Approximate Procedure for the Seismic Nonlinear Analysis of Nonstructural Components in Buildings

Roberto Villaverde

Department of Civil and Environment Engineering, University of California, Irvine, CA 92697, USA, email: rvl@eng.uci.edu

ABSTRACT: *An approximate method is proposed to estimate the seismic response of nonlinear nonstructural components attached to nonlinear building structures. The method is based on a previously developed procedure for the analysis of linear secondary systems mounted on a linear primary structure, the introduction of simplifying assumptions similar to those made in the derivation of the equivalent lateral force procedure for the seismic analysis of conventional buildings, and the use of strength reduction factors to account for the nonlinear behaviour of nonstructural component and supporting structure. Its application to any given nonstructural component only requires knowing the geometric characteristics, weights, and target ductilities of the nonstructural component and the structure to which it is connected, in addition to the fundamental natural period of the structure and the elastic response spectrum specified for the design of the structure. Presented also are a numerical example that illustrates the application of the method and the results of a comparative numerical study that is carried out to assess the method's adequacy. Based on its simplicity and rationality and the results from the comparative study, it is concluded that the proposed method represents a simple but effective procedure for the seismic design of nonstructural components in buildings.*

Keywords: Nonlinear analysis; Approximate methods; Seismic design; Secondary systems; Building attachments; Equipment design

1. Introduction

Many methods have been proposed during the last three decades for the seismic analysis of nonstructural components attached to building structures. These methods have been developed in recognition of the vulnerability of nonstructural components to the effects of earthquakes and the importance of their survivability from a safety and an economic point of view. For the most part, however, they have been limited to linear nonstructural components mounted on linear structures [1]. Most of the available methods, therefore, cannot be used directly to estimate the maximum response of a nonstructural component under an extreme seismic event since, by design, its supporting structure is supposed to incur into its nonlinear range of behaviour in such a case. They cannot take into account, either, the often advantageous fact that many nonstructural components or their anchors are capable of resisting large inelastic deformations. Furthermore, the use of linear methods in the analysis of nonstructural components may lead to unrealistic designs. As pointed out by Kawakatsu, et al [2], Viti, et al [3], Lin and Mahin [4], Aziz and Ghobarah [5], Segal and Hall [6], Toro, et al [7], Sewell, et al [8], Igusa [9], Schroeder and Bachman [10], Singh, et al [11], Adam and Fotiu [12] and Adam [13], the nonlinear behaviour of a structure and a nonstructural component may significantly affect the response of the nonstructural component. The effect is mainly in the form of a reduction over the corresponding linear response, but also in the form of an amplification in those cases for which a linear nonstructural component is tuned to a higher mode of the structure, the natural frequency of the structure in this mode is an odd integer multiple of its fundamental natural frequency, and the seismic input is narrow-banded and centered around the fundamental frequency of the structure [8, 11].

It is the purpose of this paper to introduce an approximate but rational method for the seismic analysis of nonstructural components that accounts for their nonlinear behaviour and that of their supporting structures. Also presented here is a numerical example that illustrates the application of the method and the results of a numerical comparative study that has been performed to assess the method's adequacy.

2. Derivation

The proposed method is based on the integration of a simplified procedure for the analysis of linear nonstructural components attached to linear structures with an approach that accounts in an approximate but simple way the nonlinearity of the structure and that of the nonstructural component itself. The simplified procedure for linear systems, in turn, is based on a previously developed response spectrum method for the analysis of linear secondary systems mounted on a linear primary structure, and the introduction of simplifying assumptions similar to those made in the derivation of the equivalent lateral force procedure for the seismic analysis of conventional buildings. The aforementioned response spectrum method is described in detail elsewhere [14]. It is derived by applying the conventional response spectrum technique to the combined system that a light secondary system forms with the structure to which it is attached, and by formulating equations that give the maximum response of such a combined system in terms of ordinates from a ground response spectrum. By considering small damping and mass ratios and neglecting second-order terms, these equations are then simplified to derive approximate relationships that explicitly give the maximum response of the secondary system. In the application of the response spectrum technique, however, the modal properties of the combined system are expressed first in terms of the independent dynamic properties of the two subsystems through a modal synthesis [15]. The derived expressions are therefore written in terms of the mode shapes, natural frequencies and damping ratios of the structure and the secondary system when they are independently considered. Also, the complex mode shapes and natural frequencies of the combined system are used to account for the fact that the combined system under consideration is nonclassically damped.

In the application of the response spectrum method in question, one determines first the N_p unit-participation-factor mode shapes $\{\Phi\}_i$, circular natural frequencies ω_{pi}, generalized masses M^*_i, and modal damping ratios ξ_{pi} of the structure when independently considered, and the N_s unit-participation-factor mode shapes $\{\phi\}_j$, circular natural frequencies ω_{sj}, generalized masses m^*_j, and modal damping ratios ξ_{sj} of the nonstructural component system when considered fixed at its points of attachment to the structure. Here, N_p denotes the number of degrees of freedom of the independent primary system and N_s the number of degrees of freedom of the secondary system when, once again, it is considered fixed at its points of attachment to the structure. A mode shape with a unit participation factor is attained by simply multiplying the same mode shape when normalized in any arbitrary way by the participation factor that results from considering the arbitrarily normalized mode shape.

When the secondary system is attached to the structure at two locations, one calculates, additionally, the displacements of the nonstructural component when one of its points of attachment is considered free and subjected to a unit force in the direction of the excitation, while the other point of attachment is held fixed. On the basis of these calculated displacements, one also defines a vector $\{\phi\}_c$, a displacement f_{cc}, and a vector $\{df\}$. $\{\phi\}_c$ contains the aforementioned displacements, f_{cc} is the displacement corresponding to the point subjected to the unit force, and $\{df\}$ contains the corresponding distortions in the elements of the system, after they are normalized with respect to the displacement f_{cc}. Furthermore, one calculates for each of the component modes the values of β_j and $\Phi_0(i, j)$, which are parameters defined by

$$\beta_j = \frac{1}{1 + R_{2j} / R_{1j}};$$

$$F_0(i, j) = F_m(i) + \beta_j [F_n(i) - F_m(i)] \tag{1}$$

where R_{1j} and R_{2j} are the reactions at the ends of the independent secondary system when it vibrates in its j^{th} mode shape; and $\Phi_m(i)$ and $\Phi_n(i)$ are the values in the mode shape $\{\Phi\}_i$ of the structure corresponding to the degrees of freedom to which the secondary system is connected.

In the application of the method, one also considers that the combined primary-secondary system is a system with $N_p + N_s$ degrees of freedom, that the natural frequencies of this combined system are approximately equal to those of its independent components, and that its modes of vibration may be classified into three types: (a) a resonant mode when the natural frequency in this mode is a natural frequency that is common to the two independent components, (b) a nonresonant mode with a primary system frequency when its natural frequency corresponds to a frequency of the independent primary system, and (c) a nonresonant mode with a secondary system frequency when its natural frequency corresponds to a natural frequency of the independent secondary system. For each pair of resonant modes and each nonresonant mode of the combined primary-secondary system, one thereafter calculates a vector of maximum secondary-system modal distortions and combines these vectors on the basis of the square root of the sum of the squares to estimate the corresponding maximum distortions. The formulas employed to calculate such vectors of maximum modal distortions, denoted here as $\{X_s\}_r$, are as follows:

