Research Paper

Equivalent Linear Analysis of Semi-Infinite Free-Field Column Using PML

Mohammad Davoodi[1], Reza Afzalsoltani[2*] and Abbas Pourdeilami[3]

1. Associate Professor, Geotechnical Engineering Research Centre, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran
2. Ph.D. Candidate, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran,
*Corresponding Author; email: reza.afzalsoltani@gmail.com
3. Assistant Professor, School of Engineering, Damghan University, Damghan, Iran

Received: 25/03/2023
Revised: 04/05/2023
Accepted: 24/05/2023

ABSTRACT

One of the important factors in determining the response of the structures is the correct evaluation of the input motion. This input excitation can be affected by various factors such as the propagation of waves in different layers, site effects, interaction of soil-structure, etc. Another problem with the numerical analysis is the use of appropriate absorbing boundaries to prevent the return of scattered waves into the analysis environment. Besides, the non-linear behavior of construction materials can also change the propagated waves, which makes the problem more complicated. One of the simple methods to estimate the nonlinear behavior of materials is to use the equivalent linear analysis method, which is still used due to its simplicity and ease of use. In this research, by preparing the finite element time domain dynamic analysis code using C programming language, the response of the free field, which is the first step in estimating the soil-structure interaction effect, has been evaluated using the equivalent linear analysis method. In addition, to increase the accuracy of the results, radiation damping simulation by perfectly matched layers (PML) has been implemented. This program uses four-nodded quadrilateral elements and the implicit Newmark method to solve the dynamic equation. For using PML in the equivalent linear method, the PML properties were updated based on adjacent elements to avoid reflection from boundaries. The results showed that the nonlinear behavior of materials can change responses significantly in a way that it be far away from results of the linear analysis. Furthermore, the results showed that the procedure adopted to perform equivalent linear analysis using PML is efficient.

Keywords:
Wave propagation; Perfectly matched layer; Free-field; Equivalent linear; IDAMP

1. Introduction

Wave propagation in soil plays a crucial role in the input motion arriving at the structures. Numerical models predicting the excitation should therefore account for dynamic soil-structure interaction at the source and the receiver and incorporate a model that accurately describes wave propagation in the soil.

Volume gridding methods as the finite element method, the finite difference method, and the spectral element method allow modeling arbitrary layering. As the soil is semi-infinite, however, absorbing boundary conditions are needed to consider the radiation damping. Boundary conditions limit the numerical model dimensions and preserve the banded structure of the system matrices, but may cause spurious wave reflections from the boundaries. Therefore, to prevent the effects of these reflections, large extended meshes are needed. Furthermore, there are some limitations in mesh dimensions such that the length of the elements relative to the smallest wavelength of the propagating wave should be small; This limitation imposes small elements at high frequencies. Both conditions result in models that become prohibitively large.

In numerical analysis, the only finite domain of the soil medium is modeled and get analyzed. To prevent the reflection of radiation waves from boundaries, a boundary condition (BC) is imposed. The accuracy of the numerical analysis depends on the performance of these boundary conditions and absorption of deflected waves. To simulate such boundary conditions, many approaches have been developed and the accuracy of absorbing boundary conditions has been improved over the years. Perfectly matched layer (PML) is one of these absorbing boundaries which has been used in different sciences as an absorbing boundary to simulate a semi-infinite media. By using PML, the modeling dimensions and volume of calculations are able to decrease without losing response accuracy and any consideration about incident wave frequency or direction.

In 1994, Berenger (Berenger, 1994) used perfectly matched layers to solve the governing equation of electromagnetic wave propagation in unbounded media using finite difference time domain (FDTD) method. In 1996, Hastings et al. were the first researchers who applied perfectly matched layers to the problem of elastic wave propagation problem. By introducing new functions, they decomposed the corresponding potential functions into primary and secondary waves and by applying reducing parameters; they solved the elastodynamic equations in terms of stress and velocity by the finite difference method in the time domain and in a two-dimensional environment. In 1996, Chew and Liu defined a perfectly matched medium using mixed coordinates and showed that it reduces the amplitude of the propagated waves in the defined directions.

In 1997, Chew et al. expressed Maxwell's equation in perfectly matched medium by changing the variable to the usual form of Maxwell's equation in complex coordinates and showed that many existing analytical solutions can be transferred to complex coordinates. In 1998, Turkel and Yetef investigated the performance of perfectly matched layers proposed by Berenger in solving the Helmholtz and Maxwell equations in the acoustic environment.

In 2000, Harari et al. applied the PML equations in the harmonic analysis of wave propagation in an acoustic unbounded medium in the time domain. In 2001, Collino and Tsogka explained how to use the method of decomposition of potential functions in the formulation of the PML with the split space approach, to solve the general hyperbolic equation, and applied their method to solve the elastodynamic equations in inhomogeneous environments. In 2003, Komatitsch and Tromp presented a formulation for PML for analyzing the displacement field and using it to solve the elastodynamic equations by the spectral element method. Also in 2003, Festa and Nielsen extended the studies of Collino and Tsogka to the three-dimensional system and by solving various problems; they investigated the ability to absorb Rayleigh waves by the PML. They also showed that the thickness of PML can be considered small even in low frequency studies.

In 2003 and 2004, Basu and Chopra defined perfectly matched layers for solving elastodynamic problems with harmonic excitation using mixed coordinates and solved the equations by finite element method. They also transferred the obtained equations to the time domain and presented a method for analyzing problems in the time domain. In 2004, Rylander and Jin presented a new formulation for the definition of PML in solving Maxwell's equation using the finite element method and transferred the frequency-dependent equations to the time domain and proposed a method to solve the equations in the time domain. In 2006, Harari and Albocher conducted a parametric study on PML used in harmonic elastodynamics problems using the finite element method and proposed values for the parameters of perfectly matched layers. In 2009, by using auxiliary variables, Qin et al. decomposed the equation governing the environment of PML in the time domain into two normal and reducing parts, and with this method, the convolution term units were prevented after transferring the equations to the time domain. They proposed a new method with simpler equations, easier programming and less required memory based on the finite difference method to solve the environmental equations of PML. In 2009, Basu presented an optimal method for numerical integration in solving the equations of PML in the time domain and used it in solving three-dimensional problems. In 2011, by defining additional variables corresponding to the stresses in the environment of PML, Kucukcoban and Kallivokas presented a formulation that can be used to solve the equations of perfectly adapted layers in the time domain using conventional numerical integration methods. In 2014, Khazaee and Lotfi showed that PML boundary condition is the most efficient method in time harmonic analysis in the dynamic analysis of dam reservoir systems.

Davoodi and Pourdeilami (2017) and Davoodi et al. (2018, 2020, 2021) solved the problem of wave propagation in a semi-infinite two-dimensional environment by using PML and investigated the effect of the interaction between the structure (earth-filled dam) and the soil. In 2021, Zhang and Taciroglu developed both two- and three-dimensional viscoelastic PMLs that incorporate the effects of Rayleigh damping. They implemented proposed PMLs in ABAQUS through a user defined element (UEL) subroutine.

Because of the related structures and the nature of required models in civil engineering and especially in geotechnical engineering, the use of absorbing boundaries to have an accurate result is necessary. In this study, a series of models considered investigating the wave propagation in semi-infinite media. In all considered models, PML has been used to absorb scattered waves at the base. Also, to study the effect of nonlinear behavior of the materials, the equivalent linear analysis has been done. Because of the nature of the PML, properties of these elements in equivalent linear analysis were updated based on adjacent elements to avoid reflection from boundaries. For this purpose, a code has been developed for dynamic analysis in time domain using finite element method by C programming language, called "IDAMP". In this code the modeling and analysis of the geometry, layering, and loading with an elastic material behavior is possible. This program uses four-nodded quadrilateral elements and the implicit Newmark method to solve the dynamic equation.

2. Free-Field Column Models

The application of the prepared program has been evaluated with different models in linear time domain analysis (Davoodi & Pourdeilami, 2017, (Davoodi, et al., 2020, Davoodi, et al., 2021 and Davoodi, et al., 2018). In all models used in this study, the minimum size of all elements has been considered to be less than one-tenth to one-eighth of the lowest length of the incident waves (Kuhlemeyer & Lysmer, 1973). Also, the optimum thickness of the PML has been considered based on the literature (Davoodi & Pourdeilami, 2017 and Davoodi, et al., 2020).

