Technical Note

Multiple Tuned Mass Dampers for Response Control of Multi-Storey Space Frame Structure

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ABSTRACT

Dynamic response of a multi-storey symmetrical and asymmetrical space frame structures having six degrees of freedom (three translations along x, y, z-axes and three rotations about these axes) at each node, with multiple tuned mass dampers (MTMD) on its top is obtained. Each tuned mass damper (TMD) is modeled using a two-noded element having two translational degrees of freedom at each node. MTMD with uniformly distributed frequencies is considered for this purpose. The effectiveness of MTMD in suppressing the structural response is determined by comparing the response of corresponding structure without MTMD. It is found that the MTMD can be used effectively to suppress the acceleration, base shear, bending moment, translational and rotational responses of the symmetrical and asymmetrical structures. The effect of important parameters on the effectiveness of the MTMD is also studied. The parameters include the fundamental characteristics of the MTMD such as damping, mass ratio, total number of MTMD, tuning frequency ratio, frequency spacing of the dampers and structural damping. It is shown that these parameters have considerable influence on the effectiveness of the MTMD in reducing the dynamic response of the structure.

Keywords:
Vibration control; MTMD; Base-excitation; Six degrees of freedom at each node; Symmetrical and asymmetrical space frame structures

1. Introduction

In Vibration control of structures, the tuned mass damper (*TMD*) has been accepted as an effective passive control device to attenuate undesirable vibration of a structure [1-2]. The *TMD* consists of a mass, a spring and a viscous damper attached to the structure. The natural frequency of the damper is tuned to a frequency near to the natural frequency of the structure. The vibration of the structure causes the *TMD* to vibrate in resonance; as a result, the vibration energy is dissipated through the damping of the *TMD*. The main disadvantage of a single *TMD* is its sensitivity of the effectiveness to the error in the natural frequency of the structure and/or that in the damping ratio of the *TMD*. The effectiveness of a tuned mass damper is reduced significantly by mistuning or the off-optimum damping in *TMD*. As a result, the use of more than one tuned mass damper with different dynamic characteristics has been proposed in order to improve the effectiveness. Multiple tuned mass dampers with distributed natural frequencies were proposed by Xu and Igusa [3-4] and also studied by Yamaguchi and Harnpornchai [5], Jangid and Datta [6], Abe and Fujino [7], Abe and Igusa [8], Park and Reed [9], Chunxiang [10-11] and Sadek et al [12]. It was shown that *MTMD* is more effective for vibration control as compared to single *TMD*. Almost all of these studies considered the controlled structure as a single degree of freedom (*SDOF*) system with its fundamental modal properties to design the *TMD* and *MTMD*. However, a real building usually possesses a large number of degrees of freedom and is actually asymmetric to some degree even with a nominally symmetric plan. It will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Thus, the simplified *SDOF* system, which ignores the structural lateral-torsional coupling and *TMD* effect on different modes, could overestimate the control effectiveness of *TMD* [13]. Consequently, the controllers have to be designed through taking into account the effect of transverse-torsional coupled vibration modes in such cases. Examination of the *TMD* and *MTMD* for structures, which possess transverse-torsional coupled vibration modes, has already been recently performed by Jangid and dutta [13], Chunxiang and Weilan [14], Lin et al [15], Singh et al [16] and Pansare and Jangid [17].

Jangid and dutta [13], Pansare and Jangid [17] and Chunxiang and Weilan [14] have studied the response control of two degrees of freedom (one translation and one rotation) torsional systems by a cluster of *MTMD*. Lin et al [15] studied the response reduction of a multi-storey torsional building (with two translations and one rotation at each floor) system with one and two tuned mass dampers. Singh et al [16] studied the response control of a multi-storey tensional building (with two translations and one rotation at each floor) system with four tuned mass dampers, placed along two orthogonal directions in pairs.