Corresponding to a pair of resonant modes,

$$\{X_s\}_r = \sqrt{2(\rho_{mn} - \alpha_{mn})}\,\mu_{sR}\{d\phi\}_J\sqrt{SD(\omega_m,\xi_m)SD(\omega_n,\xi_n)} \tag{2}$$

if $|\xi_{pI} - \xi_{sJ}| \geq |F_0(I,J)\sqrt{\gamma_{IJ}}|$, or

$$\{X_s\}_r = \sqrt{2(1-\alpha_{IJ})}\,\mu_{sR}\{d\phi\}_J SD(\omega_o,\xi_o) \tag{3}$$

if $|\xi_{pI} - \xi_{sJ}| \leq |F_0(I,J)\sqrt{\gamma_{IJ}}|$. In these equations,

$$\rho_{mn} = \frac{1}{2}\left[\frac{SD(\omega_m,\xi_m)}{SD(\omega_n,\xi_n)} + \frac{SD(\omega_n,\xi_n)}{SD(\omega_m,\xi_m)}\right] \tag{4}$$

$$\alpha_{mn} = 2\frac{\sqrt{\xi'_m\xi'_n}}{\xi'_m + \xi'_n}; \qquad \alpha_{IJ} = \frac{1}{1+(\frac{D}{2\xi'_o})^2} \tag{5}$$

$$\mu_{sR} = \frac{1}{2}\frac{F_0(I,J)}{D} \leq \frac{F_0(I,J)}{(\xi_{pI}-\xi_{sJ})^2} \tag{6}$$

$$D = \sqrt{|F_0^2(I,J)\gamma_{IJ} - (\xi_{pI}-\xi_{sJ})^2|} \tag{7}$$

where

$$\omega_o = \omega_{pI} = \omega_{sJ} \quad \xi_o = \frac{1}{2}(\xi_{pI}+\xi_{sJ}) \quad \gamma_{IJ} = m_J^*/M_I^* \tag{8}$$

$$\omega_m = \omega_n = \omega_o \quad \xi_m = \xi_o - D/2 \quad \xi_n = \xi_o + D/2 \tag{9}$$

and for any circular natural frequency ω_r and damping ratio ξ_r

$$\xi'_r = \xi_r + \frac{2}{\omega_r s} \tag{10}$$

in which s represents ground motion duration.

For a nonresonant mode with primary system frequency,

$$\{X_s\}_r = \mu_{sp}\{d\psi_s\}_p \; SD(\omega_{pI},\xi_{pI}) \tag{11}$$

where

$$\mu_{sp} = \frac{A_0(J)}{\sqrt{1-\delta_{IJ}^2}} \qquad \{d\psi_s\}_p = r_c\{df\} + \sum_{j=1}^{N_s} r_j\{d\phi\}_j \tag{12}$$

$$r_c = \frac{F_n(I) - F_m(I)}{A_0(J)}\sqrt{1+\delta_{IJ}^2};$$
$$r_j = sgn(1-\delta_{Ij})\frac{A_0(j)}{A_0(J)}\sqrt{\frac{1+\delta_{Ij}^2}{1+\delta_{Ij}^2}} \tag{13}$$

in which

$$A_0(j) = \frac{F_0(I,j)\omega_{pI}^2}{\omega_{sj}^2 - \omega_{pI}^2}$$
$$\delta_{Ij} = \frac{\xi_{pI}\omega_{pI} - \xi_{sj}\omega_{sj}}{\omega_{pI} - \omega_{sj}} \tag{14}$$

and "*sgn*" is a function that reads as "the sign of."

Finally, for a nonresonant mode with a secondary system frequency,

$$\{X_s\}_r = \mu_{ss}\{d\phi\}_J SD(\omega_{sJ},\xi_{sJ}) \tag{15}$$

where

$$\mu_{ss} = \sqrt{\left[1+\sum_{i=1}^{N_p}\frac{B_0(i)}{1+\delta_{iJ}^2}\right]^2 + \left[\sum_{i=1}^{N_p}\frac{B_0(i)\delta_{iJ}}{1+\delta_{iJ}^2}\right]^2} \tag{16}$$

in which

$$B_0(i) = \frac{F_0(i,J)\omega_{sJ}^2}{\omega_{pi}^2 - \omega_{sJ}^2};$$

$$\delta_{iJ} = \frac{\xi_{sJ}\,\omega_{sJ} - \xi_{pi}\,\omega_{pi}}{\omega_{sJ} - \omega_{pi}} \tag{17}$$

In all the equations above, $\{d\phi\}_j$ represents a vector of modal distortions whose elements are the distortions in the secondary system when its displacement configuration corresponds to that of $\{\phi\}_j$, its j^{th} mode shape when independently considered; subscripts I and J respectively identify the parameters of the independent primary and secondary systems in the modes whose natural frequencies are the closest or coincide with the natural frequency of the combined system mode under consideration; and $SD(\omega_k, \xi_k)$ signifies the ordinate corresponding to a natural frequency ω_k and damping ratio ξ_k in the displacement response spectrum of the specified earthquake ground motion.

On the basis of the formulas presented above and by introducing approximations similar to those introduced in the derivation of the equivalent lateral force procedure for the analysis of building structures, a less accurate but much simpler procedure may be developed as follows:

Assume, first, that the response of the combined system is approximately given by the response in the two modes that correspond to the fundamental modes of the independent primary and secondary components, and that the natural frequencies in these two modes are the same; that is, assume that these two modes are in resonance. Assume, furthermore, that for the purpose of computing the spectral displacements $SD(\omega_m, \xi_m)$ and $SD(\omega_n, \xi_n)$, $\omega_m = \omega_n = \omega_o$, and $\xi_m = \xi_n = \xi_o$. Introducing these assumptions and approximations into either Eq. (2) or Eq. (3), the maximum distortions in the elements of the secondary system may then be approximated as

$$\{X_s\} = \sqrt{\frac{1}{2}(1-\alpha_{11})}\,\frac{F_0}{D}\{d\phi\}_1 SD(\omega_o,\xi_o) \tag{18}$$

where $F_0 = F_0(1,1)$ is given by Eq. (1), D is defined by Eq. (7), $SD(\omega_o, \xi_o)$ is the ordinate in a specified displacement response spectrum corresponding to a natural frequency ω_o and a damping ratio ξ_o, and α_{11} is given by the left-hand side expression in Eq. (5) when $|\xi_{pI} - \xi_{sJ}| \geq |F_0(I,J)\sqrt{\gamma_{IJ}}|$ and $m = n = 1$, and by the right-hand side expression in this same equation when $|\xi_{pI} - \xi_{sJ}| \leq |F_0(i,J)\sqrt{\gamma_{IJ}}|$ and $I = J = 1$.

Eq. (18) offers a simple expression to compute the distortions in the elements of the secondary system in terms of the modal distortions in its first mode and a spectral displacement. For design purposes, however, it is desirable to define the response of the secondary system in terms of forces and ordinates from an acceleration response spectrum, as it is traditionally done for the seismic design of the structure. To accomplish this, then, one may note that the shear forces acting on the elements of the secondary system are equal to the distortions in the system times their respective stiffness coefficients, and that the lateral force exerted on any of its masses is equal to the sum of the shear forces acting on the elements connected to that particular mass. Accordingly, Eq. (18) may be alternatively written as

$$\{F_p\} = \sqrt{\frac{1}{2}(1-\alpha_{11})}\,\frac{F_0}{D}[k]\{\phi\}_1 SD(\omega_o,\xi_o) \tag{19}$$

where $\{F_p\}$ is a vector that contains the lateral forces acting on the masses of the secondary system and $[k]$ denotes the stiffness matrix of this secondary system. However, by virtue of the relationship between the spectral displacement $SD(\omega_o, \xi_o)$ and the spectral acceleration $SA(\omega_o, \xi_o)$ and the fact that $[k]\{\phi\}_1 = \omega_o^2\,[m]\{\phi\}_1$, where $[m]$ denotes the mass matrix of the secondary system, Eq. (19) may also be expressed as

$$\{F_p\} = \sqrt{\frac{1}{2}(1-\alpha_{11})}\,\frac{F_0}{D}[m]\{\phi\}_1 SA(\omega_o,\xi_o) \tag{20}$$