2.1. Effect of Damping Ratio on the Response of Free-Field Column

In order to control the effect of the damping ratio on the final results and also to evaluate the trend of the variety of damping ratio on the maximum free-field column displacement, a sensitivity analysis of the results has been performed, using a linear analysis on different input values of the damping ratios.

For this purpose, a 50 m column consisting of materials with a shear wave velocity of 25 m/s has been considered (as shown in Figure (1). Also, in this model and analysis, constant damping ratios of 0, 1, 2, 5, 10 and 20% have been considered. A displacement excitation as a normal distribution function with an amplitude of 25 cm was applied at the interface of the soil environment and PML. An applied excitation has been shown in Figure (2).

The results of the pick point of the column provided as shown in Figure (3).

As can be seen in this figure, the amount of the damping ratio has a direct effect on the column's displacement, but no relationship can be considered for the amount of damping ratio and the response amplitude.

2.2. Equivalent Linear Analysis

The equivalent linear method gives an

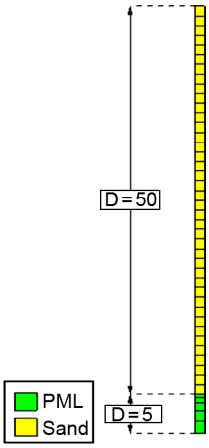


Figure 1. 50 m height free-field column based on 5m PM.

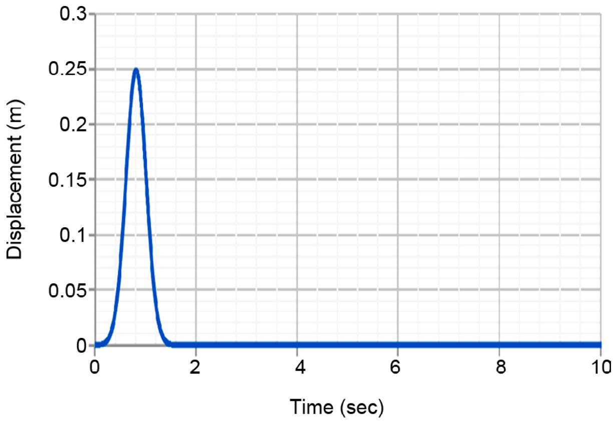


Figure 2. Applied excitation as normal distribution of the displacement with an amplitude of 25 cm.

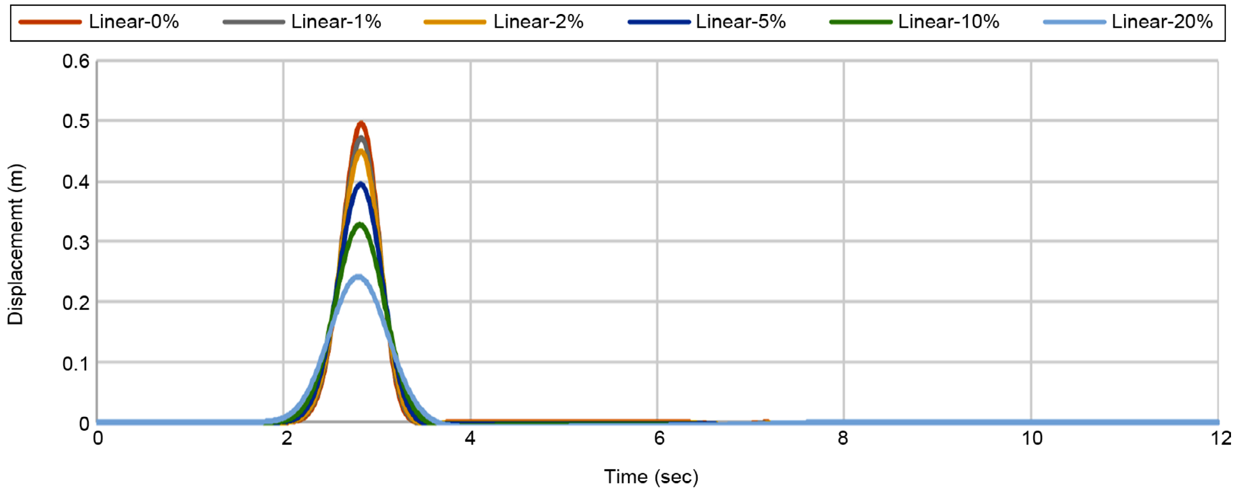


Figure 3. Displacement of peak point of the 50m height column with different damping ratios under applied load.

approximation of real nonlinear dynamic behavior. Although the accuracy of the equivalent linear method is lower than nonlinear methods, there are several reasons why equivalent linear methods are often utilized in practice. One reason is the simplicity of defining the input data, as the required data can be obtained directly from laboratory tests easily.

In order to perform equivalent linear analysis, it is necessary to determine the changes in shear modulus and damping coefficient against shear strain. For this purpose, in the present study and in examining the results of the equivalent linear analysis, four sets of diagrams of shear modulus and damping coefficient against shear strain have been used. These diagrams include: diagrams provided by Seed and Idris (1970) developed in 1970, relation developed by Darendeli (2001) and the relation developed by Roblee and Chiou (2004) in 2004. Considered relations between shear modulus and damping coefficient against shear strain have been shown in Figure (4).

3. Analysis

To investigate the performance of the prepared program for equivalent linear analysis, several analyses have been carried out, the results of which are presented below.

3.1. A Free-Field Column with 30 m Height

In the first step, for checking the performance of the prepared equivalent linear analysis program, an attempt was made to compare the results with linear analysis. To do this, a one-layered column with 30 m height and shear wave velocity of 320 m/s as shown in Figure (5) has been considered. An excitation of Gaussian distribution of displacement

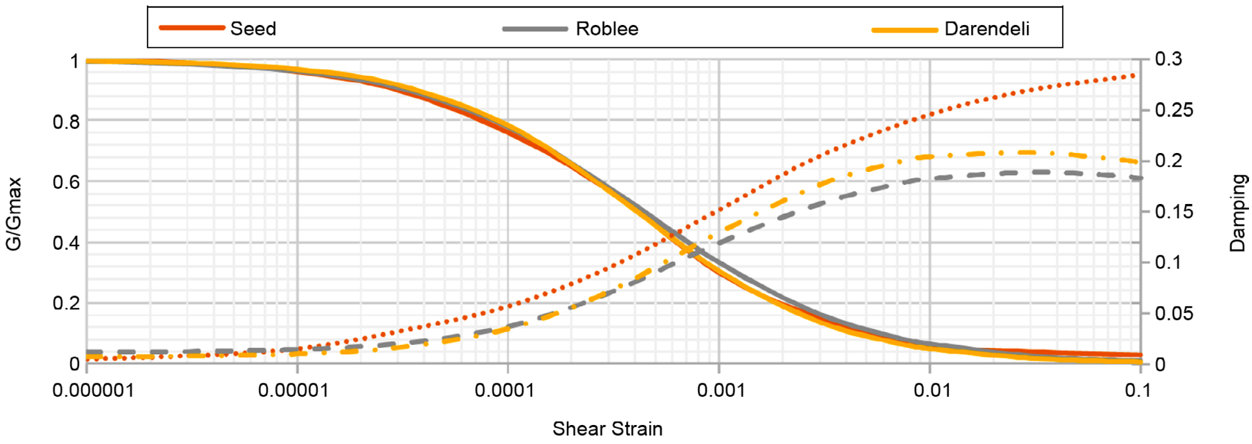


Figure 4. Considered G-D-γ relations for equivalent linear analysis.

which its velocity time history is shown in Figure (6) was applied in the boundary of the soil and PML and obtained results has been shown in Figure (7). Here, the analysis has been done using different G-D-γ relations presented in Figure (4). The linear analysis result also compared with Plaxis extended mesh in Figure (7).