The formulation of mathematical model of the structure is the most critical step in any seismic analysis, because how well the computed response agrees with the actual response of a structure during an earthquake depends primarily on the quality of the structural idealization. The quality of the structural idealization can be improved by more realistic idealizations of buildings that consider beam flexure, all translations along *x*, *y*, and *z* axes and all rotations about these axes. In the present paper, the effectiveness of *MTMD* in controlling the response of a symmetrical and asymmetrical multi-storey space frame structures having six degrees of freedom (three translations along *x*, *y*, and *z* axes and three rotations about these axes) at each node are investigated. The total degrees of freedom of the controlled structure (with *MTMD*) in this idealization are $(6 \times N) + (2 \times n)$, where N is the number of nodes and n is total number of *MTMD*. The objectives of the study are (i) to study the dynamic behaviour of a multi-storey symmetrical and asymmetrical space frame structures having six degrees of freedom at each node, with *MTMD*, (ii) to distinguish between the response characteristics of the structure with *MTMD* and single *TMD*, and (iii) to study the effect of important parameters on the effectiveness of *MTMD*.

2. Analysis

The structure is divided into a number of elements consisting of beams and columns. The beams and columns are modeled using two noded frame elements with six degrees of freedom at each node, i.e. three translations along *x*, *y*, and *z* axes and three rotations about these axes. For each element, the stiffness matrix, consistent mass matrix, and transformation matrix are obtained. The mass matrix and the stiffness matrix of each element from local direction are transformed to global direction as proposed by Paz [18]. The mass matrix and the stiffness matrix of each element are assembled by direct stiffness method to get the overall mass matrix, *M*, and overall stiffness matrix, *K*, for the entire structure. Knowing the overall mass matrix, *M*, and overall stiffness matrix, *K*, the frequencies for the structure are obtained using simultaneous iteration method. The damping matrix for structure is obtained using Rayleigh's equation, $C = \alpha M + \beta K$, where α and β are the constants. These constants can be determined easily if the damping ratio for each mode is known. The overall dynamic equation of equilibrium for the entire structure can be expressed in matrix notations as:

$$M\ddot{u} + C\dot{u} + Ku = f(t) \qquad (1)$$

where *M*, *C* and *K* are the overall mass, damping, and stiffness matrices of size $6N \times 6N$, where N is the number of nodes. $\ddot{u}, \dot{u}$ and u are the relative acceleration, velocity and displacement vectors with respect to ground and $f(t)$ is the nodal load vector. $u = u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}, ..., u_N, v_N, w_N, \theta_{xN}, \theta_{yN}, \theta_{zN}$.

The nodal load vector due to earthquake is obtained using the Eq. (2):

$$f(t) = -M I \ddot{u}_g(t) \qquad (2)$$

where M is the overall mass matrix, I is the influence vector of size $6N \times 1$, and $\ddot{u}_g(t)$ is the ground acceleration. The resulting equation of dynamic equilibrium is solved using Newmark method to obtain the displacements and acceleration at the nodes as explained in Chopra [19]. Owing to its unconditional stability, the constant average accel-eration scheme (with $\beta = 1/4$ and $\gamma = 1/2$) is adopted.

2.1. Modeling of Multiple Tuned Mass Dampers

Each tuned mass damper (*TMD*) is modeled using a two-noded element with two translational degrees of freedom (*x* and *z* direction) at each node. The natural frequencies of the *MTMD* are uniformly distributed around their average natural frequency. The natural frequency ω_j (i.e. $\omega_j = \sqrt{k_j / m_j}$) of the j^{th} *TMD* is expressed as:

$$\omega_j = \omega_T \left[1 + \left(j - \frac{n+1}{2} \right) \frac{\beta}{n-1} \right] \tag{3}$$

and

$$\omega_T = \sum_{j=1}^{n} \omega_j / n \tag{4}$$

$$\beta = \frac{\omega_n - \omega_1}{\omega_T} \tag{5}$$

where n is the total number of *MTMD*, ω_T is the average frequency of all the *MTMD*, and β is the frequency range parameter of the *MTMD*.