Consequently, the lateral seismic force in the j^{th} mass of the secondary system may be considered given by

$$F_{pj} = C_p w_{pj}\phi_{j1}\frac{SA(\omega_o,\xi_o)}{g} \tag{21}$$

where ϕ_{j1} denotes the value corresponding to the j^{th} mass of the secondary system in its fundamental (unit participation factor) mode shape, w_{pj} represents the weight of this mass, $SA(\omega_o, \xi_o)$ is the ordinate corresponding to ω_o and ξ_o in a given ground acceleration response spectrum, g is the acceleration of gravity, and C_p is an amplification factor given by

$$C_p = \sqrt{\frac{1}{2}(1-\alpha_{11})}\,\frac{F_0}{D} \tag{22}$$

To derive a simplified expression for the parameter

F_0 that appears in Eq. (22), it may be noted, first, that according to Eq. (1): (1) β_j always varies, depending on the relative value of the reactions R_{1j} and R_{2j}, between 0 and 1; (2) β_j represents a weighing parameter that makes F_0 vary between the values for F_m and F_n, the modal amplitudes corresponding to the two points of the structure to which the secondary system is attached; and (3) β_j is always equal to 0.5 for symmetric secondary systems. Thus, it is reasonable to adopt the average value of 0.5 for this parameter. As a result, F_0 may be considered equal to the average of the structural modal amplitudes F_m and F_n. That is,

$$F_0 = F_0(1,1) = \frac{1}{2}[F_m(1) + F_n(1)] \qquad (23)$$

Second, note that by definition F_0 denotes an amplitude in a mode shape of the structure when this mode shape has been normalized in a way to attain a unit participation factor. As such, it may be expressed as $F_0 = \Gamma_1 F'_0$, where F'_0 represents the same amplitude as F_0 does but when the mode shape $\{F\}_1$ is normalized in any arbitrary way, and G_1 is the participation factor corresponding to this mode shape. Also note that from the definition of participation factor and the concept of effective weight [16] which for the fundamental mode of the structure may be assumed conservatively equal to its total weight, it is possible to approximate this participation factor as

$$G_1 = \frac{W}{\sum_{i=1}^{N_p} W_i F'_i(1)} \qquad (24)$$

where W is the total weight of the structure; W_i is the weight concentrated at its i^{th} floor; $\Phi'_i(1)$ is the amplitude in the fundamental mode shape of the structure corresponding to the ith floor when this mode shape is normalized in any arbitrary way; and N_p is the total number of floors in the structure. If it is assumed now that the fundamental mode shape of the structure varies linearly with height, such that

$$F'_i(1) = F'_{Np}(1)\frac{h_i}{h_{Np}} \qquad (25)$$

where $\Phi'_{Np}(1)$ denotes the modal amplitude corresponding to the top of the structure in such a mode shape, and h_i and h_{Np} are the elevations above ground of its ith and top floors, respectively, then, by virtue of Eqs. (23), (24) and (25), F_0 may be approximated as

$$F_0 = G_1 F'_0 = \frac{W F'_0}{\sum_{i=1}^{N_p} W_i F'_i(1)} = \frac{W\, h_{av}}{\sum_{i=1}^{N_p} W_i h_i} \qquad (26)$$

where h_{av} represents the average of the elevations above ground of the points of the structure to which the secondary system is attached and, as before, W_i and h_i respectively denote the weight and elevation of the ith floor of the structure.

To derive now a simplified expression for the parameter D in Eq. (22) and defined by Eq. (7), one can resort first to the common approximation of substituting the generalized mass of a structure in its fundamental mode by its total mass. In this way, the mass ratio γ_{11}, see Eq. (8), may be considered approximately equal to the ratio between the total weight of the secondary system, w_p, and the total weight of the structure, W; that is, $\gamma_{11} = w_p/W$. Second, one can set the damping ratio of the structure in its fundamental mode equal to 5 per cent, as it has been traditionally assumed in the seismic design of buildings. By the same token, it is reasonable and conservative to assume the damping ratio for the secondary system equal to zero, as damping in nonstructural components is usually negligibly small. As a result, D may be considered approximately equal to

$$D = \sqrt{\left| F_0^2 \frac{w_p}{W} - 0.0025 \right|} \qquad (27)$$

As in the case of the structure, a simplified expression for the modal amplitude ϕ_{j1} may be obtained by assuming that the fundamental mode shape of the secondary system varies linearly along its length. In the case of the secondary system, however, it is necessary to make a distinction between those systems with a single point of attachment and those with two. For a secondary system with a single point of attachment, it may be assumed that its fundamental mode shape varies linearly from zero at its fixed end to a maximum value at the level of its top mass, see Figure (1a). On the other hand, for a secondary system with two points of attachment, it may be assumed that its fundamental mode varies linearly from a zero at the level of its fixed ends to a maximum value at the point in which it attains its maximum displacement when each of its masses is subjected to a force equal to its own weight, see Figure (1b). In such a way, and in similarity with Eq. (26), the modal amplitude ϕ_{j1} may be approximated as

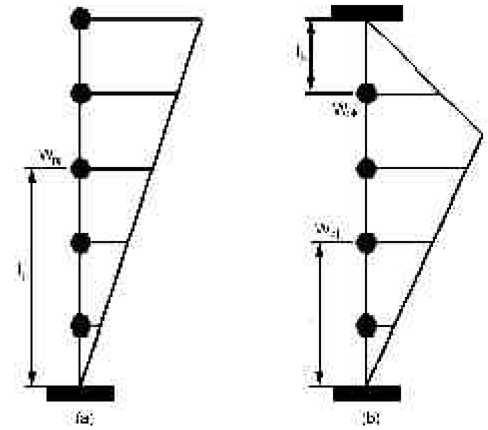

Figure 1. Assumed mode shapes for components with (a) one and (b) two points of attachment.

$$\phi_{j1} = \frac{w_p \, l_j}{\sum_{j=1}^{N_S} w_{pj} \, l_j} \tag{28}$$

where, as before, w_p denotes the total weight of the secondary system, and l_j represents the distance to the j^{th} mass of the secondary system measured from its lower end if this mass is located below the secondary system's point of maximum deflection, or from its upper end if it is located above, see Figure (1). In the case of a mass located directly on the point of maximum deflection, measure l_j from the support that is farther away from that mass.

Finally, simplified expressions for the factor α_{11} and the amplification factor C_p may be obtained as follows. Consider first the expression in the left-hand side of Eq. (5), which defines α_{11} when $|\xi_{p1} - \xi_{s1}| \geq |F_0(1,1)\sqrt{\gamma_{11}}|$. According to Eqs. (9) and (10), one has that

$$\xi'_m + \xi'_n = \xi'_o - D/2 + \xi'_o + D/2 = 2\xi'_o \tag{29}$$

and similarly,

$$\xi'_m \xi'_n = (\xi'_o - D/2)(\xi'_o + D/2) = (\xi'_o)^2 - D^2/4 \tag{30}$$

Consequently, α_{11} may be written as

$$\alpha_{11} = \frac{\sqrt{(\xi'_o)^2 - D^2/4}}{\xi'_o} = \sqrt{1 - \frac{D^2}{4(\xi'_o)^2}} \approx 1 - \frac{D^2}{8(\xi'_o)^2} \tag{31}$$

Upon substitution into Eq. (22), one obtains, thus, the following approximate expression for the amplification factor C_p:

$$C_p = \sqrt{\frac{1}{2}(1 - 1 + \frac{D^2}{8(\xi'_o)^2})} \frac{F_0}{D} = \sqrt{\frac{D^2}{16(\xi'_o)^2}} \frac{F_0}{D} = \frac{F_0}{4\xi'_o} \tag{32}$$

where, in view of Eq. (10) and after considering the approximations introduced above,

$$\xi'_o = \xi_o + \frac{2}{\omega_o s} = \frac{1}{2}(\xi_{p1} + \xi_{s1}) + \frac{T}{\pi s} = 0.025 + \frac{T}{\pi s} \tag{33}$$

in which T denotes the fundamental natural period of the structure and, as before, s denotes strong motion duration. Or, if a strong motion duration of 25 seconds is conservatively assumed, ξ'_o may be considered alternatively equal to

$$\xi'_o = 0.025 + \frac{T}{\pi(25)} = 0.025 + 0.025\frac{T}{2} = 0.025\,(1 + 0.5T) \tag{34}$$

and thus, explicitly,

$$C_p = \frac{F_0}{4(0.025)(1 + 0.5T_r)} = \frac{10F_0}{1 + 0.5T} \tag{35}$$

It may be seen that this expression is independent of the mass ratio w_p/W and thus, for secondary systems with a mass ratio of less than $0.0025/F_o^2$, the amplification factor C_p does not vary, at least within the order attained with the introduced approximations, with such ratio.