As can be seen in this figure, linear response of prepared program perfectly matched with Plaxis results. Also, in the very small shear strains criteria and near the linear behavior of material, the equivalent linear analysis results have a very

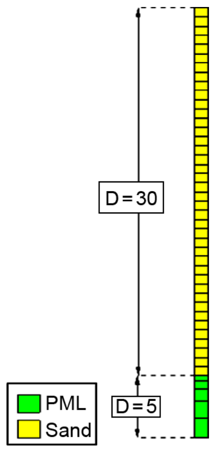


Figure 5. 30 m height free-field column with 5m PML in the base.

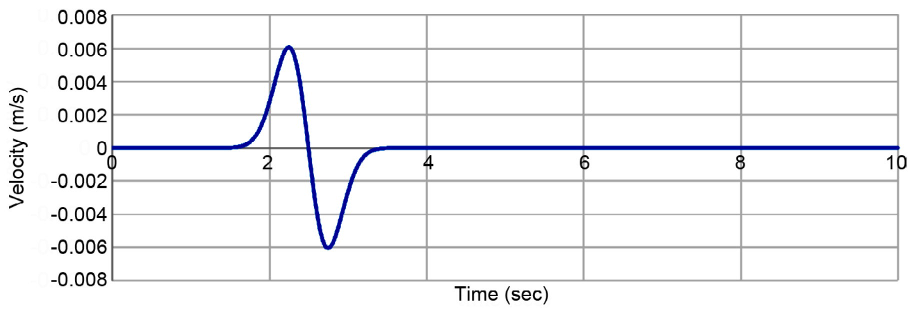


Figure 6. Applied velocity time history to the base of the 30m height free-field column.

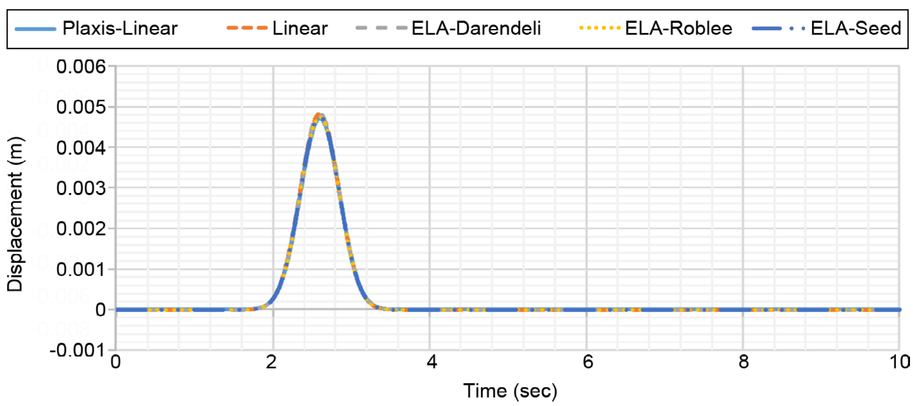


Figure 7. Results of a 30m height column linear and equivalent linear analysis under a very low amplitude excitation.

good compliance with linear analysis and the performance of the prepared program is suitable in these criteria.

For more investigation, this column was again analyzed under excitation with an amplitude 10 times of the previous step, the results of which are presented in Figure (8).

In small strains, due to the non-linear behavior of the materials, the results are far from the linear analysis, but a good agreement can still be seen in the results. Differences in the equivalent linear analysis results are due to the fact that the pattern of changes in shear modulus and damping coefficient against shear strain is not the same.

In order to further investigation, the same 30 m column was analyzed under the excitation shown in Figure (9). In this case, the materials somewhat enter into nonlinear behavior and deviate from linear behavior. To control the results, it was taken advantages of Abaqus software and the semi-infinite boundary in it. The results of IDAMP were obtained and compared with Abaqus, which is shown in Figure (10). It should be mentioned that the analysis in Abaqus and IDAMP in this case was done only using Seed's G-D-γ relations introduced in Figure (4).

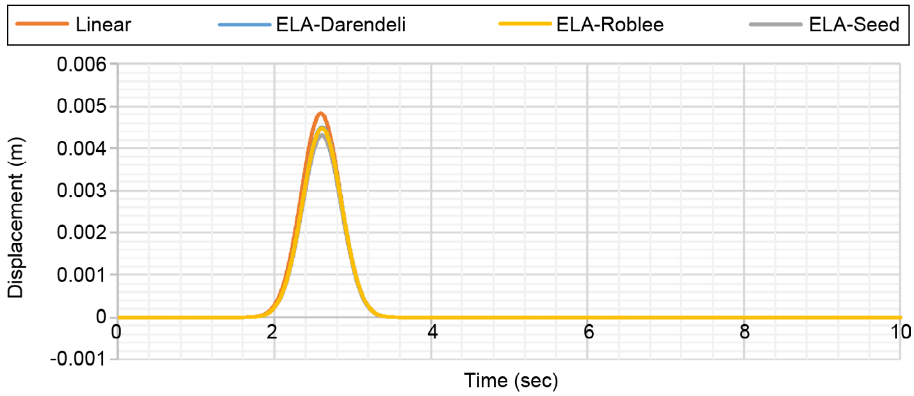


Figure 8. Results of a 30m height column linear and equivalent linear analysis under a low amplitude excitation.

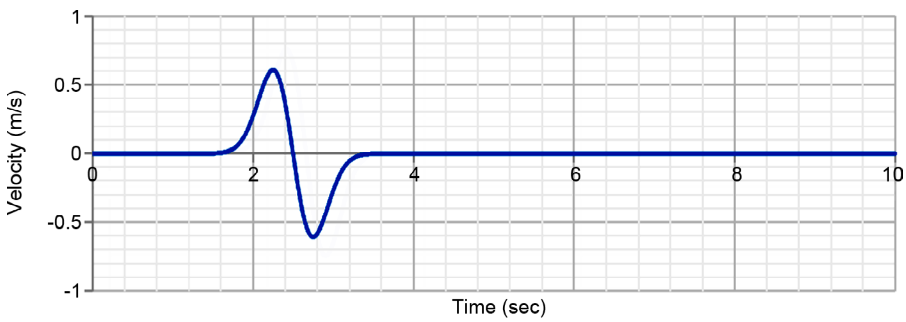


Figure 9. Applied velocity time history in IDAMP and ABAQUS.

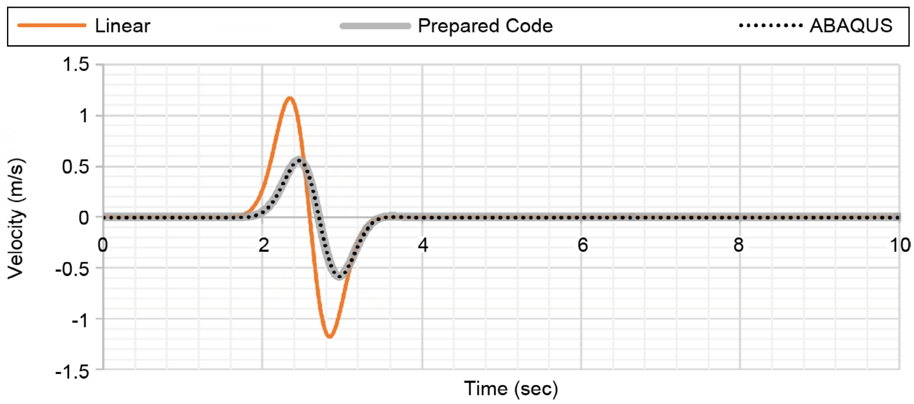


Figure 10. Comparison of the Abaqus and IDAMP responses.

As can be seen in this figure, the response of IDAMP and Abaqus software completely match with each other using the same G-d-γ diagrams. It can also be seen that the nonlinear behavior of the material changed the column responses con-siderably in comparison with linear analysis.

What can be seen in all the analyzes done here, the use of PML has caused to not have a wave reflection from the borders.

3.2. A free-Field Column with 50 m Height

In order to check the performance of the prepared program and PML efficiency in different frequencies, a free-field column with 50 m height and shear wave velocity of 320 m/s has been con-sidered as shown in Figure (1). Two types of velo-city time histories were considered. The first group of wavelets with a Gaussian distribution with a dominant frequency of 0.9, 1.5 and 3 Hz, which have an amplitude of 0.6 m/s (Figure (11)) and the second group wavelets with a Gaussian distribution and a dominant frequency of 0.9, 1.5 and 3 Hz, whose displacement amplitude is scaled to 0.25 m (Figure (12)).

