xi_i)S_{d,l}(\omega_i, \xi_i)\Bigg]^{1/2}, \tag{12}$$

in which, following from Eq. (1),

$$\alpha_{kli} = c_{ki}c_{li} + \frac{1}{2}\sum_{j=1, j\neq i}^{n}\left(c_{ki}c_{lj} + c_{kj}c_{li}\right)A_{ij},$$

$$\theta_{kli} = \frac{2}{\omega_i}\sum_{j=1}^{n}\left(c_{ki}c_{lj} - c_{kj}c_{li}\right)A_{ij},$$

$$\phi_{kli} = \frac{2}{\omega_i^3}\sum_{j=1}^{n}\left(c_{ki}c_{lj} - c_{kj}c_{li}\right)B_{ij}.$$

Eq. (12) can be simplified further by neglecting the differences among the total response peak factor and those for the ground displacements and modal responses.

4. EFFECT OF SPATIAL VARIATION OF GROUND MOTION ON MS-MDOF RESPONSE

The response analysis of a continuous two-span beam was performed to illustrate the uses of the random-vibration, methodology and investigate the influence of uniform versus support-dependent local soil conditions, and of wave passage and incoherence effects on the response of the two-span beam. Several parametric studies of the response are performed to: (1) evaluate the significance of these effects; (2) examine the relative importance of the pseudo-static and dynamic components of the response, and (3) analyse the validity of commonly-adopted assumptions in engineering practice that the ground motions are either fully coherent or statistically independent.

We considered the two-span continuous beam shown in Figure 1 subjected to vertical ground motions at the supports. It is a steel beam with elastic modulus E= 2043050 *kg/cm²*, mass density ρ =0.00080 *kg.sec²/cm⁴*, area of cross-section A=1330 *cm²*, span L=50*m* and moment of Inertia I=1.315x10⁹*cm⁴*. The beam deforms due to bending only as shear and axial deformations are

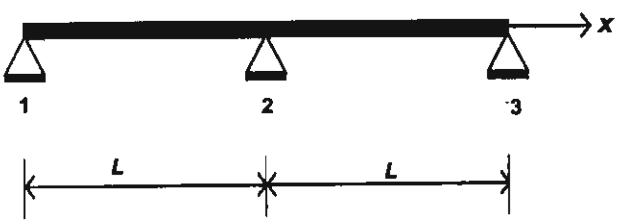

Figure 1. Two-span continuous beam.

neglected. The bending moment M has been taken as the response quantity of interest and its variance at different locations along the axis of the beam has been obtained. Based on the results of Zerva (1990), Harichandran and Wang (1990), and Der Kiureghian and Neuenhofer (1991) for similar two-span continuous beams and frequency content of the ground motions, only the first four modes were considered in the analysis. The fundamental natural frequency is $\omega_1 = 2\pi$ *rad/sec* and the first four mode shapes are shown in Figure 2; Table 1 lists the corresponding four modal frequencies. The odd-numbered modes are antisymmetric and the even-numbered ones symmetric. A critical damping ratio of 5% is chosen for all modes. The effective influence coefficients a_k and the modal participation factors c_{ki} are given in Tables 2 and 3. For convenience in the computations, M is scaled to the dimensionless bending moment 10³x*LM/EI*.

Table 1. Modal frequencies (*rad/sec*).

Mode No.	1	2	3	4
ω_i	6.28	9.81	25.12	31.79

The local spatial variation of ground motion due to wave passage, loss of coherence, and local soil conditions is accounted in the cross-spectral density function $S_{\ddot{U}_k\ddot{U}_l}(\omega)$,

$$S_{\ddot{U}_k\ddot{U}_l}(\omega) = \gamma_{kl}(\omega)\sqrt{S_{\ddot{U}_k\ddot{U}_k}(\omega)S_{\ddot{U}_l\ddot{U}_l}(\omega)}, \tag{13}$$

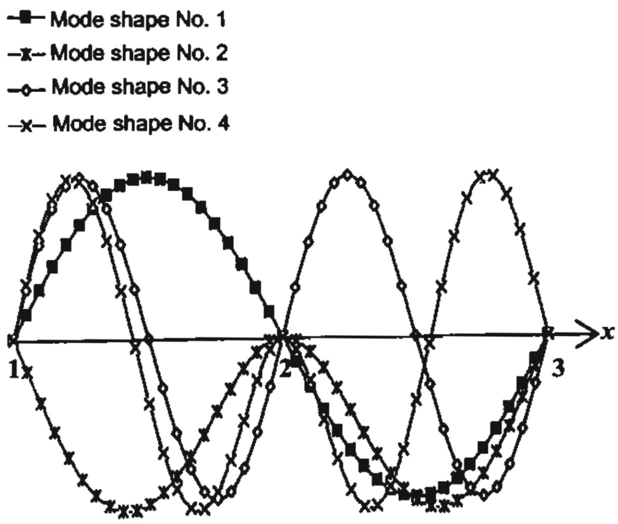

Figure 2. First four mode shapes.

Table 2. Effective influence coefficients[a].

Location	Support 1	Support 2	Support 3
L/8	3.75	-7.50	3.75
L/4	7.50	-15.00	7.50
3L/8	11.25	-22.50	11.25
L/2	15.00	-30.00	15.00
5L/8	18.75	-37.50	18.75
3L/4	22.50	-45.00	22.50
7L/8	26.25	-52.50	26.25
L	30.00	-60.00	30.00

[a] *These coefficients are symmetric with respect to the middle support.*

Table 3. Effective modal participation factors[a].

Location	Support	Mode *No.*1	Mode *No.*2	Mode *No.*3	Mode *No.*4
L/8	1	23.989	34.767	87.772	107.875
	2	0.000	101.541	0.000	-307.947
	3	-23.989	34.767	-87.772	107.875
L/4	1	44.438	61.184	125.640	139.258
	2	0.000	178.692	0.000	-397.536
	3	-44.438	61.184	-125.640	139.259
3L/8	1	57.861	71.245	87.772	66.882
	2	0.000	208.077	0.000	-190.925
	3	-57.861	71.245	-87.772	66.882
L/2	1	62.820	63.208	0.000	-51.301
	2	0.000	184.605	0.000	146.448
	3	-62.820	63.208	0.000	-51.301
5L/8	1	57.861	35.989	-87.772	-126.290
	2	0.000	105.109	0.000	360.517
	3	-57.861	35.989	87.772	-126.291
3L/4	1	44.473	-5.068	-125.640	-100.897
	2	0.000	-14.803	0.000	288.029
	3	-44.473	-5.068	125.640	-100.898
7L/8	1	23.989	-55.013	-87.772	27.494
	2	0.000	-160.671	0.000	-78.486
	3	-23.989	-55.013	87.772	27.494
L	1	0.000	-107.612	0.000	198.066
	2	0.000	-314.290	-0.000	-565.414
	3	0.000	-107.612	0.000	198.067