Consider now the expression in the right-hand side of Eq. (5), which defines the factor α_{11} when $|\xi_{p1} - \xi_{s1}| \leq |F_0(1,1)\sqrt{\gamma_{11}}|$. By substitution of that expression into Eq. (22), one obtains

$$C_p = \sqrt{\frac{1}{2}(1 - \frac{1}{1 + (\frac{D}{2\xi'_o})^2})} \frac{F_0}{D} = \frac{F_0}{2\xi'_o}\sqrt{\frac{1}{2[1 + (\frac{D}{2\xi'_o})^2]}} = \frac{F_0}{\sqrt{2[D^2 + (2\xi'_o)^2]}} \tag{36}$$

which, after substituting the approximations for D and ξ'_0 defined by Eqs. (27) and (34), becomes

$$C_p = \frac{F_0}{\sqrt{2[F_0^2 \frac{w_p}{W} - 0.0025 + 4(0.025)^2(1 + 0.5T)^2]}} = \frac{1}{\sqrt{2\frac{w_p}{W} + \frac{(1 + 0.5T)^2 - 1}{200 F_0^2}}} \tag{37}$$

Since this formula is only valid for those cases in which $|\xi_{p1} - \xi_{s1}| \leq |F_0(1,1)\sqrt{\gamma_{11}}|$, it may be noted that an upper limit to the value of C_p is obtained when $w_p/W = 0.0025/F_0^{\,2}$ and that this upper limit is equal to

$$(C_p)_{max} = \frac{1}{\sqrt{\dfrac{1}{200\,F_0^2} + \dfrac{(1+0.5T)^2-1}{200\,F_0^2}}} = \frac{\sqrt{200}\,F_0}{1+0.5T} \tag{38}$$

It may be noted, too, that for mass ratios of less than $0.0025/F_0^{\,2}$, the amplification factor C_p is given by Eq. (35) and that in this case, as noted previously, C_p does not vary with the mass ratio w_p/W. For simplicity, therefore, it may be conservatively assumed that C_p in this latter case is given by the upper limit defined by Eq. (38). This way, the amplification factor C_p may be defined, independently of the relationship between $|\xi_{p1} - \xi_{s1}|$ and $|F_0\sqrt{\gamma_{11}}|$, by the following single expression:

$$C_p = \frac{1}{\sqrt{2\dfrac{w_p}{W} + \dfrac{(1+0.5T)^2-1}{200\,F_0^2}}} \le \frac{\sqrt{200}\,F_0}{1+0.5T} \tag{39}$$

The derivation of the formulas established above is based on the assumption that the fundamental natural period of the secondary system is in resonance with the fundamental natural period of the structure; i.e., that the values of these two natural periods are equal or are very close to one another. Although this assumption offers the advantage of not having to know the natural periods of the secondary system to carry out its seismic design, it may be nonetheless overly conservative for those cases in which the two natural periods in question are not close to one another. As a means to reduce the conservatism involved in those cases for which the fundamental natural period of the secondary system is known, C_p may be substituted by a modified amplification factor C_m equal to the amplification factor that corresponds to fundamental mode of the secondary system, when this mode is a nonresonant one. That is, the amplification factor given by Eq. (16), which, according to Eq. (17), and by observing that $B_0(i) \approx 0$ for all $i \neq 1$ and $\delta_{iJ} \approx 0$ for all i if ω_{pi} and ω_{s1} are not too close to one another, may be taken approximately equal to

$$\mu_{ss} = 1 + \frac{F_0(1,1)\,\omega_{s1}^2}{\omega_{s1}^2 - \omega_{p1}^2} \tag{40}$$

of, for simplicity, just equal to

$$\mu_{ss} = \frac{F_0(1,1)\,\omega_{s1}^2}{\omega_{s1}^2 - \omega_{p1}^2} \tag{41}$$

In consequence, the modified amplification factor C_m may be considered given by

$$C_m = \frac{F_0}{\left|\left(\dfrac{T_p}{T}\right)^2 - 1\right|} \le C_p \tag{42}$$

where, as before, F_0 is given by Eq. (26), and T_p and T respectively denote the fundamental natural period of the secondary system and the fundamental natural period of the structure.

Note that this modified amplification factor accurately reflects the fact that $C_m = F_0$ when $T_p = 0$ (i.e., for rigid secondary systems) and $C_m = 0$ for $T_p = \infty$ (i.e., for extremely flexible secondary systems).

The approach followed to account for the nonlinearity of the structure and the secondary system is similar to the approach used in current design practice to consider the nonlinear behaviour of building structures. That is, the nonlinear behaviour of the structure and the secondary system is taken into account by reducing by a strength reduction factor the lateral strength that is required when the structural elements of the two sub-systems are kept into their linear range of behaviour at all times. In the case of a secondary system, however, such a strength reduction factor is considered equal to the product of two other reduction factors. One of these reduction factors accounts for the nonlinear behaviour of the structure and the fact that the motion at the supports of the secondary system is affected by this nonlinear behaviour. The other accounts for the nonlinearity of the secondary system itself and the fact that it is possible to reduce the lateral strengths of its structural elements when these elements are capable of resisting inelastic deformations. The first factor is selected on the basis of the capacity of the components of the structure to resist inelastic deformations, while the second is chosen on the basis of the capacity of the elements of the secondary system to withstand inelastic deformations. In essence, this approach is equivalent to consider one subsystem at the time, independent of each other. It is adopted after observing from the results of numerical simulations that the deformation demands on the elements of a secondary system are reduced when the supporting structure is allowed to incur into its nonlinear range of behaviour, and reduced again when the secondary system itself is also allowed to go into its nonlinear range of behaviour. A reduction in response is considered because in the method being derived it is assumed that the fundamental natural frequency of the secondary system is tuned to the fundamental natural frequency of the structure, and because it has been observed that in such a case the nonlinearity of the structure always leads to a reduction in the response of the secondary system [8, 11].

As in the case of building structures, it is also assumed that the aforementioned strength reduction factors are approximately equal to the average values that have been obtained for single-degree-of-freedom systems. This assumption is justified on the grounds that for the purpose of accounting for nonlinear effects the structure and the nonstructural component are being considered as independent systems and that the reduction factors for single-degree-of-freedom systems are approximately valid for multi-degree-of-freedom systems that have relatively uniform properties and vibrate predominantly in their fundamental modes. In particular, it is assumed that the strength reduction factors in question are those suggested by Newmark and Hall [17], or when more refined values are warranted, those proposed more recently by Miranda [18]. The strength reduction factors proposed by Newmark and Hall are given by

$$R_\mu = \begin{cases} 1 + \dfrac{T - 0.03}{0.095}\left(\sqrt{2\mu - 1} - 1\right) & \text{if } 0.03 \le T \le 0.125\ sec \\ \sqrt{2\mu - 1} & \text{if } 0.125 < T < 0.5 sec \\ \mu & \text{if } T \ge 0.5 sec \end{cases} \tag{43}$$

in which μ is a predetermined target ductility ratio and T denotes the initial fundamental natural period of the system in seconds. In terms of these same parameters and given site soil conditions, the strength reduction factors proposed by Miranda are of the form

$$R_\mu = 1 + \frac{\mu - 1}{F} \tag{44}$$

where, for rock sites,

$$F = 1 + \frac{1}{(10 - \mu)T} - \frac{1}{2T} exp[-\frac{3}{2}(lnT - \frac{3}{5})^2] \tag{45}$$

for alluvium sites,

$$F = 1 + \frac{1}{(12 - \mu)T} - \frac{2}{5T} exp[-2(lnT - \frac{1}{5})^2] \tag{46}$$

and for soft soil sites

$$F = 1 + \frac{T_g}{3T} - \frac{3T_g}{4T} exp[-3(ln\frac{T}{T_g} - \frac{1}{4})^2] \tag{47}$$

where T_g is the predominant period of the ground motion expected at the site under consideration, defined as the period that corresponds to the peak ordinate in the ground motion's 5% damping linear velocity response spectra.

In the application of the strength reduction factors presented above, it is important to keep in mind that they represent average values obtained from a statistical analysis with a large number of ground motions and there is thus an inherent dispersion associated with them. As such, they are useful to estimate on the average sense the lateral strength that is required to keep the inelastic deformations in the elements of a nonstructural component within specified limits when subjected to the earthquake ground motions expected at a given site, but not to predict the level of the inelastic deformations generated in them by a single ground motion.