For a 50 m height column under the excitation of wavelets with velocity of 0.6 m/s and different dominant frequencies (group one), the results are presented in Figure (13).

For the second group of wavelets, the displace-ment of a surface point of the column has been shown in Figure (14).

As can be seen in the figures above, the per-formance of PML in absorbing reflected waves from boundaries for different frequencies and analysis using the equivalent linear method, is suitable and can be used in the analysis of earth-quake records which have different frequency content.

4. One-Layered Column Analysis

To investigate the effect of excitation's intensity on the response of the behavior of the materials in semi-infinite free-field, a Gaussian distribution with dominant frequency of 0.9 Hz was scaled

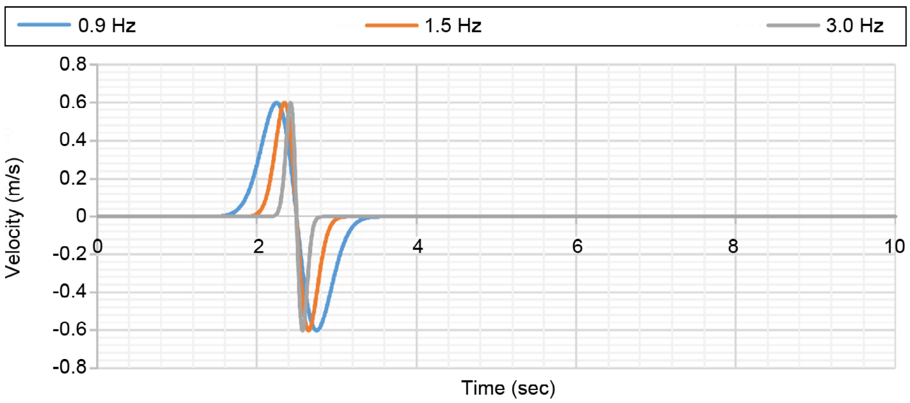


Figure 11. Considered time histories with different dominant frequency and velocity of 0.6 m/s (group 1).

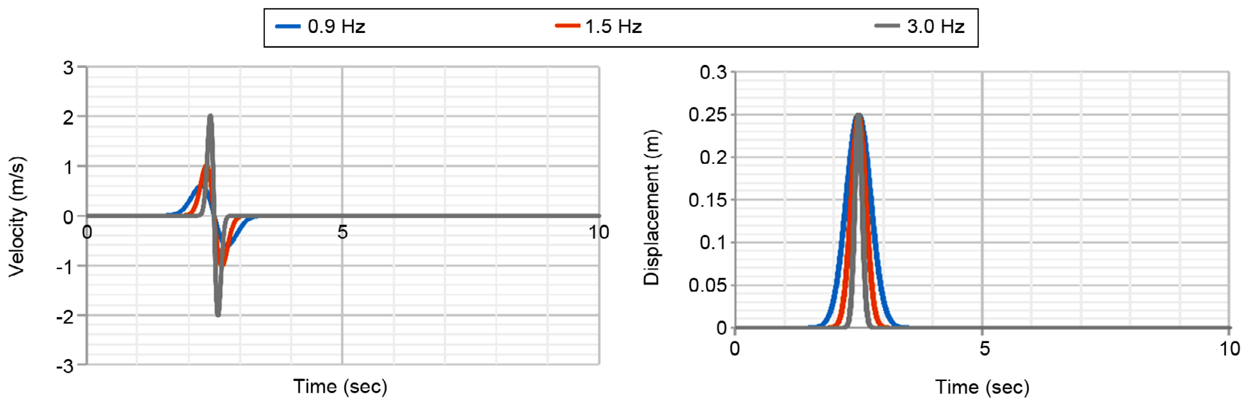


Figure 12. Considered time histories with dominant frequency of 0.9, 1.5 and 3 Hz and displacement amplitude of 0.25 meters (group 2).

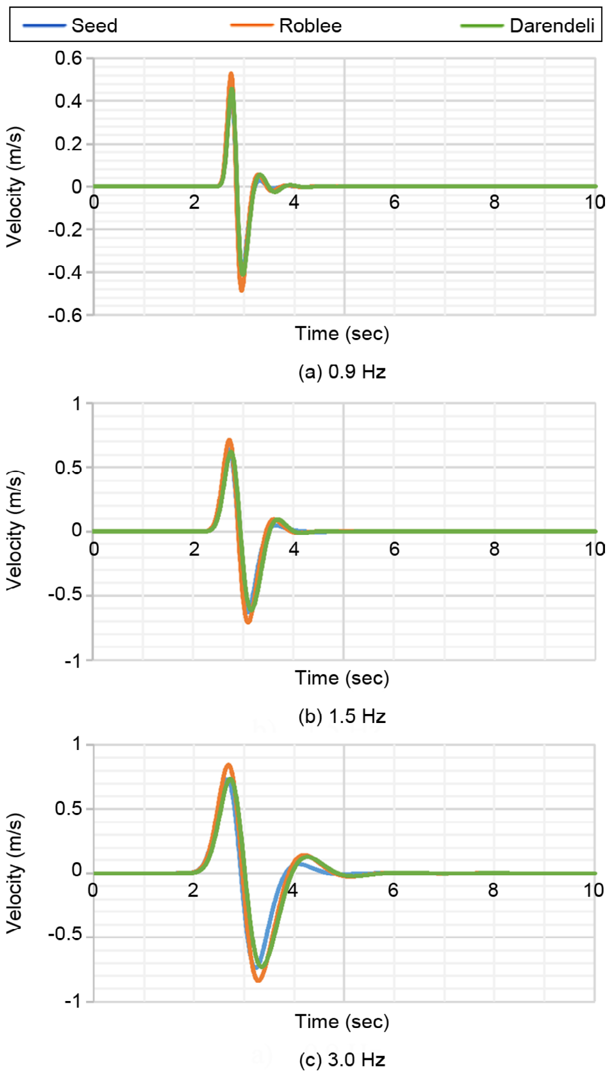


Figure 13. Response of the surface point of a 50-meter column under velocity time history with an amplitude of 0.6 m/s.

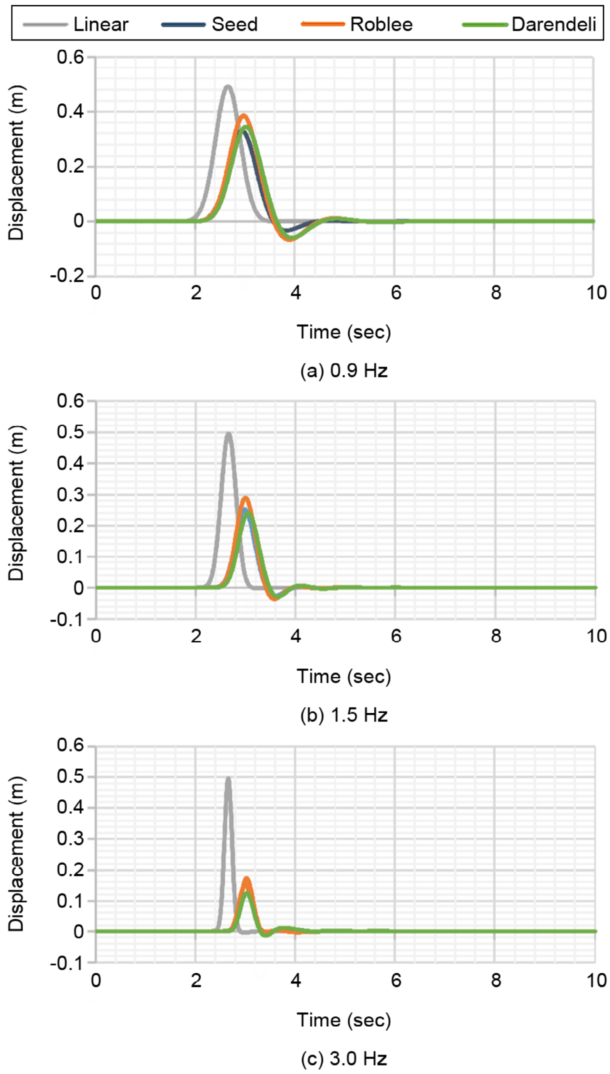


Figure 14. Response of the surface point of a 50-meter column under displacement time history with an amplitude of 0.25 m.