[a] *These factors are antisymmetric with respect to the middle support.*

where $\gamma_{kl}(\omega)$ is the coherency spectrum. Here we assume the following functional form for $\gamma_{kl}(\omega)$:

$$\gamma_{kl}(\omega) = \exp\left\{-\left\{\frac{\eta\omega\ |x_{kl}|}{v_s}\right\}^2\right\}\exp\left\{-i\frac{\omega v . x_{kl}}{v^2}\right\}, \qquad (14)$$

in which η is an incoherence factor, x_{kl} is the relative position vector, v_s is a representative shear wave velocity of the medium, and v is the apparent wave velocity vector (Luco and Wong 1986). The phase spectrum in the argument of the second exponential accounts for wave passage effects (Harichandran and Vanmarcke 1986). Whenever wave passage effects are included, it will be assumed that the direction of propagation is from support 1 to support 3.

The possible variation of local soil conditions is modeled through location-dependent power spectral density functions $S_{\ddot{U}_k\ddot{U}_k}(\omega), k = 1,2,...m,$ of the modified Kanai-Tajimi type (Clough and Penzien 1975):

$$S_{\ddot{U}_k\ddot{U}_k}(\omega) = \frac{\left[1+4\xi_{fk}^2(\omega/\omega_{fk})^2\right]G_{0k}}{\left[1-(\omega/\omega_{fk})^2\right]^2+4\xi_{fk}^2(\omega/\omega_{fk})^2}\times$$

$$\frac{(\omega/\omega_{gk})^4}{\left[1-(\omega/\omega_{gk})^2\right]^2+4\xi_{gk}^2(\omega/\omega_{gk})^2}, \qquad (15)$$

where the parameters ω_{fk} and ω_{gk}, ξ_{fk} and ξ_{gk}, may be thought of as characteristic ground frequencies and dampings, respectively. This model does not cause the variances of the velocity and ground displacement to be unbounded. Recently, some work has been conducted to study the effect of soil conditions on the coherency spectrum (Somerville et al 1991; Schneider et al 1992). As models are developed for different soil types, they could be used in consistent way with the corresponding spectral density functions.

Sites with different soil conditions have been modeled here using the modified Kanai-Tajimi spectral density in Eq. (15). Table 4 lists the values of the natural frequencies and dampings for sites with "firm" and "soft" soil conditions; these values are taken from Der Kiureghian and Neuenhofer (1991) and produce reasonable shapes for the one-sided spectral density functions shown in Figure 3.

4.1. Uniform Local Soil Conditions

Consider first the case where the soil conditions, either soft or firm, at the three supports of the beam are identical; thus $S_{\ddot{U}_k\ddot{U}_l}(\omega) = S_{\ddot{U}\ddot{U}}(\omega)\gamma_{kl}(\omega), k,l = 1,2,3,$ where $S_{\ddot{U}\ddot{U}}$ is the ground acceleration spectral density. To investigate the relative influence of incoherence and wave passage effects, the following coherency models were examined:

- Case I. Fully coherent motions at all supports: $v_s/\eta = \infty, v = \infty$ and $\gamma_{kl}(\omega) = 1$, for any k,l.

Table 4. Spectral density function parameters for soil conditions.

Soil Type	ω_f	ξ_f	ω_g	ξ_g
Firm	15.0	0.6	1.5	0.6
Soft	5.0	0.2	0.5	0.6

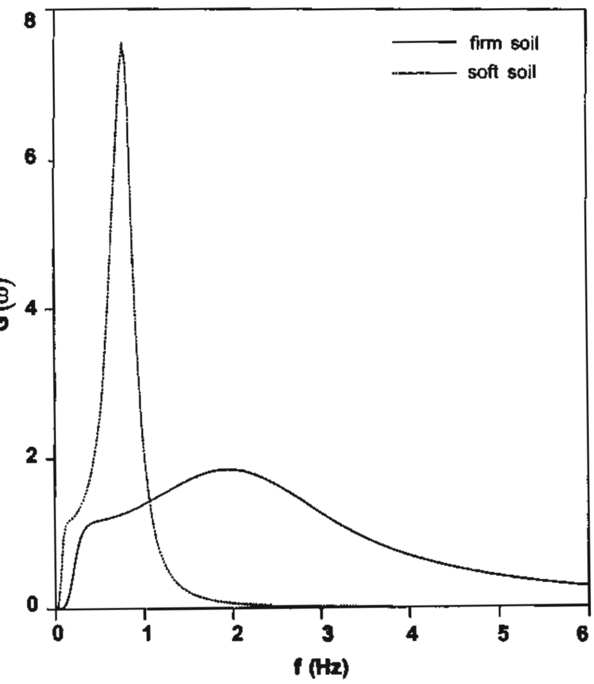

Figure 3. Ground acceleration one-sided power spectral density function for sites with firm and soft soil conditions.

- Case II. Only wave passage effects: $v_s/\eta = \infty$ and $v = 400$ *m/sec*.
- Case III. Only incoherence effects: $v_s/\eta = 500$ *m/sec* and $v = \infty$.
- Case IV. Both wave passage and incoherence effects: $v_s/\eta = 500$*m/sec*, $v = 400$ *m/sec*.

The first case is included for comparison purposes and is based on the usual assumption in deterministic structural analysis of equal support motions.

The results are presented in terms of bending moment standard deviations normalized with respect to the maximum standard deviation along the beam for the case of fully coherent ground motion (Case I). Assuming that the peak factor is the same for the bending moment at all locations along the axis of the beam, the normalized standard deviation represents the *ratio* of the peak bending moment at each location to the maximum peak moment along the beam for fully coherent ground motions (see Eq. (11)). The normalized bending moment standard deviation may therefore be referred to here as a ratio of peak bending moments or simply as a normalized peak response.

Figures 4 and 5 show plots of the normalized bending moment standard deviation at points x, $x=(L/8)x_L$, along the axis of the beam for soft and firm soil conditions, respectively. Although the structure is symmetric about the middle support, the response is not symmetric in the presence of wave passage effects (Case II and IV). The symmetry or antisymmetry of the response can be shown from the cross-spectral density function of the ground motion for each case considered.