As suggested by Xu and Igusa [3], the manufac-turing of *MTMD* with uniform stiffness is simpler than those with varying stiffness. In this study, the distribution of natural frequencies of the *MTMD* is achieved by keeping the stiffness constant (i.e., with $k_1 = k_2 = ...k_n = k_T$), but allowing the mass of each *TMD* to vary.

The mass and the damping constant of the j^{th} *TMD* are expressed as:

$$m_j = \frac{k_T}{\omega_j^2} \tag{6}$$

$$c_j = 2 m_j \zeta_T \omega_j \tag{7}$$

where ζ_T is the damping ratio which is kept con-stant for all the *MTMD*.

The ratio of total mass of *MTMD* to the total mass of the structure is defined as the mass ratio, i.e.

$$\mu = \frac{\sum_{j=1}^{n} m_j}{m_s} = \frac{m_T}{m_s} \tag{8}$$

The constant stiffness required for each *TMD* can be evaluated as:

$$k_T = \frac{\mu\, m_s}{\sum_{j=1}^{n} 1/\omega_j^2} \tag{9}$$

The average frequency of *MTMD* corresponds only to the lateral mode of vibration. In case of an asymmetrical building, the translations in *x* and *z* directions have different dominant modes. Keeping them in view, two different tuning frequency ratios are considered in the study, namely;

$$f_1 = \frac{\omega_T}{\omega_{s1}} \quad \text{and} \quad f_2 = \frac{\omega_T}{\omega_{s2}} \tag{10}$$

where ω_{s1} and ω_{s2} are the natural frequency of lateral vibration of the structure corresponding to the dominant mode in *x* and *z* direction respectively.

The natural frequency, stiffness, damping and mass parameters of the dampers in *x*-direction are denoted by $\omega_{jx}, k_{Tx}, c_{jx}$, and m_{jx}. Similar parameters for the dampers along the *z*-direction are denoted by $\omega_{jz}, k_{Tz}, c_{jz}$, and m_{jz}. It is to be noted that the stiffness and damping parameters of the j^{th} *TMD* in *x* and *z* directions are different whereas mass parameter of the j^{th} *TMD* in *x* and *z* directions are the same $(m_{jx} = m_{jz} = m_j)$.

The stiffness, damping and mass matrices of each *TMD* is expressed as:

$$k_{TMD} = \begin{pmatrix} k_{Tx} & 0 & -k_{Tx} & 0 \\ 0 & k_{Tz} & 0 & -k_{Tz} \\ -k_{Tx} & 0 & k_{Tx} & 0 \\ 0 & -k_{Tz} & 0 & k_{Tz} \end{pmatrix} \tag{11}$$

$$c_{TMD} = \begin{pmatrix} c_{jx} & 0 & -c_{jx} & 0 \\ 0 & c_{jz} & 0 & -c_{jz} \\ -c_{jx} & 0 & c_{jx} & 0 \\ 0 & -c_{jz} & 0 & c_{jz} \end{pmatrix} \tag{12}$$

$$m_{tmd} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_j & 0 \\ 0 & 0 & 0 & m_j \end{pmatrix} \tag{13}$$

In case of a symmetrical structure, the transla-tions in *x* and *z* directions have identical dominant modes $(\omega_{s1} = \omega_{s2})$. Consequently, the natural fre-quency, stiffness, damping and mass parameters of j^{th} *TMD* in *x* and *z* directions are the same $(\omega_{jx} = \omega_{jz} = \omega_j; k_{Tx} = k_{Tz} = k_T; = c_{jx} = c_{jz} = c_j; m_{jx} = m_{jz} = m_j)$.

The stiffness, damping and mass matrices of each *TMD* are added to the overall stiffness matrix, overall damping matrix and overall mass matrix of the structure at corresponding global degrees of freedom.

2.2. Determination of Member Forces

The displacement obtained at each node is assigned to each member. The forces in each member are then obtained by multiplying the element stiffness matrix with the nodal displacement vector.