Table 1. General characteristics of the scaled records.

	PGA	PGV (m/s^2)	PGD (m)
1	0.1 g	0.149	0.061
2	0.2 g	0.298	0.123
3	0.3 g	0.446	0.184
4	0.4 g	0.595	0.245
5	0.5 g	0.744	0.307
6	0.6 g	0.893	0.368
7	0.7 g	1.042	0.429
8	0.8 g	1.190	0.491
9	0.9 g	1.339	0.552
10	1.0 g	1.488	0.613

in such a way that the characteristics of the obtained records are according to Table (1) and Figure (15).

Based on these records, a 50 m height column with an initial shear wave velocity of 320 m/s was analyzed, and the results are presented in Figure (16).

In order to investigate the effect of the records' intensity on the response of the considered soil column, in Figure (17), the results of considered relation based on the Seed and Idriss (1986) is presented.

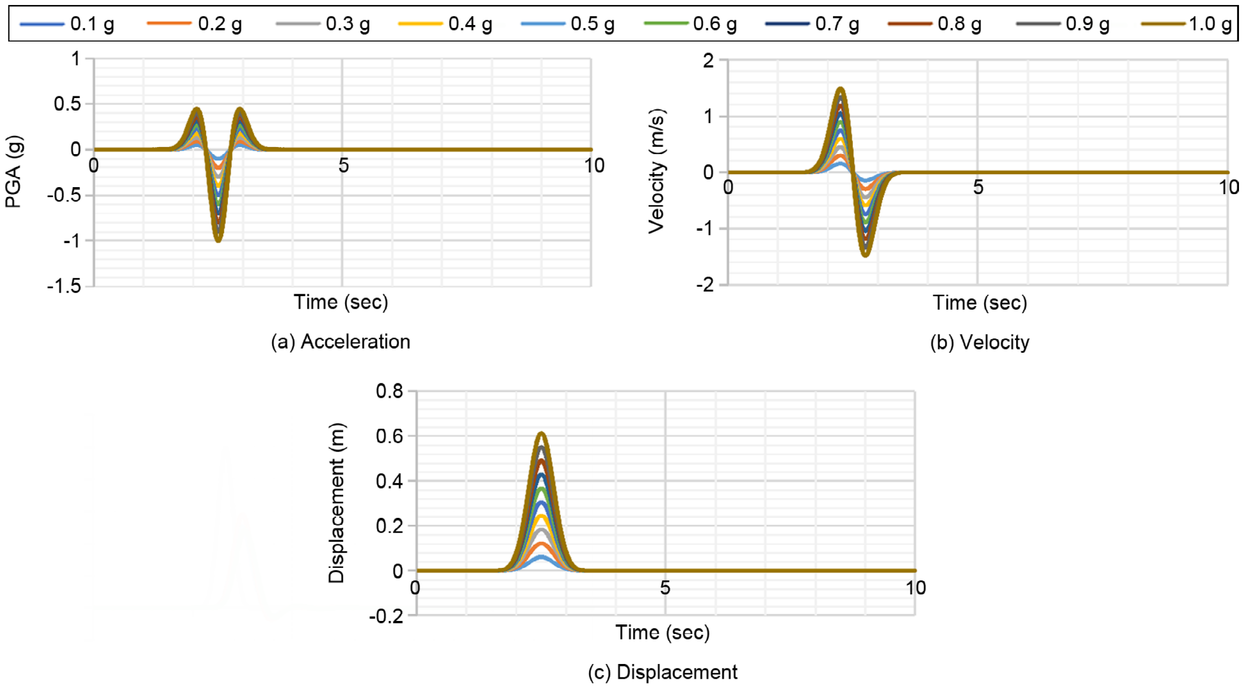


Figure 15. Considered scaled records time history.

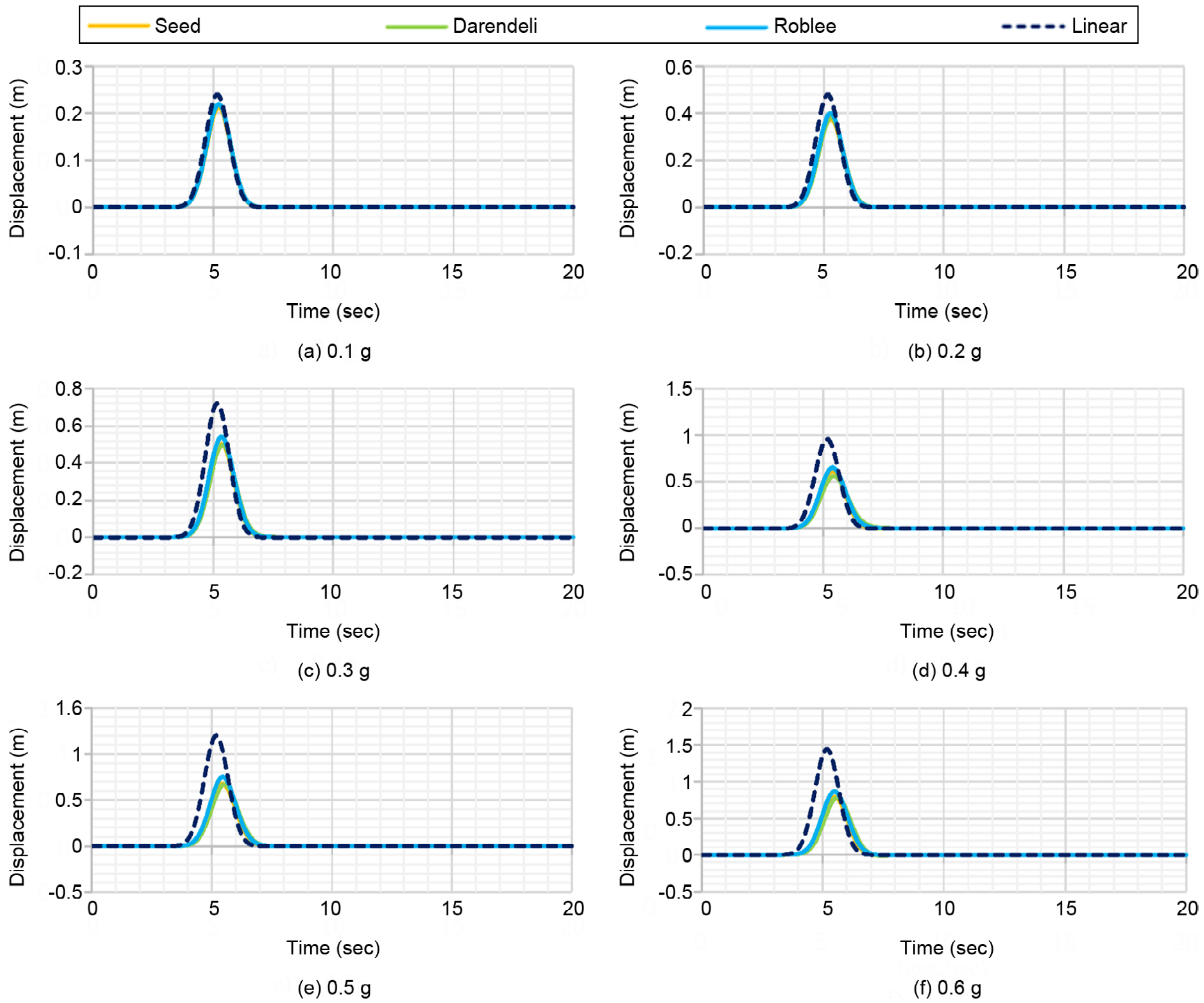


Figure 16. Response of the surface point of the 50 m column under scaled records.