For fully coherent ground motion (Case I) and for both soil conditions, the maximum peak response occurs at the middle support. This response is also the largest peak response among all four cases. When the beam is on soft soil the peak response at most locations along the spans is greater for Cases II, III and IV than for the fully coherent case (see Figure 4). The maximum peak response at the midspans when considering wave passage and incoherence effects is 66% greater than that to fully coherent motion. By comparing Cases II and III, it seems that wave passage effects have a greater influence on the response at locations along the right span (between supports 2 and 3). Although Case IV (both wave passage and incoherence effects included) produced the highest peak moment at the right span, there is not a significant difference in the response between Case II and Case IV. The results in Figure 5 for firm soils show that the local soil conditions have an important effect on the response. In this case, fully coherent ground motion produced the highest peak response at most locations along the spans. It is important to notice also that there is not a significant difference in the response between cases II to IV.

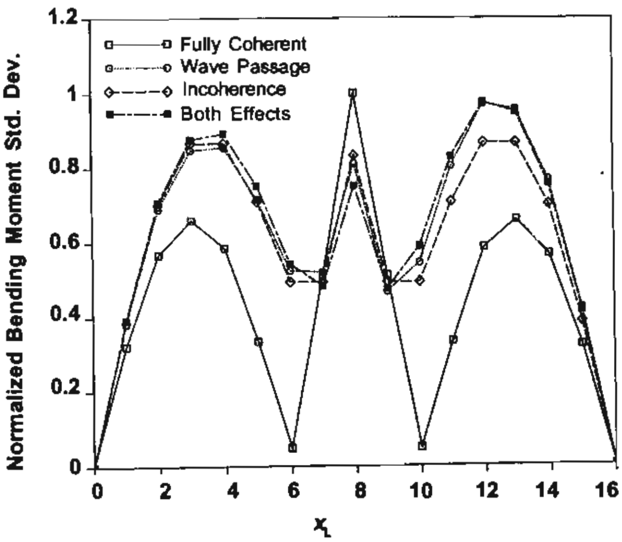

Figure 4. Normalized bending moment standard deviation for location *x* along the axis of the beam. Uniform soft soil conditions.

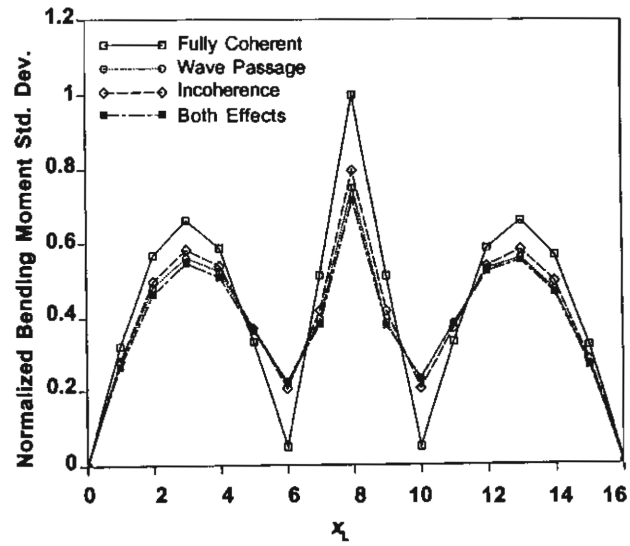

Figure 5. Normalized bending moment standard deviation for location *x* along the axis of the beam. Uniform soft soil conditions.

Tables 5 and 6 list the contributions to the total response variances at the midspans and the middle support from the pseudo-static and purely dynamic responses, first and third terms in Eq. (7), and the cross-correlation between them, second term in Eq. (7). The response to fully coherent ground motion consists only of purely dynamic terms since rigid body motions do not induce internal forces in the beam. It is seen that for both soil conditions the pseudo-static contribution is in general very small. Der Kiureghian and Neuenhofer (1991) found similar results in their response spectrum analysis where the ground motion density function is consistent with a prescribed mean response spectrum. Here, it is shown that the response at all three locations, is controlled by the combination of the purely dynamic and cross-correlation terms. These results suggests that for Cases II, III and IV, ground displacements at the three supports are highly correlated and therefore introduce small internal forces in the beam. Notice also that, as observed by Harichandran and Wang (1990), the cross-correlation contributions to the response at the middle support are negative.

In the analysis that follows only a dynamic response consisting of the purely dynamic and cross-correlation terms is considered. The contributions from each mode to the dynamic response variance are given in Tables 7 and 8 for soft and firm soil conditions, respectively. It is seen that fully coherent ground motion (Case I) excites only the symmetric modes of the purely dynamic response; at the midspans and middle support the second mode constitutes 99.9% of the response. Case I produces the highest peak response at the middle support for both soil conditions, which exceeds the sum of a dominant second mode and a small fourth mode contribution in the other cases.

For Cases II, III and IV, the response at the midspans consists mainly of contributions from the first and second modes, and from the second mode for the response at the middle support. These three locations are inflection points for the third mode shape and their corresponding bending moments are zero. The middle support is also an inflection point for the first mode shape and therefore the response there is dominated by the second mode. Notice that although the first mode (antisymmetric) is excited and contributes to the total response, the response is symmetric

Table 5. Contribution to total response from the purely dynamic, pseudo-static, and cross-correlation components. Uniform soft soil conditions.

Location	Case	Purely Dynamic	Cross-Terms	Pseudo-Static
L/2	I	100	0	0
	II	98.69	1.23	0.08
	III	97.01	2.71	0.28
	IV	98.25	1.10	0.65
L	I	100	0	0
	II	110.32	-10.67	0.35
	III	109.20	-10.42	1.22
	IV	116.02	-19.68	3.66
3L/2	I	100	0	0
	II	96.62	3.32	0.06
	III	97.01	2.71	0.28
	IV	93.84	5.62	0.54

Table 6. Contribution to total response from the purely dynamic, pseudo-static, and cross-correlation components. Uniform firm soil conditions.

Location	Case	Purely Dynamic	Cross-Terms	Pseudo-Static
L/2	I	100	0	0
	II	99.88	0.06	0.06
	III	99.19	0.7	0.11
	IV	99.67	0.08	0.25
L	I	100	0	0
	II	101.27	-1.38	0.11
	III	101.06	-1.25	0.19
	IV	101.90	-2.39	0.49
3L/2	I	100	0	0
	II	98.69	1.26	0.05
	III	99.19	0.70	0.11
	IV	97.52	2.25	0.23

about the middle support when no wave passage is considered (Case III). For the fourth mode, the modal frequency is located past the dominant frequencies of both soft and firm soils, and thus its contribution to the response is very small. Notice that for firm soils, where the fourth modal frequency is closer to the peak spectral density frequency, the participation of this mode is higher than for soft soils.

Recalling the results in Figures 4 and 5 we see that at the midspans, where fully coherent motions on soft soils underestimate the peak response, the first mode is dominant. In order for the peak response at the midspans to be greater than that for fully coherent motion, the first mode has to dominate, so that the sum of its contribution and that from the second mode exceeds the second mode term that constitutes largely the peak response to fully coherent ground motion. This is partly because of the fact that for soft soils, the first natural frequency falls closer than the second modal frequency to the dominant frequencies of the ground power spectral density, and thus its contribution to the total response is larger. In the case of firm soils the opposite occurs, the second modal frequency falls within the range of the motion's dominant frequencies and has a larger contribution than that of the first mode. An important conclusion here is that the soil conditions play a crucial role in evaluating the response to spatially varying ground motion.