3. Proposed Method

The proposed procedure involves thus the calculation of the maximum lateral forces that may be generated by a specified earthquake ground motion on the masses of a nonstructural component attached to a building structure. These forces are determined according to

$$F_{pj} = \frac{w_{pj}\, l_j}{\sum_{j=1}^{n} w_{pj}\, l_j} V_p \tag{48}$$

where F_{pj} denotes the force acting on the j^{th} mass of the nonstructural component; w_{pj} is the weight of this j^{th} mass; and l_j is the distance to the same mass measured in the case of a single attachment point from the attachment point, see Figure (1a). In the case of a nonstructural component with two attachment points, l_j is measured from its lower end if the mass is located below the point at which the component attains its maximum deflection when each mass is subjected to a lateral force equal to its own weight, and from its upper end otherwise, see Figure (1b). In the case of a mass located directly at such point of maximum deflection, l_j is measured from the attachment point that is the farthest away from that mass. In addition, n represents the total number of masses in the nonstructural component and V_p is defined as

$$V_p = \frac{C_p}{RR_p} S_a w_p \tag{49}$$

in which S_a is the ordinate corresponding to the fundamental natural period and damping ratio of the structure in the acceleration response spectrum specified for the design of the structure, expressed as a fraction of the acceleration of gravity. However, when the fundamental natural period of the component is known, S_a represents the average of the spectral ordinates corresponding to the fundamental natural periods and damping ratios of the structure and the nonstructural component. Additionally, R and R_p are strength reduction factors that account for the nonlinear behaviour of the supporting structure and the nonstructural component, respectively, computed using either Eq. (43) or Eqs. (44) through (47). R is obtained based on the target ductility ratio for the structure and R_p based on the target ductility ratio for the nonstructural component. Finally, w_p denotes the total weight of the nonstructural component, and C_p is a component amplification factor calculated according to

$$C_p = \frac{1}{\sqrt{\frac{2w_p}{W} + \frac{(1+0.5T)^2 - 1}{200 F_0^2}}} \le \frac{\sqrt{200}\, F_0}{1+0.5T} \tag{50}$$

where, once again, w_p denotes the total weight of the nonstructural component, W is the total weight of the building, T is the fundamental natural period of the structure, and

$$F_0 = \frac{W\, h_{av}}{\sum_{i=1}^{N} W_i h_i} \tag{51}$$

in which W_i and h_i respectively denote the weight and elevation above ground of the building's i^{th} floor, h_{av} is the average of the elevations above ground of the points of the building to which the nonstructural component is connected, and N denotes the number of floors in the building. For those cases in which the fundamental natural period of the nonstructural component is known, the amplification factor C_p may be alternatively considered equal to a modified amplification factor C_m given by

$$C_m = \frac{F_0}{\left| \left(\frac{T_p}{T}\right)^2 - 1 \right|} \le C_p \tag{52}$$

where T_p represents the fundamental natural period of the nonstructural component and, as before, T denotes the fundamental natural period of the structure.

The method is intended to be valid for systems and ground motions that do not deviate significantly from the major assumptions made in its derivation. As described in the previous section, these assumptions are:

1. The response of the combined structural-nonstructural system is approximately given by the response in the two modes of the system that correspond to the fundamental natural periods of the two independent subsystems.
2. The fundamental natural period of the nonstructural element coincides with the fundamental natural period of the structure; that is, the fundamental mode of the nonstructural element is in resonance with the fundamental mode of the structure.
3. The fundamental mode shape of the structure varies linearly from zero at its base to a maximum value at its top.
4. The fundamental mode of the nonstructural element varies linearly along its height. In the case of a single point of attachment, it varies from zero at its point where it is connected to the structure to a maximum value at its other end. In the case of two points of attachment, it varies from zero at these two attachment points to a maximum value at the point where it attains its maximum displacement when each of its masses is subjected to a lateral force equal to its own weight, see Figure (1).
5. The generalized masses in the fundamental modes of the structure and the nonstructural element are equal to their respective total masses.
6. The damping ratios in the fundamental modes of the structure and the nonstructural element are equal to 5 and 0 per cent, respectively.
7. The strong part of the ground motions exciting the base of the structure exhibit a duration of 25 seconds.

4. Illustrative Example

To illustrate its use, the proposed method is employed to determine the design lateral forces for the three-mass nonstructural component shown in Figure (2), when the component is rigidly connected to the fourth and sixth stories of the six-story building shown in this same figure. The building is structured with ordinary steel moment-resisting

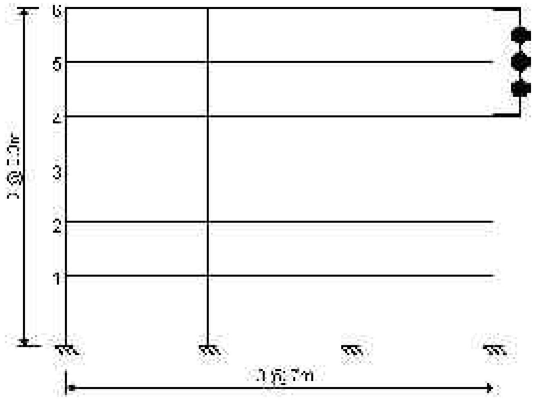

Figure 2. Building and nonstructural component considered in illustrative example.

frames and has a fundamental natural period of 0.6 seconds. A target ductility of 4.0 is considered in its design. Its weight per floor is 2,200*kN* and its total weight is thus equal to 13,200*kN*. The nonstructural component is an ordinary architectural fixture which may be modelled as a three-degree-of-freedom shear beam with four equal segments, each with a length of 1.65 meters. Each of its three masses weighs 4.4*kN*, and hence its total weight is 13.2*kN*; that is, 0.1% of the total weight of the building. Its fundamental period is estimated to be 0.5 seconds when its two ends are assumed fixed. A target ductility factor of 2.0 is considered appropriate for its seismic design. The earthquake input to the building is defined by the design spectrum shown in Figure (3).

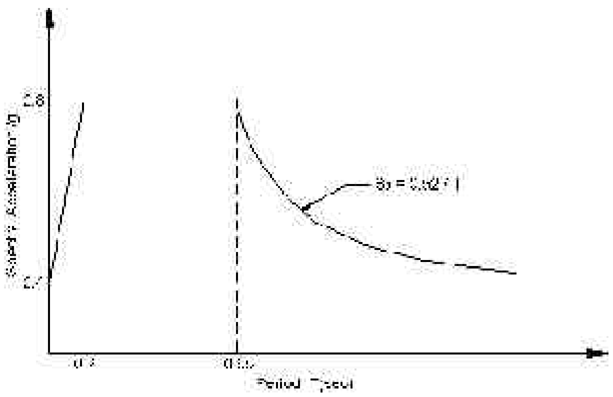

Figure 3. Design spectrum specified for building in illustrative example.