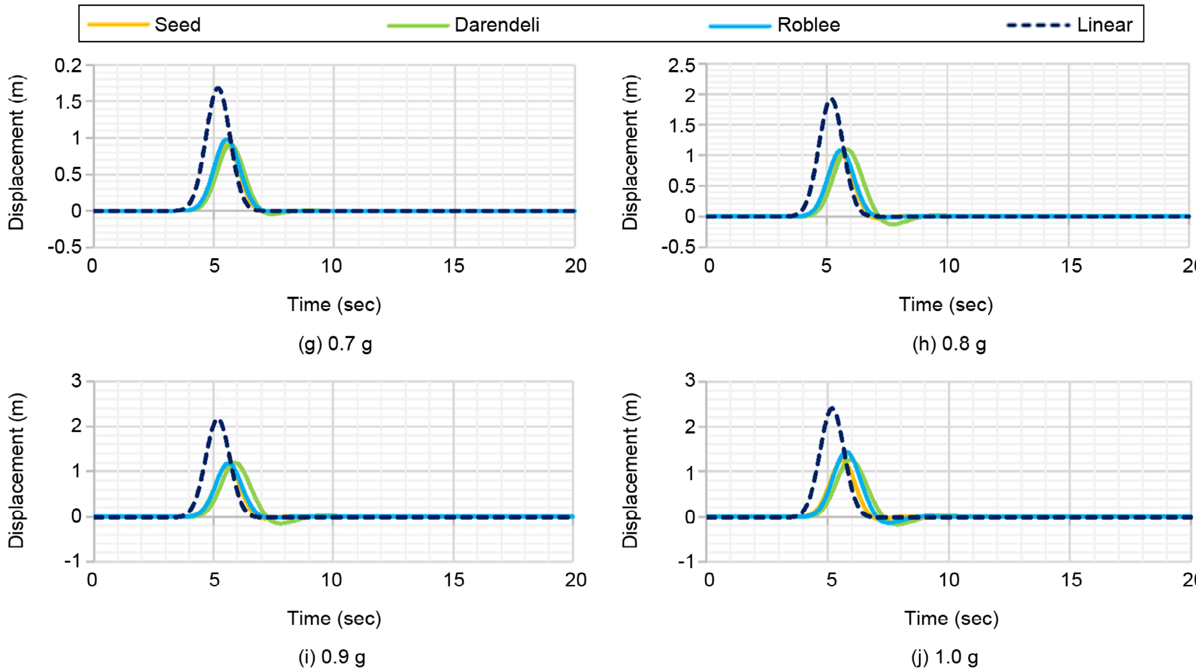


Figure 16. Continue.

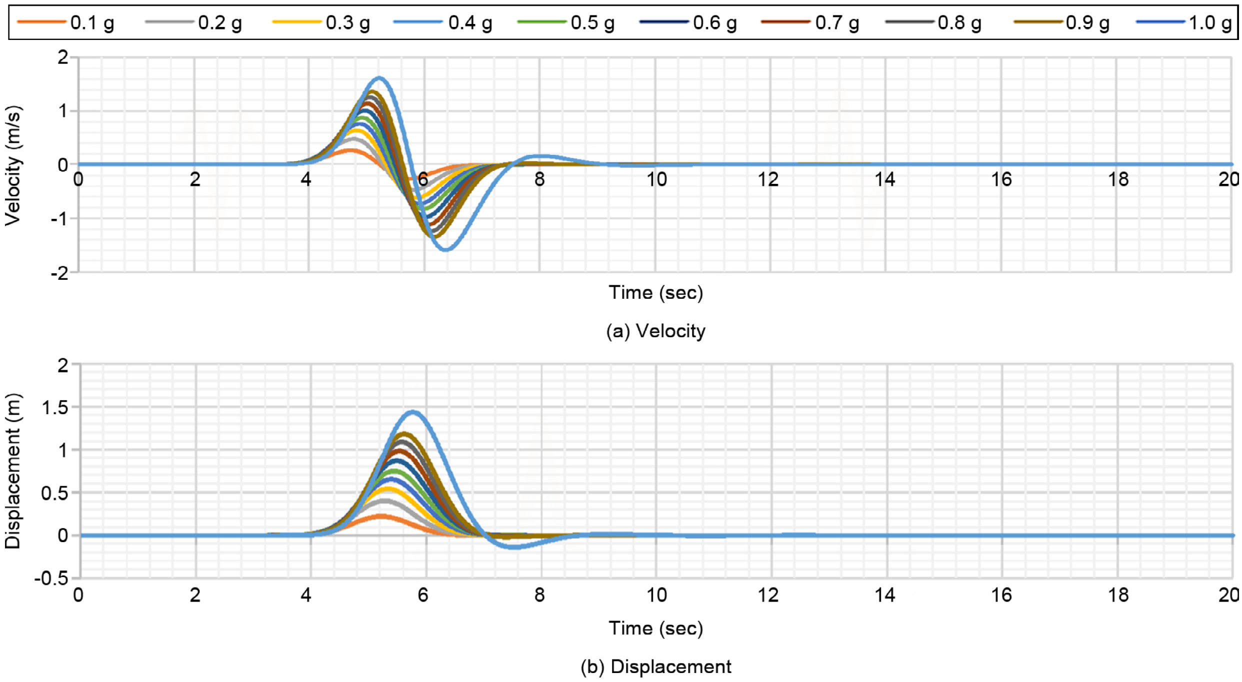


Figure 17. Response of the considered soil column under scaled excitations.

As can be seen in this figure, by increasing the intensity of the input excitation, the material experiences more shear strain and the stiffness of the elements decreases accordingly. As a result, the time for arriving the wave to the top of the column increases. Also, with the increase of magnitude, the results of the equivalent linear analysis are further away from the results of the linear analysis.

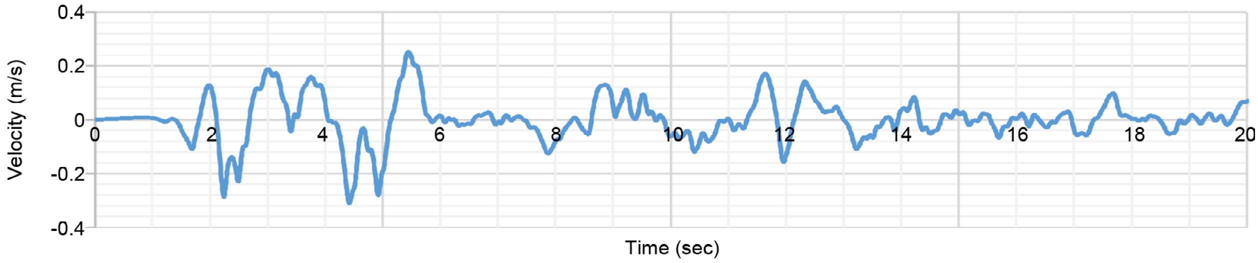
For more analysis, the same 50 m column with 320 m/s shear wave velocity has been considered and the Imperial Valley earthquake record whose characteristics are presented in Table (2) has been applied.

The velocity time history and Fourier spectrum of considered Imperial Valley earthquake has been shown in Figure (18).

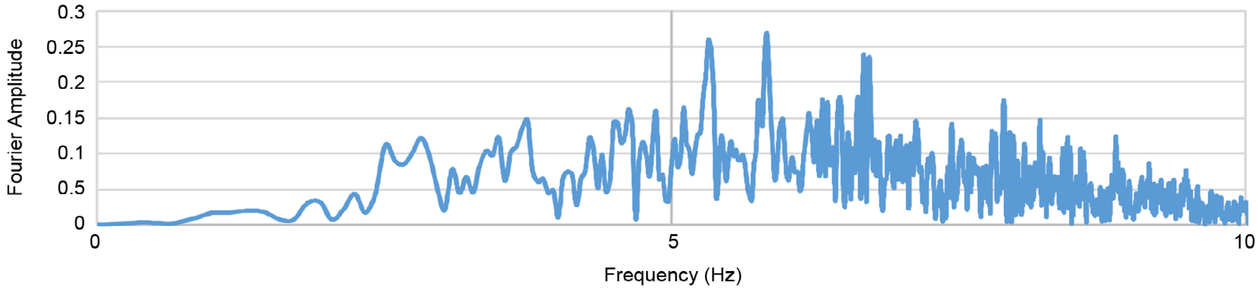
The response of the ground surface of the column has been shown in Figure (19). Also, in Figure (20) the frequency content of the results

Table 2. Characteristics of Imperial Valley earthquake record.