4.2. Varying Local Soil Conditions

In this section we study the effect of different soil conditions at the different supports of the beam on the response. We consider the case where the end supports are on firm

Table 7. Modal contribution to dynamic response. Uniform soft soil conditions.

Location	Case	Mode *No.*1	Mode *No.*2	Mode *No.*3	Mode *No.*4
L/2	I	0	99.99	0	0.01
	II	62.15	37.79	0	0.06
	III	62.65	37.26	0	0.09
	IV	67.32	32.52	0	0.16
L	I	0	99.96	0	0.04
	II	0	100.15	0	-0.14
	III	0	100.44	0	-0.44
	IV	0	101.27	0	-1.26
3L/2	I	0	99.99	0	0.01
	II	72.16	27.79	0	0.05
	III	62.65	37.26	0	0.09
	IV	73.24	26.63	0	0.13

Table 8. Modal contribution to dynamic response. Uniform firm soil conditions.

Location	Case	Mode *No.*1	Mode *No.*2	Mode *No.*3	Mode *No.*4
L/2	I	0	99.98	0	0.02
	II	24.77	74.65	0	0.58
	III	23.29	76.36	0	0.35
	IV	28.33	71.22	0	0.45
L	I	0	99.91	0	0.09
	II	0	96.87	0	3.13
	III	0	98.51	0	1.49
	IV	0	98.25	0	1.75
3L/2	I	0	99.98	0	0.02
	II	36.75	62.64	0	0.61
	III	23.29	76.36	0	0.35
	IV	35.23	64.35	0	0.42

soil and the middle one rests on soft soil conditions. This example is intended to resemble the case of a two-span bridge modeled as a continuous beam, with the middle support found on a stratum much softer than that for the end supports. It is assumed that the shear wave velocity v_s and the apparent wave velocity v in the coherency model are average velocities representative of the medium with varying soil conditions.

The same cases, I to IV, for the coherency spectrum in the previous section have been considered here. Figure 6 shows plots of the normalized bending moment standard deviation along the axis of the beam for each case. Two important observations can be made from the results shown. First, the peak response increases steadily from the end supports towards the middle one, where the maximum occurs. Second, there is very little difference in the response of the beam to the four cases considered; the peak bending moments are slightly higher only around the middle support for fully coherent motions. Based on these results, it seems that the response is controlled by the varying soil conditions, and that the incoherence and wave passage effects are not important. In fact, since the apparent wave velocities for Cases III and IV are $v = \infty$ and, $v = 400$ *m/sec*, respectively, wave passage effects seem

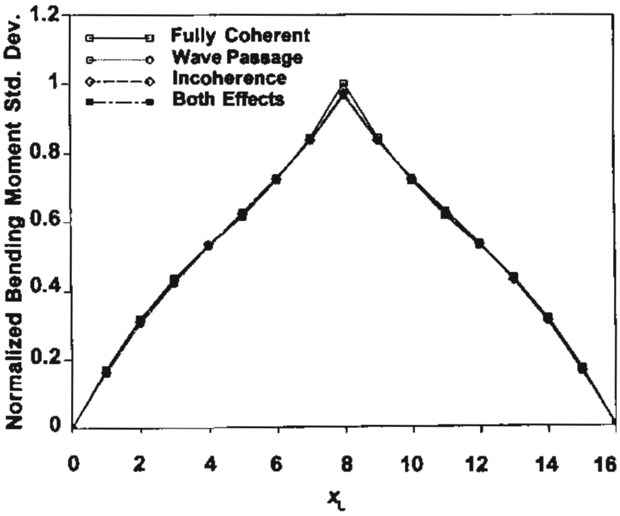

Figure 6. Normalized bending moment standard deviation for location x along the axis of the beam. Varying soil conditions: soft soil at middle support.

to have very little influence on the response. The same comment holds for the incoherence effects in Cases II and IV.

These results suggest that the response is controlled by the pseudo-static component. Table 9 shows the contributions to the total response variance from the pseudo-static, purely dynamic, and cross-correlation terms. It is seen that indeed, the response is dominated by the pseudo-static terms which constitute about 80% and at least 91% of the total response at the midspans and middle support, respectively. This implies that for the wave passage and incoherence effects considered, the ground displacements between soft and firm soils are not highly correlated and introduce large internal forces in the beam.

To study the influence of varying the soil conditions at the middle support from firm to soft on the response of the beam, two intermediate soil conditions, termed here medium "A" and "B", have been used. Their natural frequencies and critical damping ratios are given in Table 10. For each case I to IV, four soil cases are considered depending on the conditions at the middle support (the end supports on firm soil): Case (1) uniform firm soil; Case (2) medium A; Case (3) medium B; and Case (4) soft soil. Plots of normalized response standard deviations along the axis of the beam are shown in Figure 7. These results are normalized with respect to the maximum standard deviation along the beam obtained in each case I to IV for uniform firm soil conditions. It is seen that the peak response increases as the soil conditions at the middle support become softer. When the middle support rests on soft soil, the peak response is more than two times that in cases II to IV for uniform firm soils. Thus, accounting for different local soil conditions is of major importance in the presence of wave passage and/or incoherence effects.

Table 10. Spectral density function parameters for medium soil conditions.

Soil Type	ω_f	ξ_f	ω_g	ξ_g
Medium A	10.0	0.4	1.0	0.60
Medium B	7.0	0.25	0.7	0.60

Table 11 shows the contributions from the pseudo-static and dynamic terms to the total response variance for Case IV (wave passage and incoherence effects) for varying soil conditions at the middle support. As the soil at the middle support approaches the soft condition, the dynamic terms become less dominant and the pseudo-static response becomes the controlling factor. Whereas for uniform firm soil conditions the dynamic terms are responsible for 99% of the total response at the three locations considered, for soft soil conditions at the middle support,

Table 9. Contribution to total response from the purely dynamic, pseudo-static, and cross-correlation components. Soft and firm soil conditions at middle and end supports, respectively.

Location	Case	Purely Dynamic	Cross-Terms	Pseudo-Static
L/2	I	15.42	3.91	80.67
	II	14.43	3.82	81.75
	III	15.80	4.12	80.08
	IV	15.16	3.84	81.00
L	I	12.71	-3.97	91.26
	II	6.24	-4.51	98.27
	III	8.07	-4.45	96.38
	IV	6.73	-4.66	97.93
3L/2	I	15.42	3.91	80.67
	II	16.41	4.41	79.18
	III	15.80	4.12	80.08
	IV	16.47	4.61	78.92

Table 11. Contribution to total response from the purely dynamic, pseudo-static, and cross-correlation components. Varying soil conditions at middle support and firm soil conditions at end supports. Case IV: wave passage and incoherence effects.