For the calculation of the design lateral forces, it may noted that in the case under consideration the average of the elevations above ground of the nonstructural component's two attachment points is equal to 16.5*m*. Hence, substitution into Eq. (51) of this value and the floor weights given above leads to

$$F_0 = \frac{W\, h_{av}}{\sum_{i=1}^{6} W_i\, h_i} =$$

$$\frac{13200\,(16.5)}{2200\,(3.3+6.6+9.9+13.2+16.5+19.8)} = 1.43$$

Similarly, after substituting into Eq. (50) the given total weights of the structure and the nonstructural component and F_0 = 1.43, one obtains

$$C_p = \frac{1}{\sqrt{\dfrac{2w_p}{W} + \dfrac{(1+0.5T)^2 - 1}{200 F_o^2}}} =$$

$$\frac{1}{\sqrt{\dfrac{2(13.2)}{13200} + \dfrac{[1+0.5(0.6)]^2 - 1}{200\,(1.43)^2}}} = 16.5$$

which exceeds the limit of $\sqrt{200}\,F_0/(1+0.5T) = 15.6$ and thus C_p will be considered equal to 15.6. In this case, however, the fundamental natural period of the nonstructural component is known. It is possible, therefore, to use a reduced value of this amplification factor by using instead the formula given by Eq. (52). By considering that for the systems under consideration T = 0.60*s* and T_p = 0.5*s*, such a reduced amplification factor results as

$$C_m = \frac{F_0}{\left|\left(\dfrac{T_p}{T}\right)^2 - 1\right|} = \frac{1.43}{\left|\left(\dfrac{0.5}{0.6}\right)^2 - 1\right|} = 4.7$$

which is less than the valued of C_p determined above. It may be noted, too, that from the given design spectrum and for the fundamental natural periods of the structure and the nonstructural component,

$$S_a = \frac{1}{2}(0.8 + 0.8) = 0.8$$

Finally, note that, if Eq. (43) is used, the strength reduction factor for the structure, R, is in this case equal to 4.0 since for the structure μ = 4.0 and T ≥ 0.5*s*. Similarly, the strength reduction factor for the nonstructural component, R_p, is equal to 2.0 since for the nonstructural component μ = 2.0 and T ≥ 0.5*s*. Upon substitution into Eq. (49) of the total weight of the component and the values of C_m, S_a, R, and R_p determined above, one obtains, thus,

$$V_p = \frac{C_m}{RR_p} S_a w_p = \frac{4.7}{(4.0)(2.0)}(0.8)(13.2) = 6.204\ kN$$

To determine now the value of the lateral forces on the masses of the nonstructural component using Eq. (48), it may be observed that it is necessary to obtain first its point of maximum deflection under lateral forces equal to the weight of its masses and define the distances l_j that appear in this equation. In the case under consideration, however, the nonstructural component is symmetric in mass and geometry and thus such a point of maximum deflection is located at its geometric centre. By inspection, therefore, it may be determined that $l_1 = l_3 = 1.65m$ and $l_2 = 3.3m$, where l_1, l_2, and l_3 correspond, respectively, to the lower, middle, and upper masses. As a result, the desired lateral forces are equal to

$$F_{p1} = \frac{w_{p1} l_1}{\sum_{j=1}^{3} w_{pj} l_j} V_p = \frac{4.4(1.65)}{4.4(1.65+3.30+1.65)}(6.204) =$$

$$0.25(6.204) = 1.551\ kN$$

$$F_{p2} = \frac{w_{p2} l_2}{\sum_{j=1}^{3} w_{pj} l_j} V_p = \frac{4.4(3.30)}{4.4(1.65+3.30+1.65)}(6.204) =$$

$$0.50(6.204) = 3.102\ kN$$

$$F_{p3} = \frac{w_{p3} l_3}{\sum_{j=1}^{3} w_{pj} l_j} V_p = \frac{4.4(1.65)}{4.4(1.65+3.30+1.65)}(6.204) =$$

$$0.25(6.204) = 1.551\ kN.$$

5. Comparative Study

To assess whether or not nonstructural components designed with the proposed method would resist a critical earthquake ground motion, a comparative analysis is performed with two different nonstructural components mounted alternatively on a 10-story building and a 13-story one. In this analysis, the deformation ductility demands imposed on the resisting elements of the considered nonstructural components when the base of their supporting building is excited by a severe ground motion are compared against the deformation ductility capacities assumed in the components' design. To this end, the shear force capacities (yield strength) of the elements of the non-structural components are considered to be equal to the shear forces that act on the components' resisting elements when the components' masses are subjected to the equivalent lateral forces computed with the proposed method. Two different cases are considered in the determination of such shear force capacities. In the first case, the components are assumed able to resist deformation ductilities of up to two, while in the second case these ductilities are assumed to be up to six. The ductility demands are obtained by means of a nonlinear time-history analysis in which nonstructural component and supporting structure are considered together as a single unit. The beams and columns of both buildings are assumed to possess elastoplastic behaviour with yield moments defined by their ultimate moments. Similarly, it is assumed that the nonstructural components behave as elastoplastic shear beams rigidly attached to their supports and yield shear strengths equal to their shear force capacities. Each building is considered with a damping matrix proportional to its own stiffness matrix and a damping ratio of five per cent in its fundamental mode. The seismic design of the 10-story building is carried out considering a target ductility of 4.0, whereas that of the 13-story building is performed considering a target ductility of 6.0. In each case, a component is considered satisfactorily designed if the deformation ductility demands imposed on its resisting elements by the ground motion selected for the analysis are equal or less than the deformation ductility capacities assumed in its design; that is, a deformation ductility capacity of 2.0 in the first case and 6.0 in the second case.

The characteristics of the two buildings considered are depicted in Figures (4) and (5). The first building represents an actual reinforced-concrete office building located in the soft-soil area of Mexico City. The building experienced significant damage during the September 19, 1985 earthquake [19] and incurred thus into its nonlinear range of behaviour during this earthquake. Its floor system is formed with a 100*mm* reinforced concrete slab supported by reinforced concrete beams and girders. The materials used in the building's design are concrete with a nominal 28-day strength of 24*MPa* and reinforcing steel with a nominal yield strength of 400*MPa*. The dead and live loads considered in its design yield a load per floor of 2,031*kN* for floors 1 to 9, a roof load of 1,591*kN*, and a total building weight of 19,870*kN*. The properties of its beams and columns are listed in Table (1). On the basis of the effective moments of inertia given in this table, the first three natural frequencies of the building are

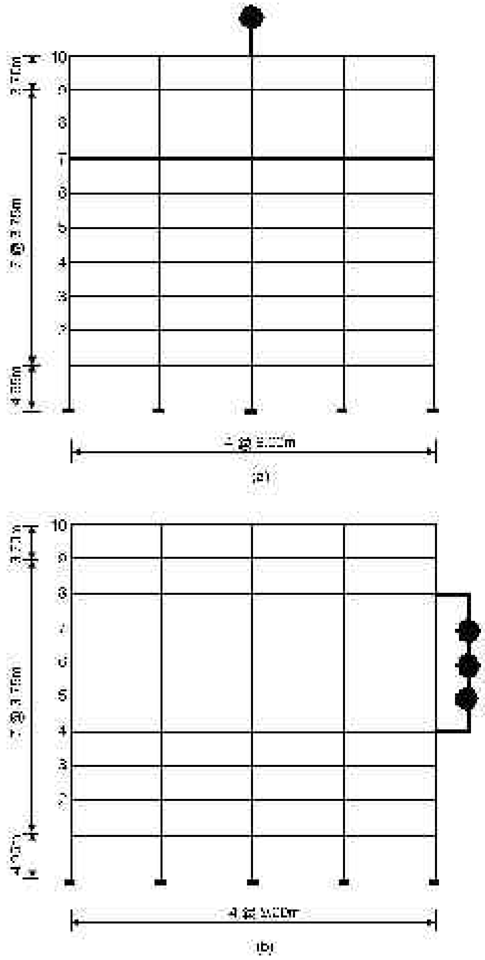

Figure 4. Ten-story building in comparative study and attachment configuration of nonstructural components.

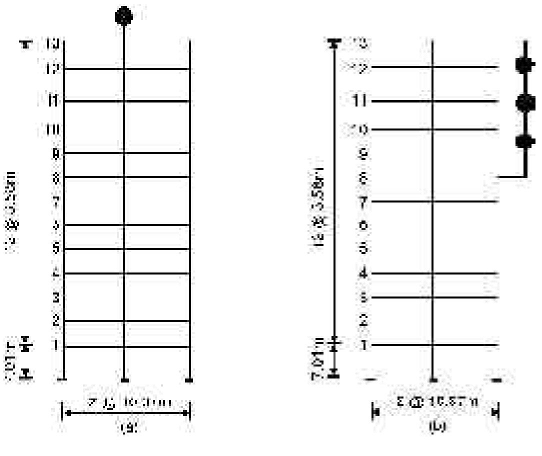

Figure 5. Thirteen -story building in comparative study and attachment configuration of nonstructural components.