3. Dynamic Response of a Multi-Storey Symmetrical Space Frame Structure with and without MTMD

Figure (1a) shows a symmetrical four-storey space frame structure with *MTMD* on its top. Since the uncontrolled structure (without *MTMD*) is symmetrical, the natural frequency of lateral vibration of the structure corresponding to the dominant mode in *x* and *z* directions are identical, i.e., $\omega_{s1} = \omega_{s2} = \omega_s$ = 8.376 *rad/sec*. The *MTMD* with uniformly distributed frequencies are placed on the top of four-storey space frame structure as shown in Figure (1b). Since the mass parameter of each *TMD* is not the same, the controlled (with *MTMD*) structure is an asymmetrical one. The structure is subjected to bi-directional (*x* and *z* directions) harmonic ground excitation equal to $a_0 \sin(\omega t)$ (where a_0 is equal to 20% of acceleration due to gravity and ω is the excitation frequency). The horizontal displacements, rotations, absolute acceleration at nodes and the base shear, bending moment in the member are computed. The damping ratio of structure is taken as 2% of critical for all modes, damping ratio of *MTMD* is taken as 1% of critical, mass ratio is taken as 1%, the total number of *MTMD* is taken as 10, the frequency range parameter of the *MTMD* is taken as 0.2, and tuning frequency ratio ($f = f_1 = f_2$) is taken as unity. Figure (2a) shows the variation of maximum horizontal displacement and maximum rotations against the frequency ratio (ω/ω_s) for a structure with and without *MTMD*. The response of the uncontrolled structure is sharply peaked, and the peak is centered around the fundamental natural frequency of the uncontrolled structure. This peak is due to resonating effect. Since the uncontrolled structure is symmetrical, the response at node 1 and node 2 are identical. Further, it can also be seen from the Figure (2a) that there is a significant reduction in the peak value of the horizontal displacements and rotations of the structure due to *MTMD*. Hence the *MTMD* can be used effectively to suppress the translational and rotational responses of the structure. It is also found that, in the frequency range, $0.8 > \omega/\omega > 1.3$, the response curves of structure with and without *MTMD* are almost the same; this indicates that *MTMD* are effective only near the fundamental natural frequency of structure. Thus, the effectiveness of *MTMD* is dependent on the frequency characteristics of ground motion.

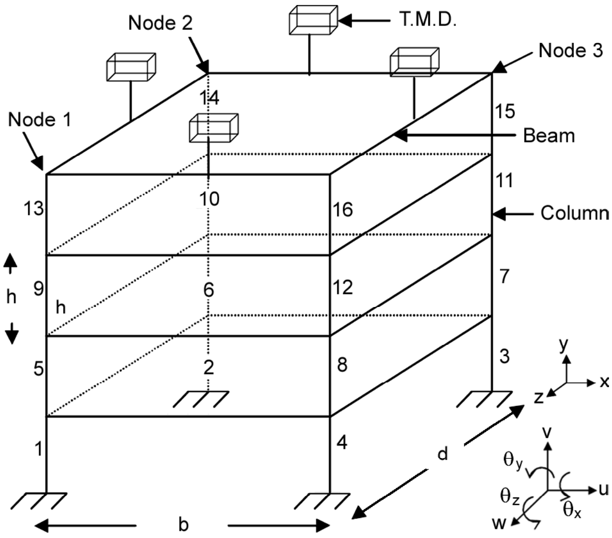

Figure 1a. Four-storey space frame structure with MTMD on its top.

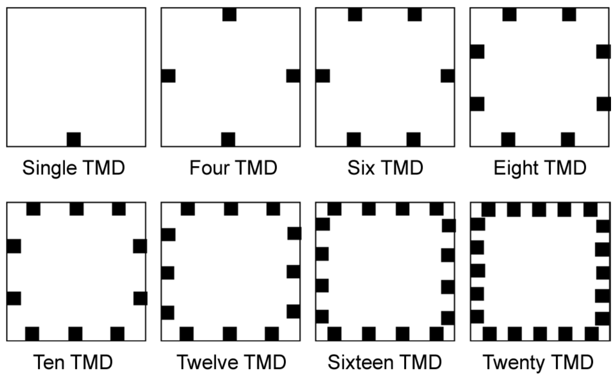

Figure 1b. The placement of MTMD on the top of the structure.