Location	Case	Purely Dynamic	Cross-Terms	Pseudo-Static
L/2	1	99.67	0.08	0.25
	2	87.32	2.44	10.24
	3	48.27	4.65	47.08
	4	15.16	3.84	81.00
L	1	101.90	-2.39	0.49
	2	87.05	-5.67	18.62
	3	36.47	-8.06	71.59
	4	6.73	-4.66	97.93
3L/2	1	97.51	2.26	0.23
	2	85.94	4.31	9.75
	3	49.37	6.73	43.90
	4	16.47	4.61	78.92

the dynamic terms account for only about 20% and 2% of the total response at the midspans and middle support, respectively.

4.3. Wave Passage Effects

To study the influence of wave passage effects on the response of the beam, four apparent wave velocities have been used: Case (a) $v = \infty$, Case (b), $v = 1600$ *m/sec*, Case (c) $v = 400$ *m/sec*, Case (d) $v = 100$ *m/sec*. The value of the incoherence effect has been kept constant, $v_s/\eta = 500$ *m/sec*.

Figures 8 and 9 show plots of normalized bending moment standard deviation along the axis of beam for uniform soft and firm soil conditions, respectively. It can be noted that for both soil conditions the peak response at the middle support is reduced due to the effect of wave passage; the smaller the apparent wave velocity, the smaller the peak response. This is consistent with the results obtained previously by other researchers. Hao (1989) analyzed the effect of ground motion phase differences on the response of a three-span continuous beam by simulating spatially correlated ground motions with different apparent wave velocities. The results of his time history-based analysis at the midpoint of the central span showed that the response was reduced with increasing apparent velocities. Here, however, it is shown that this effect depends on the location along the beam where the response is evaluated. For soft soils Figure 8 shows that at locations along the inner half-spans of the beam, such relationship is reversed and smaller apparent velocities yield higher peak bending moments. For each case (a) to (d), the maximum peak response for soft soils occurs now at the midspan between supports 2 and 3; Case (c), $v = 400$ *m/sec* produces the highest overall response. At locations along the inner half-spans the peak response is the smallest when wave passage effects are not considered.

In contrast with soft soil conditions, Figure 9, the maximum peak response on firm soil occurred for all cases at the middle support, and the highest peak bending moment was obtained when no wave passage effects were considered. Notice that for firm soils, decreasing the apparent wave velocity reduces the response at almost all locations along the axis of the beam.

The same four apparent wave velocities were used to study the influence of wave passage for the case of different soil conditions where the middle support rests on soft soil. The results are shown in Figure 10. As expected from the results shown in Figure 6, and the discussion in the previous section, it is seen that in fact wave passage effects have very little influence on the response due to the dominance of the pseudo-static terms. For instance, while the pseudo-static terms were found to be responsible for about 80% of the total response variance at the midspans in all four wave passage cases, in the case on uniform soil conditions the largest pseudo-static contributions obtained were only about 9% and 3% for soft and firm soil conditions, respectively, when $v = 100$ *m/sec*.

4.4. Incoherence Effects

We now analyse the influence of coherence between input motions at different supports on the response of the two-span beam, by widely varying the incoherence parameter v_s/η. Uniform soft and firm soil conditions are considered and wave passage is not included $(v = \infty)$.

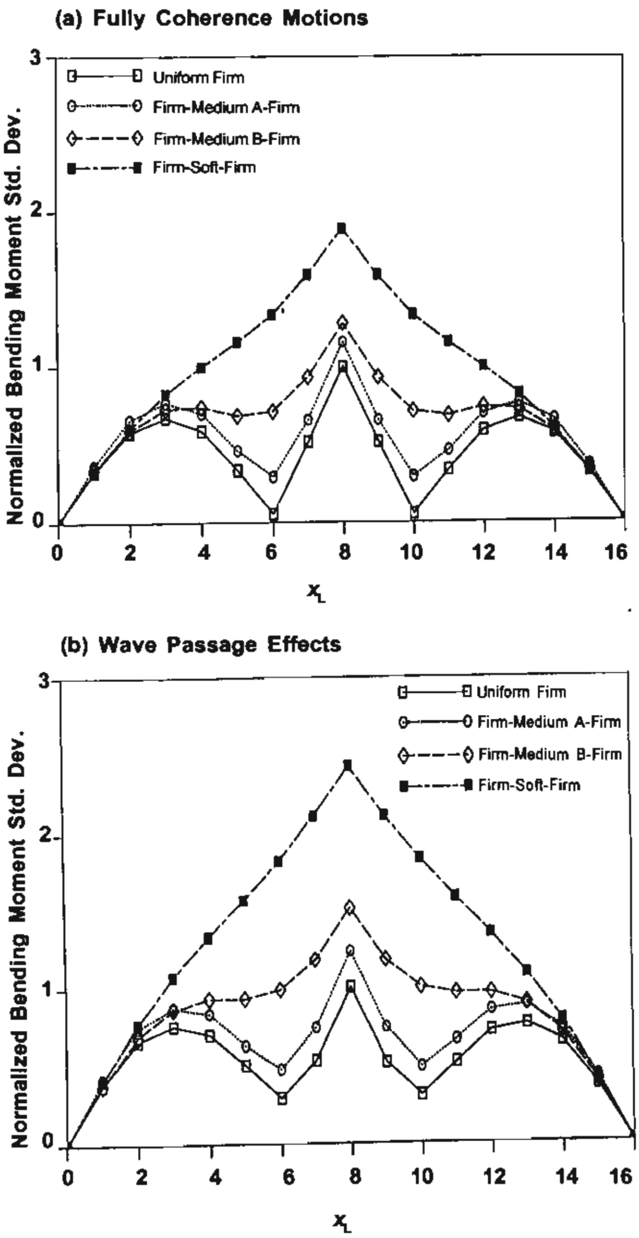

Figure 7(a), (b). Normalized bending moment standard deviation for (a) Fully coherent motions, (b) Wave passage effects. Varying soil conditions at middle support.