Station	Earthquake Date	Magnitude	Epicentral Distance (km)	Component	PGA (g)
El Centro- Array #9	Imperial Valley, May 19, 1940	6.3 (ML)	8	180	0.28



(a) Velocity Time History of Imperial Valley Earthquake



(b) Fourier Spectrum of Imperial Valley Record

Figure 18. Applied Imperial Valley record to the base of the 50m height free-field column.

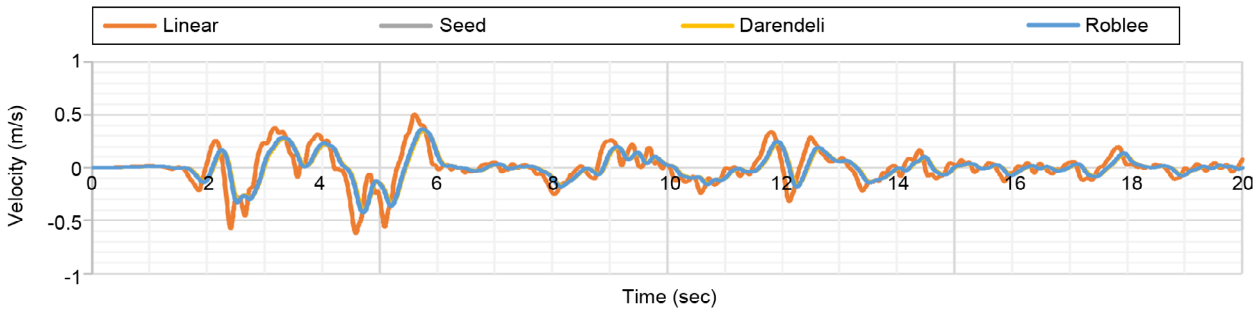


Figure 19. Response of the surface point of the 50m height column under Imperial Valley earthquake.

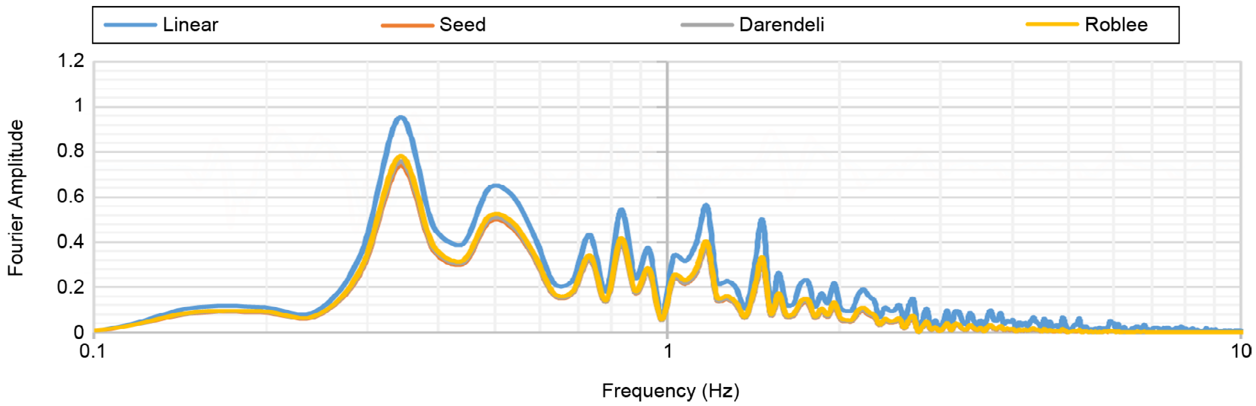


Figure 20. Fourier amplitude of the responses of 50m height column with soft soil under Imperial Valley earthquake

has been presented.

As can be seen in this figure, the response of the analysis using the equivalent linear method based on the different relations considered are completely consistent with each other, and due to the softness and damping of the materials in the equivalent linear analysis compared to the linear analysis, we can see a phase difference in the

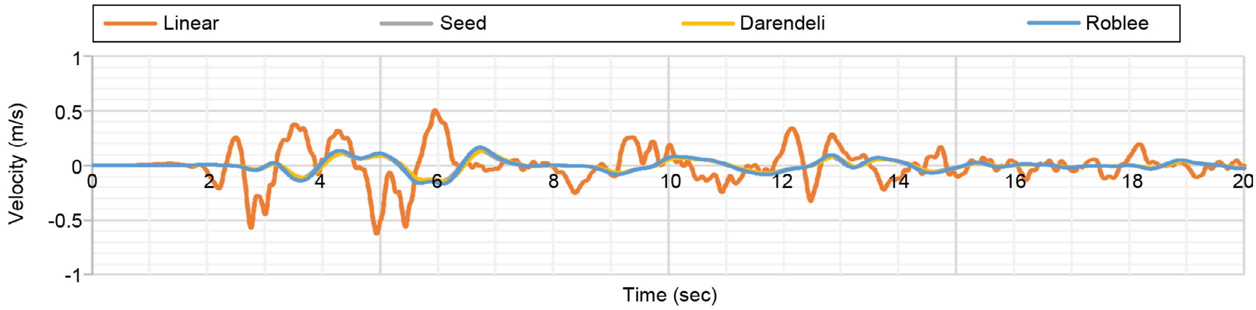


Figure 21. Response of the peak point of the 50m height column with soft soil under Imperial Valley earthquake.

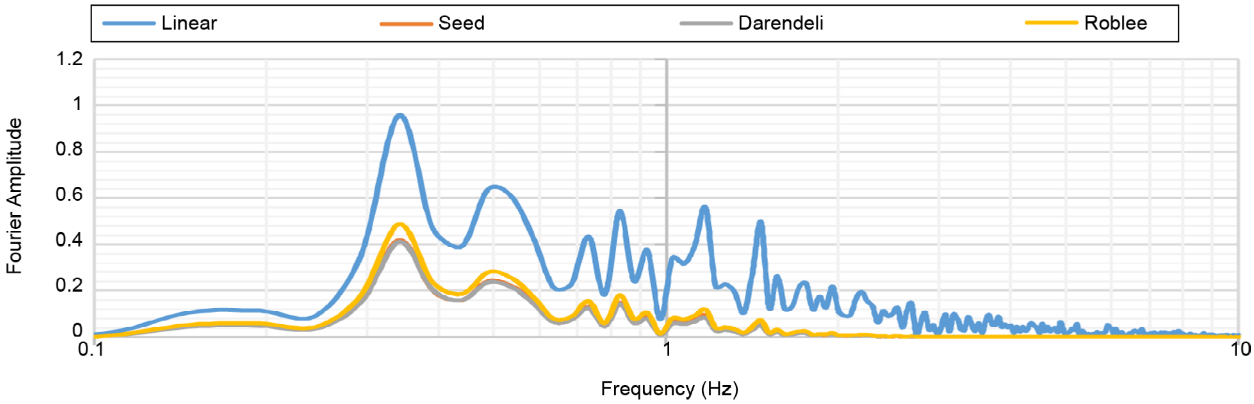


Figure 22. Fourier amplitude of the responses of 50m height column with soft soil under Imperial Valley earthquake.

response of the two methods.

To investigate the effect of the nonlinear behavior of materials, the previous analysis repeated using the material with initial shear wave velocity of 100 m/s. the response of the surface point of the column and its frequency content has been shown in Figures (21) and (22).

As can be seen from the comparison of Figure (19) and Figure (21), as the stiffness of the material decreases, the shear strain of the elements increases and as a result the non-linear behavior of the material increases. Also, as the initial materials wave velocity considered lower and accordingly materials gets softer, frequency content of the responses gets vary from what linear analysis obtained.