0.542, 1.439, and 2.421*Hz*. The second building corresponds to an existing commercial building located in Sherman Oaks, California. This building was instrumented by the California Division of Mines and Geology and acceleration records were obtained at its base and several other locations during the 1994 Northridge, California earthquake [20]. It reportedly suffered some structural damage during this earthquake [21], which is an indication that the building incurred into its inelastic range of behaviour during this event. Its floor system consists of a 102-mm reinforced concrete slab supported by reinforced-concrete beams and girders. Its beams and columns have the dimensions and properties listed in Table (2). The concrete and reinforcing steel used in its design have strengths of 27.5*MPa* and 412*MPa*, respectively. Using the dead and live loads assumed in its design, the building is considered with a load per floor of 820*kN* for floors 1 to 12, a roof load of 760*kN*, and a total building weight of 10,600*kN*. The first three natural frequencies of the building, calculated on the basis of these loads and the effective moments of inertia listed in Table (2), are 0.415, 1.29, and 2.34*Hz*.

The two nonstructural components studied are: (a) a single-mass system with a single point of attachment; and (b) a three-mass system with two points of attachment. They are considered with the masses and stiffnesses listed in Table (3) and connected to the buildings in the way shown in

Table 1. Properties of beams and columns in 10-story building.

	Columns				Beams				
Story	b (m)	h (m)	I_{eff} (m^4)	M_y (kN-m)	Floor	b (m)	h (m)	I_{eff} (m^4)	M_y (kN-m)
1-2	0.5	0.9	0.01309	497.0	1-3	0.4	0.8	0.03151	780.9
3-4	0.5	0.8	0.00953	397.0	4-7	0.4	0.7	0.02549	627.6
5-6	0.5	0.7	0.00669	255.0	8-9	0.4	0.6	0.01742	463.4
7-8	0.5	0.6	0.00447	217.0	10	0.4	0.5	0.01330	210.9
9-10	0.5	0.5	0.00279	180.0	-	-	-	-	-

Note: b = width; h = height; I_{eff} = effective moment of inertia; M_y= yield moment

Table 2. Properties of beams and columns in 13-story building.

	Columns								Beams				
	Exterior				Interior								
Story	b (m)	H (m)	I_{eff} (m^4)	M_y (kN-m)	b (m)	h (m)	I_{eff} (m^4)	M_y (kN-m)	Floor	b (m)	H (m)	I_{eff} (m^4)	M_y (kN-m)
1-4	0.61	0.91	0.0388	1480	0.91	0.91	0.0583	3304	1-4	0.61	0.81	0.0273	1485
5-8	0.61	0.91	0.0388	1480	0.91	0.91	0.0583	3304	5-8	0.61	0.81	0.0273	1192
9-13	0.61	0.91	0.0388	1480	0.91	0.91	0.0583	3304	9-13	0.61	0.81	0.0273	1128

Note: b = width; h = height; I_{eff} = effective moment of inertia; M_y= yield moment

Table 3. Masses and stiffnesses of nonstructural components in comparative study.

Component	Masses		Stiffnesses	
	Mass No.	Mass (Mg)	Element No.	Stiffness (kN/m)
1-mass	1	0.00450	1	0.04440
3-mass	1	0.00300	1	0.05584
	2	0.00150	2	0.03722
	3	0.00100	3	0.01861
			4	0.00931

Figures (4) and (5). Their fundamental natural frequencies when their ends connected to the structure are held fixed are equal to 0.5*Hz* in both cases. To be consistent with the assumptions made in the derivation of the method being evaluated, each component is assumed to have zero-percent damping.

The ground motion selected for the analysis of the 10-story building corresponds to the first 80 seconds of the acceleration time history which results from combining vectorially along the direction that maximises its peak value the two horizontal ground acceleration records obtained at the *SCT* station during the 1985 Mexico City earthquake. The peak ground acceleration of the resulting time history, which occurs at a time of 60.34 seconds, is 0.188*g*. The spectral accelerations corresponding to this acceleration time history and the fundamental natural frequencies and damping ratios of the building and the nonstructural components are 0.79*g* and 4.02*g*, respectively. In this case, therefore, the equivalent lateral forces on the nonstructural components are calculated using the average spectral acceleration of 2.41*g*. For the 13-story building, the ground motion used corresponds to the acceleration time history recorded at the base of the building (Channel 11) during the 1994 Northridge earthquake [20]. This time history exhibits a peak ground acceleration of 0.87*g* at a time of 3.04 seconds. The spectral accelerations corresponding to this ground motion and the fundamental natural frequencies and damping ratios of the building and the nonstructural components are respectively equal to 0.215g and 0.287g. Thus, in this case the equivalent lateral forces on the nonstructural components are computed considering the average spectral acceleration of 0.251*g*.

As mentioned before, Newmark and Hall's and Miranda's strength reduction factors represent average values for a large number of ground motions and may thus deviate considerably from the values obtained for individual ground motions (Miranda [18] reports coefficients of variation as large as 0.6). For example, the strength reduction factors that lead to deformation ductility demands of 2 and 6 in a single-degree-of-freedom system with a natural frequency of 0.5*Hz* and a damping ratio of zero percent are respectively equal to 5.3 and 13.6 when subjected directly to the ground motion from the Mexico City earthquake and 2.1 and 7.9 when subjected directly to the ground motion form the Northridge earthquake. In contrast, if computed with Miranda's formulas, these strength reduction factors are 2.4 and 8.0 for the soil conditions of the site where the ground motion from the Mexico City earthquake was recorded, and 2.1 and 6.2 for the soil conditions of the site where the ground motion from the Northridge earthquake was obtained. Thus, comparing against a target ductility ratio the ductility demands imposed by a single ground motion on a nonstructural component designed considering average strength reduction factors is a meaningless exercise. A similar problem arises in regard to the use of the target ductility ratios specified for the design of the supporting structures in the determination of the shear force capacities of the nonstructural components. The problem is that, as shown later on, the ground motions selected for the analysis generate in the structures ductility demands that are larger than the target ductility ratios assumed in their designs. For the purpose of a comparative analysis, it makes thus no sense to use the "nominal" target ductility ratios considered in the design of the structures in the calculation of the shear force capacities of the components.

For a meaningful comparison, therefore, in this comparative study the shear force capacities of the nonstructural components are computed using the suggested approximate formulas but considering the "exact" reduction factors for each of the considered ground motions. These exact reduction factors are determined from a nonlinear time-history analysis in which the shear force capacities of the structural components' elements are reduced iteratively from the peak shear forces acting on them when the structure and the nonstructural component are both assumed to behave linearly until the target ductility ratio is attained. The exact reduction factors obtained this way are listed in Table (4). It is important to note that these exact reduction factors represent the factors by which the shear forces capacities calculated on the basis of a linear structure and a

Table 4. Exact strength reduction factors considered in comparative study.

Nonstructural Component	Mexico City Earthquake $\mu = 2$	Mexico City Earthquake $\mu = 6$	Northridge Earthquake $\mu = 2$	Northridge Earthquake $\mu = 6$
1-mass	133.5	693.0	5.2	12.3
3-mass	41.0	174.0	6.1	13.7

Note: μ = component target ductility demand.

linear nonstructural component need to be reduced to obtain a ductility demand on the element subjected to the largest deformation equal to the target ductility. Note also that they are equivalent to the product RR_p that appears in Eq. (49). Furthermore, note that the reduction factors for the Mexico City earthquake are what it seems exaggeratedly large because for this ground motion: (a) the actual reduction factors deviate considerably from the average reduction factors (as reported in the previous paragraph), and (b) the actual ductility demands on the analysed structure are much larger than the nominal target ductility ratios assumed in its design (as reported in the following paragraph).

The results of the study are summarised in Tables (5) and (6). Table (5) lists the shear force capacities and deformation ductilities obtained for the resisting elements of the nonstructural components when these are connected to the 10-story building. Table (6) lists the corresponding parameters when the nonstructural components are attached to the 13-story building. Worthwhile to note for the interpretation of these results is the fact that both buildings undergo significant yielding in the performed time-history analyses. The 10-story building experienced yielding in all its beams and all its columns up to the 9^{th} story, with rotational ductility demands of up to 9.5 in its beams and 12.5 in its columns, and story deformation ductility demands of up to 22.7. Similarly, the 13-story building experienced yielding in all its beams up to the 11^{th} story and all its columns up to the 3^{rd} story, with rotational ductility demands of up to 3.1 in the beams and 4.9 in the columns, and story deformation ductility demands of up to 11.2.