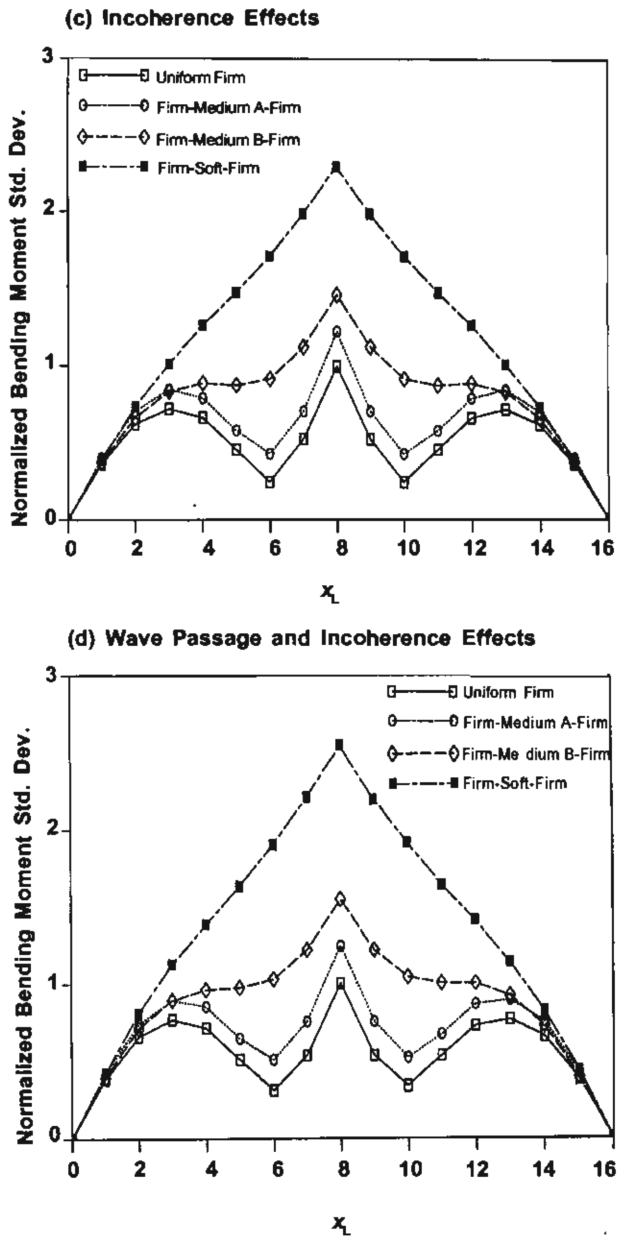

Figure 7(c), (d). Normalized bending moment standard deviation for (c) Incoherenc, and (d) Wave pasage and incoherence effects. Varying soil conditions at middle support.

Results were obtained for values of v_s / η ranging from 0.5 *m/sec* to 6250 *m/sec*. Since wave passage effects are not taken into account, the response of the beam is symmetric about the middle support and results need to be presented for locations along one span only. The bending moment standard deviation at each location along the beam has been normalized with respect to its value corresponding to the case of statistically independent ground-motion inputs. In other words, the normalized response standard deviation equals the ratio of the peak bending moment to the peak moment for statistically independent ground motions.

Results are shown in Figures 11 and 12 for several locations along the left span of the beam for soft and firm soil conditions respectively. The results show that the peak response of the beam progresses from that for fully coherent to that statistically independent ground motion.

For soft soil conditions, Figure 11 shows that the peak response under the full coherence assumption is smaller than under statistically independent motions. As correlation decreases the peak response increases steadily from that for fully coherent towards that for statistically independent ground motion. At the middle support the peak response is smaller than that for full coherence if v_s / η is greater than 80 *m/sec*. At all locations considered along the span of the beam, assuming fully coherent ground motion would give unconservative results for an incoherence parameter v_s / η less than 50 *m/sec*; whereas assuming statistical independence would greatly overestimate the response if v_s / η is greater than about 50 *m/sec*.

In the case of firm soils, see Figure 12, the results

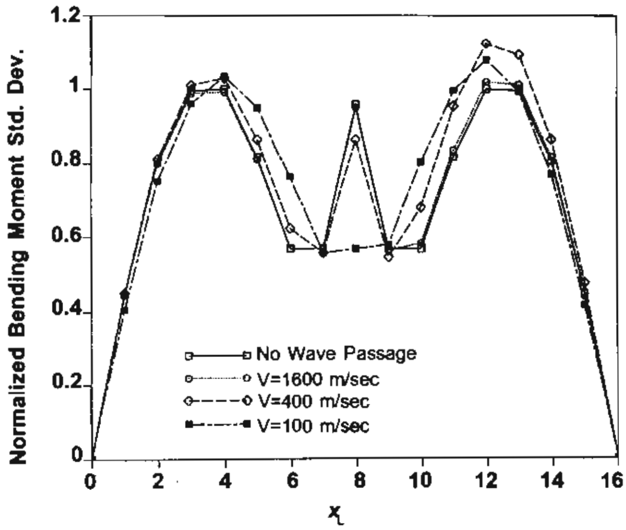

Figure 8. Normalized bending moment standard deviation for location x along the axis of the beam. Influence of wave passage effects. Uniform soft soil conditions.

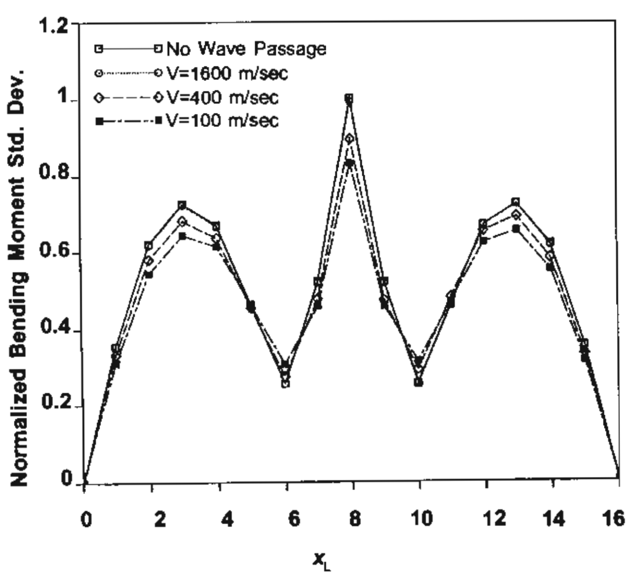

Figure 9. Normalized bending moment standard deviation for location *x* along the axis of the beam. Influence of wave passage effects. Uniform soft soil conditions.

show that the response varies significantly depending on the location , *x*, along the axis of the beam. At some locations (*x=L/8, L/4, 3L/8*, and at the middle support, *x=L*), the peak response under full coherence is greater than that for statistically independent ground motion. At other locations *x=3L/4* and *5L/8*, it increases steadily as correlation decreases, from the fully coherent to the statistically independent ground motion case. The peak response may also be less than for both the cases of statistically independent and fully coherent ground motions, as when *x=7L/8, L/2* and *L*. Hence, assuming either for the analysis of the beam, might yield conservative or unconservative results depending on the location *x*. For instance, for $v_s/\eta = 200$ *m/sec*, assuming statistical independence would give reasonable results for the peak bending moments at locations *x=L/8, L/4* and *3L/8*; but would greatly overestimate the response at *x=3L/4* and