5. Conclusion

In this study, wave propagation in semi-infinite soil column and therefore free-field response has been investigated. A finite element time domain code has been developed using the C programing language called "IDAMP". This code uses four-nodded quadrilateral elements and the implicit Newmark method to solve the dynamic equation. It can simulate semi-infinite media by using perfectly matched layers at the boundaries. Different soil columns have been considered, and various excitations have been applied to the models. The results of linear and equivalent linear analysis have been verified using different models. To do equivalent linear analysis, the maximum strain of each element is obtained at the end of each linear analysis and an efficient strain has been calculated. After that, based on the given variation of the material shear modulus and damping ratio by the shear strain, for each model element, the material properties have been updated and analysis is repeated until the relative difference between the material properties, such as the shear modulus, in two successive iteration steps become less than 5% for all model elements. The PML parameters have also been updated in each analysis to be compatible with the adjacent elements to avoid wave reflection from the boundaries. The results show that:

- By increasing the intensity of the input excitation, the material experiences more shear strain and the stiffness of the elements decreases accordingly. As a result, the time for the wave arrival at the top of the column increases, and it can be seen that there is a phase difference in the responses. Also, with the increase of the magnitude, the results of the equivalent linear analysis are further away from the results of the linear analysis.

- The efficiency of PML in absorbing different frequencies has been investigated in both linear and equivalent linear analysis, and it has been shown that it works perfectly. There is not any refraction or reflection in the considered models. So as a result, PML is a suitable absorbing boundary to use in the analysis under earthquake excitation, which contains a variety of frequencies.
- The considered procedure in the prepared program for equivalent linear analysis worked well. PML properties are completely matched with adjacent materials and propagating wave easily enters the PML environment.
- Analysis of a column under earthquake excitation shows that the nonlinear behavior of materials can change the responses considerably in comparison with linear analysis.

Generally, all the results show that PML is an efficient absorbing boundary and is able to eliminate the unwanted scattered motions regardless of the frequency or direction of the waves. Also, the ability of prepared code (IDAMP) was investigated and showed that it is capable to do equivalent linear analysis considering PML as an absorbing boundary. IDAMP can consider different layers with various materials and variations of the material shear modulus and damping ratio with the shear strains for each layer. This manuscript is a part of an ongoing study and the research team will attempt to eventually cover phenomena such as the nonlinear behavior of geomaterial, multiphase medium, etc., using PML as an absorbing boundary.

Acknowledgment

The authors would like to acknowledge the technical and financial support of IIEES under the research project "Investigation of non-linear behavior of embankment dams considering dam-foundation interaction using perfectly matched layers PML", (CODE- 779). In addition, we thank Fariborz Sanjari for his assistance in development of verification models in ABAQUS.

References

Basu, U. (2009). Explicit finite element perfectly matched layer for transient three-dimensional elastic waves. *International Journal for Numerical Methods in Engineering*, *77*(2), 151-76.

Basu, U., & Chopra, A. (2003). Perfectly matched layers for time-harmonic elastodynamics of unbounded domains: theory and finite-element implementation. Computer *Methods in Applied Mechanics and Engineering*, *192*(11-12), 1337-75.

Basu, U., & Chopra, A. (2004). Perfectly matched layers for transient elastodynamics of unbounded domains. *International Journal for Numerical Methods in Engineering*, *59*(8), 1039-74.

Berenger, J.-P. (1994). A perfectly matched layer for the absorption of electromagnetic waves. *Journal of computational physics*, *114*(2), 185-200.

Chew, W., & Liu, Q. (1996). Perfectly matched layers for elastodynamics: a new absorbing boundary condition. *Journal of Computational Acoustics*, *4*(4), 341-59.

Chew, W., Jin, J., & Michielssen, E. (1997). Complex coordinate stretching as a generalized absorbing boundary condition. *Microwave and Optical Technology Letters*, *15*(6), 363-9.

Collino, F., & Tsogka, C. (2001). Application of the perfectly matched absorbing layer model to the linear elastodynamic problem in anisotropic heterogeneous media. *Geophysics*, *66*(1), 294-307.

Darendeli, M. (2001). *Development of a New Family of Normalized Modulus reduction and Material Damping Curves*. The university of Texas at Austin.

Davoodi, M., & Pourdeilami, A. (2017). *Application of Perfectly Matched Layer (PML) in Earthfill Dam-Foundation Interaction Analysis (in Persian)*. Ph.D. Degree thesis, International Institute of Earthquake Engineering and Seismology (IIEES), Tehran, Iran.

Davoodi, M., Afzalsoltani, R., & Pourdeilami, A.

(2021). Using Perfectly Matched Layer (PML) for evaluating earth dam deformations considering equivalent-linear analysis. *12th International Congress on Civil Engineering*, Mashhad, Ferdowsi University.

Davoodi, M., Afzalsoltani, R., & Pourdeilami, A. (27-28 May, 2020). Investigating effect of earthquake input method in seismic behavior of the earth dam-foundation interaction (in Persian). 12th *National Congress of Civil Engineering*. Tabriz: Sahand University.

Davoodi, M., Pourdeilami, A., Jahankhah, H., & Jafari, M. (2018). Application of perfectly matched layer to soil-foundation interaction analysis. *Journal of Rock Mechanics and Geotechnical Engineering*.

Festa, G., & Nielsen, S. (2003). PML absorbing boundaries. *Bulletin of the Seismological Society of America*, *93*(2), 891-903.

Harari I, S.M. (2000). Analytical and numerical studies of a finite element PML for the Helmholtz equation. *Journal of Computational Acoustics*, *8*(1), 121-37.

Harari, I., & Albocher, U. (2006). Studies of FE/PML for exterior problems of time-harmonic elastic waves. *Computer Methods in Applied Mechanics and Engineering*, *195*(29-32), 3854-79.

Hastings, F., Schneider, J., & Broschat, S. (1996). Application of the perfectly matched layer (PML) absorbing boundary condition to elastic wave propagation. *The Journal of the Acoustical Society of America*, *100*(5), 3061-9.

Khazaee, A., & Lotfi, V. (2014). Application of perfectly matched layers in the transient analysis of dam-reservoir systems. *Soil Dynamics and Earthquake Engineering*, *60*, 51-68.

Komatitschm, D., & Tromp, J. (2003). A perfectly matched layer absorbing boundary condition for the second-order seismic wave equation. *Geophysical Journal International*, *154*(1), 146-53.

Kucukcoban, S., & Kallivokas, L. (2011). Mixed perfectly-matched-layers for direct transient analysis in 2D elastic heterogeneous media. *Computer Methods in Applied Mechanics and Engineering*, *200*(1-4), 57-76.

Kuhlemeyer, R., & Lysmer, J. (1973). Finite element method accuracy for wave propagation problems. *Journal of the Soil Mechanics and Foundations Division*, *99*(5), 421-7.

Qin, Z., Lu, M., Zheng, X., Yao, Y., Zhang, C., & Song, J. (2009). The implementation of an improved NPML absorbing boundary condition in elastic wave modeling. *Applied Geophysics*, *6*(2), 113-21.

Roblee, C., & Chiou, B. (2004). A proposed geoindex model for design selection of non-linear properties for site response analyses. *International Workshop on Uncertainties in Nonlinear Soil Properties and Their Impact on Modeling Dynamic Soil Response*. PEER Headquarters, UC Berkeley.

Rylander, T., & Jin, J.-M. (2004). Perfectly matched layer for the time domain finite element method. *Journal of Computational Physics*, *200*(1), 238-250.

Seed, H., & Idriss, I. (1970). *Soil Module and Damping Factors for Dynamic Response Analysis*. EERC. Berkelely: University of California.

Seed, H., Wong, R., Idriss, I., & Tokimatsu, K. (1986). Moduli and damping factors for dynamic analyses of cohesionless soils. *Journal of Geotechnical Engineering*, *112*(11), 1016-32.

Turkel, E., & Yefet, A. (1998). Absorbing PML boundary layers for wave-like equations. *Applied Numerical Mathematics*, *27*(4), 533-57.

Zhang, W., & Taciroglu, E. (2021). A novel rayleigh-type viscoelastic Perfectly-Matched-Layer for wave propagation analysis: Formulation, implementation and application. *Computer Methods in Applied Mechanics and Engineering*, 383.