The results presented in Tables (5) and (6) reveal that the ductility demands imposed on the resisting elements of the nonstructural components by the selected ground motions are in every case less than or approximately equal to the target deformation ductilities considered in their design. These results indicate thus that the proposed procedure in combination with the calculated exact strength reduction factors properly accounted for the post-yield deformations in the buildings and the nonstructural components and led in each instance to adequate designs. They also indicate that the proposed procedure may be useful to determine, on the average sense, the design lateral strengths of nonstructural components on buildings when used in conjunction with average strength reduction factors such as those proposed by Newmark and Hall [17] and Miranda [18].

Table 5. Shear force capacities and deformation ductility demands in elements of nonstructural components in 10-story building.

Component	Element	Components with Target Ductility of 2: Shear Force Capacity (kN)	Components with Target Ductility of 2: Ductility Demand	Components with Target Ductility of 6: Shear Force Capacity (kN)	Components with Target Ductility of 6: Ductility Demand
1-mass	1	0.00815	1.9	0.00157	6.7
3-mass	1	0.01597	1.4	0.00377	5.2
	2	0.00743	2.1	0.00175	4.3
	3	0.00111	2.1	0.00026	4.6
	4	0.00396	1.4	0.00093	3.4

Table 6. Shear force capacities and deformation ductility demands in elements of nonstructural components in 13-story building.

Component	Element	Components with Target Ductility of 2: Shear Force Capacity (kN)	Components with Target Ductility of 2: Ductility Demand	Components with Target Ductility of 6: Shear Force Capacity (kN)	Components with Target Ductility of 6: Ductility Demand
1-Mass	1	0.01182	1.4	0.00503	4.4
3-Mass	1	0.01068	1.3	0.00476	3.0
	2	0.00497	1.5	0.00221	3.7
	3	0.00075	2.1	0.00033	3.8
	4	0.00265	1.8	0.00118	3.6

6. Summary and Conclusions

An approximate method that accounts for the nonlinear behaviour of building and nonstructural component has been proposed for the seismic analysis of nonstructural components in buildings. The method is based on a simplified procedure for the analysis of linear primary-secondary systems and the use of strength reduction factors to account for the nonlinearity of the two subsystems. It is simple to use since the only information required for its application to any given nonstructural component is the geometric characteristics, weights, and target ductilities of the nonstructural component and its supporting structure, in addition to the fundamental natural period of the structure and the response spectra specified for the design of the structure. A numerical example that illustrates the use of the method and a numerical comparative study that verifies its adequacy have also been presented. Based on its simplicity and rationality and the results from the comparative study, it is concluded that the proposed method represents a simple but effective procedure for the seismic design of nonstructural components in buildings.

The adequacy of the method, nevertheless, needs to be investigated further considering nonstructural components with different characteristics, mounted on different buildings, and subjected to different ground motions. The adequacy of the recommended strength reduction factors also deserves a careful examination. It has been assumed here that the strength reduction factors for single-degree-of-freedom systems are also a good approximation for multi-degree-of-freedom nonstructural components subjected to motions filtered by the dynamic characteristics of their supporting structures, but it is not known at this point how good this approximation is. In any case, it is important to keep in mind that the intended purpose of the proposed method is not to predict accurately the seismic response of nonstructural components, but, rather, to obtain in a straightforward manner conservative estimates of the forces they may be subjected to during a severe earthquake. It is through these estimates that a designer may assess how seriously a nonstructural component may be affected by an earthquake and decide whether or not a refined analysis is warranted.

References

1. Villaverde, R (1997). "Seismic Design of Secondary Structures: State of the Art", *Journal of Structural Engineering, ASCE*, **123**(8), 1011-1019.

2. Kawakatsu, T., Kitada, K., Takemory, T., Kuwabara, Y., and Okiwara, Y. (1979). "Floor Response Spectra Considering Elasto-Plastic Behavior of Nuclear Facilities," *Trans. 5th Int. Conf. Struc. Mech. Reactor Tech.*, Berlin, Fed. Rep. Germany, North-Holland Pub. Co., Amsterdam, K9/4.

3. Viti, G, Olivieri, M, and Travi, S. (1981). "Development of Nonlinear Floor Response Spectra", *Nuclear Engineering and Design*, **64** (1), 33-38.

4. Lin, J. and Mahin, S.A. (1985). "Seismic Response of Light Subsystems on Inelastic Structures," *Journal of Structural Engineering, ASCE*, **111**(2), 400-417.

5. Aziz, T. and Ghobarah, A. (1988). "Equipment Design: Future Directions," *Proceedings 9th World Conference on Earthquake Engineering*, Tokyo-Kyoto, Japan, **6**, 261-266.

6. Segal, D. and Hall, W.J. (1989). *Experimental Seismic Investigation of Appendages in Structures*, SRS Report No. 545, Department of Civil Engineering, University of Illinois at Urbana-Champaign.

7. Toro, G.R., McGuire, R.K., Cornell, C.A., and Sewell, R.T. (1989). "Linear and Nonlinear Response of Structures and Equipment to California and Eastern United States Earthquakes", EPRI Report NP-5566, Electric Power Research Institute, Palo Alto, Calif.

8. Sewell, R.T., Cornell, C.A., Toro, G.R., McGuire, R.K., Kassawara, R.P., Singh, A., and Stepp, J.C. (1989). "Factors Influencing Equipment Response in Linear and Nonlinear Structures," *Transactions 9th International Conference on Structural Mechanics and Reactor Technology*, Lausanne, Switz., K2, 849-856.

9. Igusa, T. (1990). "Response Characteristics of Inelastic 2-DOF Primary-Secondary System",

Journal of Engineering Mechanics, ASCE, **116**(5), 1160-1174.

10. Schroeder, M.E. and Backman, R.E. (1994). "Analytical Studies in Support of the 1994 NEHRP Provisions for Nonstructural Components", *5th U.S. National Conf. on Earthquake Engineering*, Chicago, Ill., **4**, 755-764.

11. Singh, M.P., Chang, T-S., and Suarez, L.E. (1996). "Floor Response Spectrum Amplification Due to Yielding of Supporting Structure", *Proceedings 11th World Conf. on Earthquake Engineering*, Acapulco, Mexico, Paper No. 1444.

12. Adam, C. and Fotiu, P.A. (2000). "Dynamic Analysis of Inelastic Primary-Secondary Systems", *Engineering Structures*, **22**, 58-71.

13. Adam, C. (2001). "Dynamics of Elastic-Plastic Shear Frames with Secondary Structures: Shake Table and Numerical Studies", *Earthquake Engineering and Structural Dynamics*, **30**(2), 257-277.

14. Villaverde, R. (1991). "Approximate Formulas to Calculate the Seismic Response of Light Attachments to Buildings", *Nuclear Engineering and Design*, **128**(3), 349-368.

15. Villaverde, R. and Newmark, N.M. (1980). *Seismic Response of Light Attachment to Buildings*, SRS Report No. 469, Department of Civil Engineering, University of Illinois at Urbana-Champaign.

16. Chopra, A.K. (1982). *"Dynamics of Structures: A Primer"*, Monograph MNO-2, Earthquake Engineering Research Institute, Oakland, Calif.

17. Newmark, N.M, and Hall, W.J. (1982). *"Earthquake Spectra and Design"*, Earthquake Engineering Research Institute, Oakland, Calif.

18. Miranda, E. (1993). "Site-Dependent Strength-Reduction Factors", *Journal of Structural Engineering, ASCE*, **119**(12), 3503-3519.

19. Villaverde, R. (1991). "Explanation for the Numerous Upper Floor Collapses During the 1985 Mexico City Earthquake", *Earthquake Engineering and Structural Dynamics*, **20**(3), 223-241.

20. Shakal, A.F. and Huang, M.J. (1995). "Recorded Ground and Structure Motions", Chapter 2, Northridge Earthquake Reconnaissance Report, **1**, *Earthquake Spectra*, Supplement C, **11**.

21. Hall, J.F., Technical editor (1994). Preliminary Reconnaissance Report, Northridge Earthquake, Earthquake Engineering Research Institute.