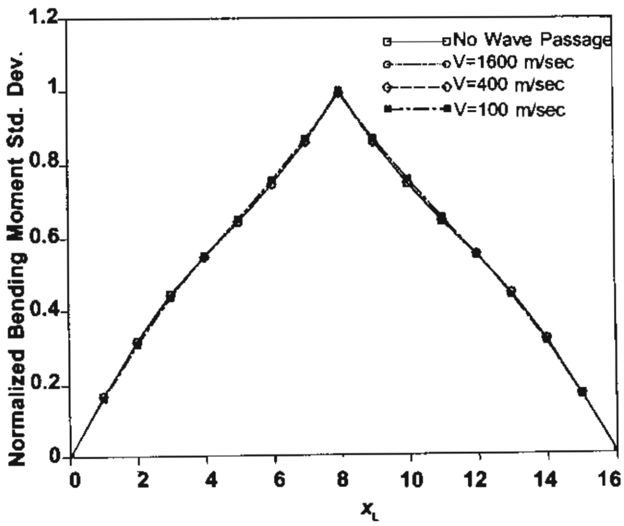

Figure 10. Normalized bending moment standard deviation for location *x* along the axis of the beam. Influence of wave passage effects. Varying soil conditions: soft soil at middle support.

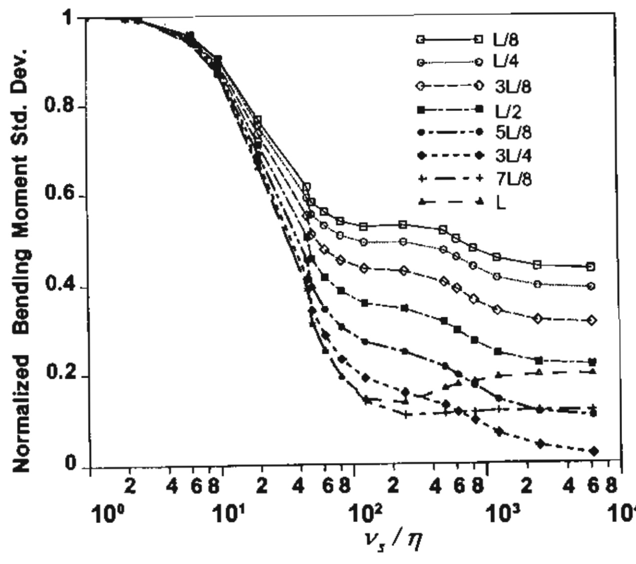

Figure 11. Normalized bending moment standard deviation as a function of the incoherence parameter for uniform soft soil conditions.

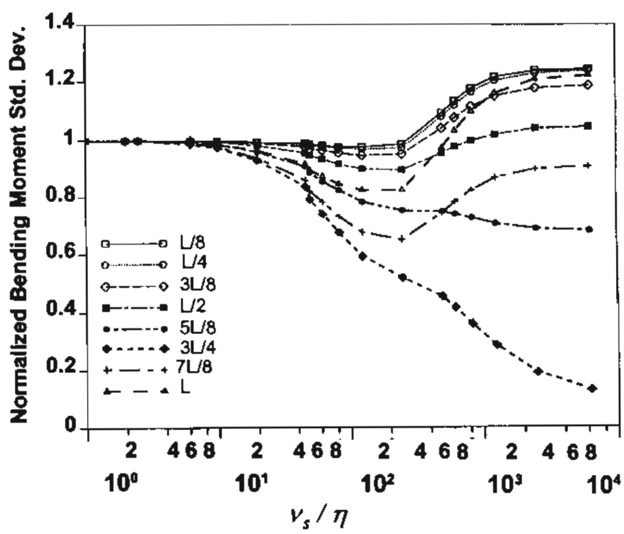

Figure 12. Normalized bending moment standard deviation as a function of the incoherence parameter for uniform firm soil conditions.

5L/8. Moreover, it would be unconservative to assume fully coherent ground motions to compute the response at these two locations, while for the other three locations such an assumption would overestimate the peak response.

For a similar two-span beam with $L = 60m$, Zerva (1990) found that on firm soils, the response variance at the middle support decreases as the incoherence parameter v_s / η takes the values 1000, 500 and 200*m/sec*. The results in Figure 12 indicate, however, that for $v_s / \eta <$ 200*m/sec*, the peak response does not necessarily keep decreasing as $v_s / \eta \rightarrow 0$. For instance, at the middle support the peak bending moment increases as the statistically independent case is approached.

Figures 13 and 14 show the pseudo-static and purely dynamic contributions to the total response variance, and the contribution from the cross-correlation between them, at the midspan and middle support for soft and firm soil conditions, respectively. Each response variance has been normalized by its corresponding value for statistically independent ground motions, and thus all of them approach one as $v_s / \eta \rightarrow 0$. The pseudo-static and cross-correlation terms decrease steadily as the correlation

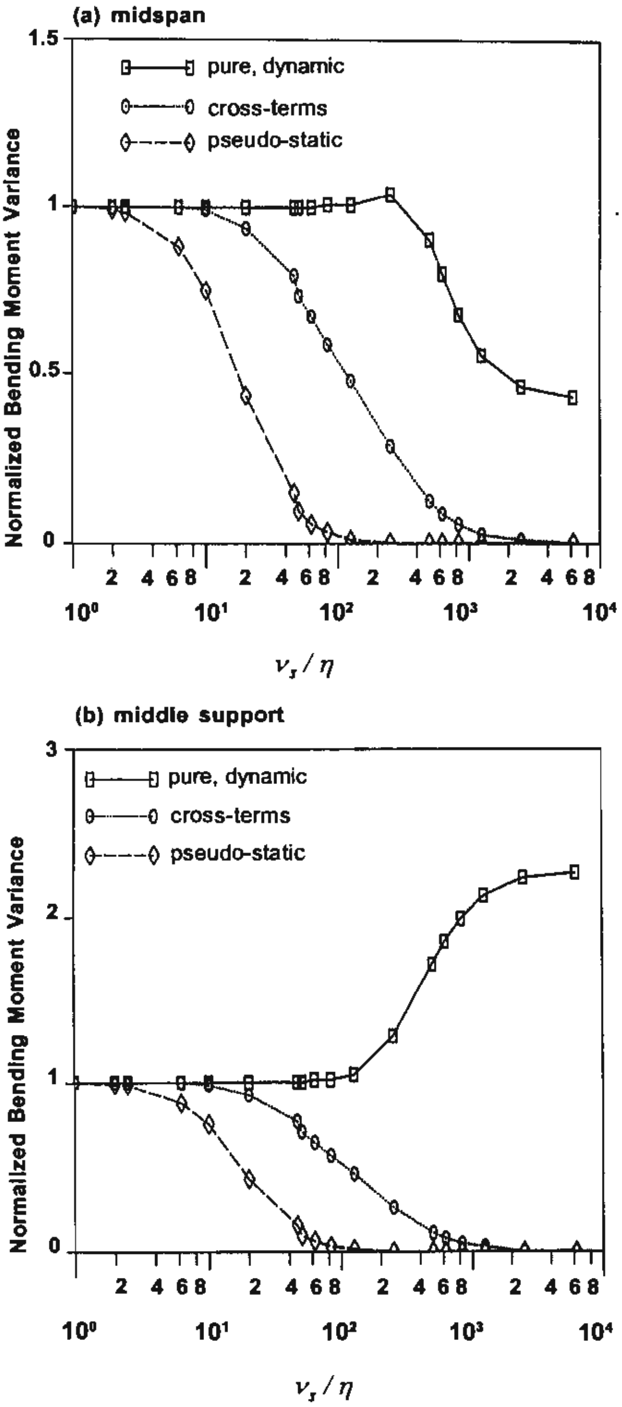

Figure 13. Purely dynamic, pseudo-static and cross-correlation terms as a function of the incoherence parameter for uniform soft soil conditions. Location (a) midspan, (b) middle support.

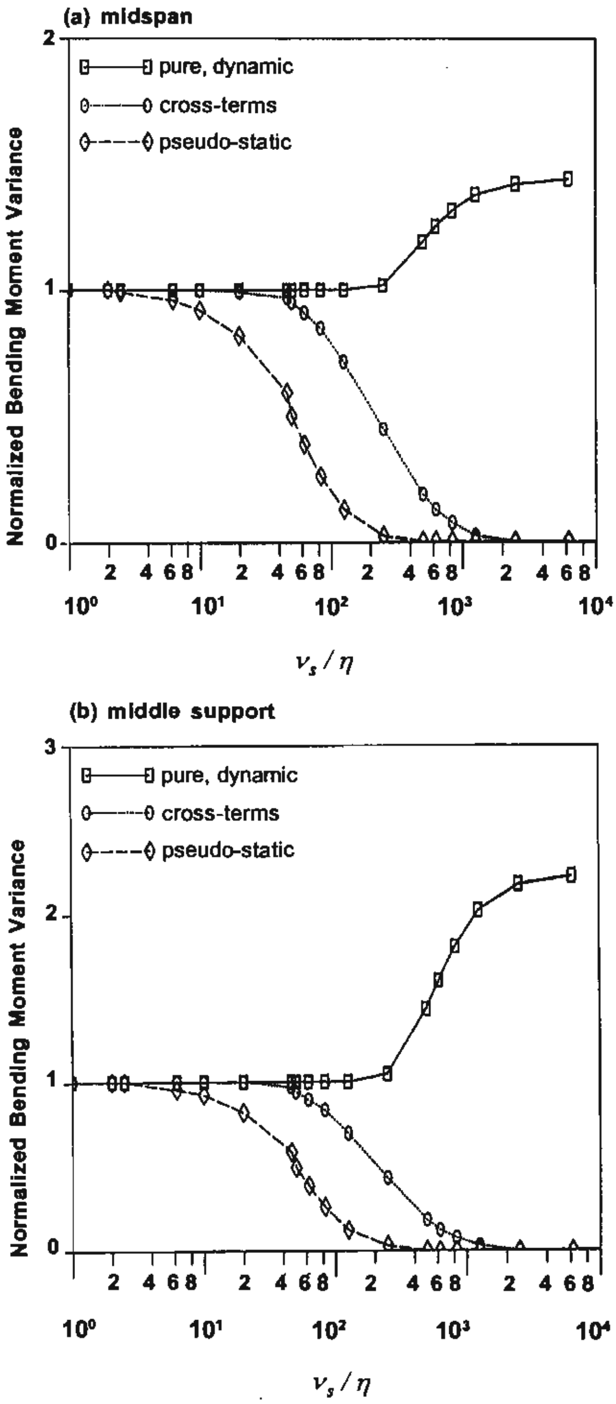

Figure 14. Purely dynamic, pseudo-static and cross-correlation terms as a function of the incoherence parameter for uniform firm soil conditions. Location (a) midspan, (b) middle support.

increases, and approach zero as $v_s / \eta \rightarrow \infty$. Comparing these results with those in Figures 11 and 12, indicates that the total response often can not be represented adequately by the purely dynamic response only.

5. CONCLUSIONS

A random vibration approach previously developed by the authors for the response analysis of linear multi-support structural systems has been briefly outlined. It allows the seismic response of such systems to be expressed in terms of a series of independent one-degree modal oscillators accounting fully for the multi-support input and the ground motion spatial variation. The approach serves as the basis for a response spectrum method for the analysis of multi-support structural systems. Peak factors are used to predict the spectral values of response quantities of interest in terms of their standard deviations. The seismic response of a two-span continuous beam subjected to vertical ground motions has been evaluated. Normalized bending moment standard deviations were obtained at different locations along the axis of the beam. The influence of uniform and varying local soil conditions, wave passage, and incoherence effects on the response were studied. The response-spectrum method presented here enables comprehensive parametric studies of the beam response with a relatively computational ease.

Soil conditions were found to play an important role in the response to spatially varying ground motion. Whereas for soft soils the peak bending moment when considering wave passage and incoherence effects is 66% greater than that for fully coherent motions, for firm soils the largest peak response is obtained under the assumption of full coherence. For the cases examined the pseudo-static contribution was in general very small for both soil conditions and the response was controlled by the purely dynamic and cross-correlation terms. The influence of varying soil conditions at the middle support was also studied. As the soil conditions at the middle support become softer the response increases and the maximum peak response is obtained at the middle support. In the presence of wave passage and/or incoherence effects the maximum peak response for soft soil at the middle support is at least two times that for uniform firm soil conditions. Varying ground motion coherence allows one to assess the validity of the common assumptions of statistical independence and full coherence. These assumptions may give conservative or unconservative results depending on the soil conditions, the location along the axis of the beam, and the degree of correlation between ground motions at the supports.

The significant and systematic effect of local soil conditions on the response of MS-MDOF systems should be further investigated using site-dependent models for the coherency spectrum of ground motion. Before general recommendations for the design of multi-support systems can be made, other factors such as the influence of wave propagation direction and the effect of multiple components of ground motion should be studied. Also, more complex structures need to be analyzed to gain further insight into the effect of spatial variation of ground motion on structural response.

ACKNOWLEDGMENTS

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