Figure (2b) shows the variation of maximum absolute acceleration, maximum base shear and maximum bending moment for the member along member *y*-axis (M_y) and member *z*-axis (M_z) against the frequency ratio. It can be observed from Figure (2b) that there is a significant reduction in the peak value of the acceleration, base shear and bending moment of the structure due to *MTMD*.

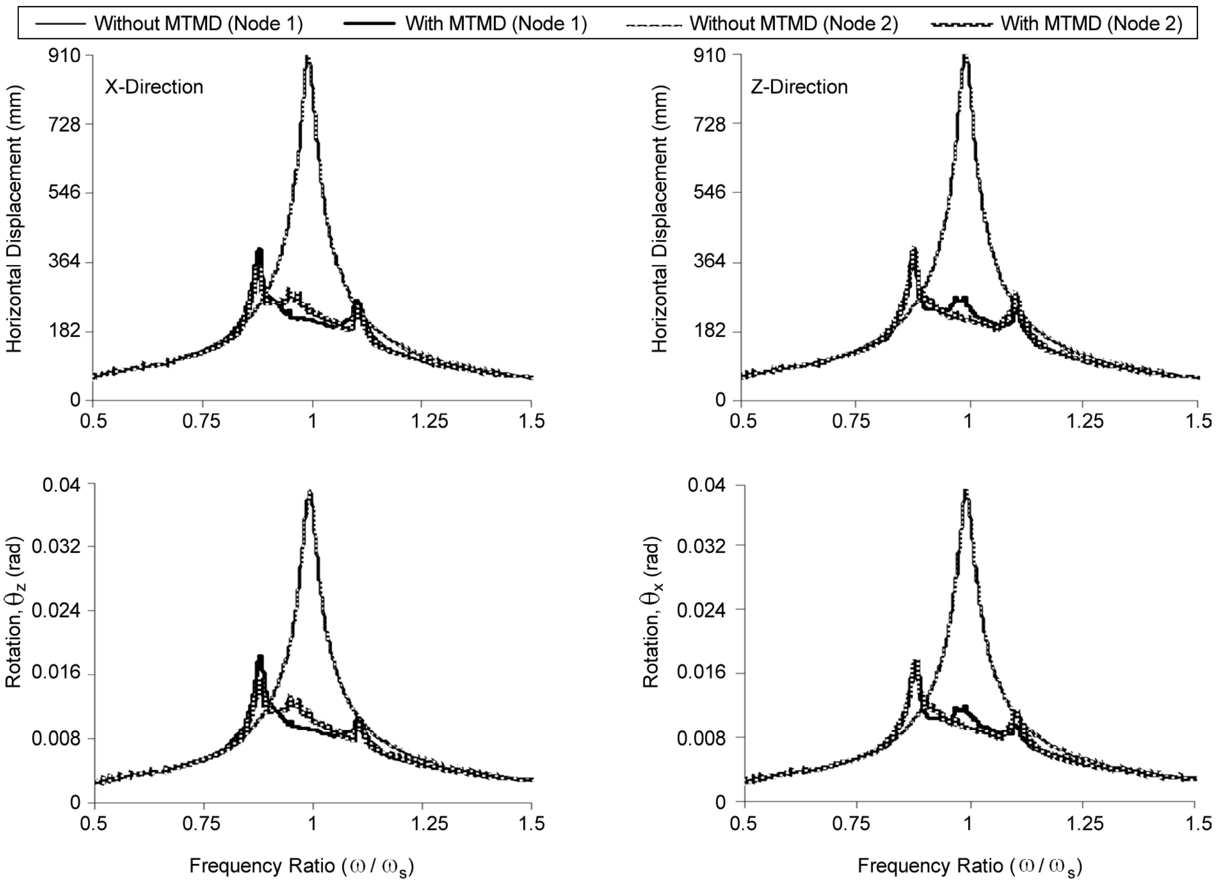

Figure 2a. Variation of translational and rotational response of the structure against frequency ratio.

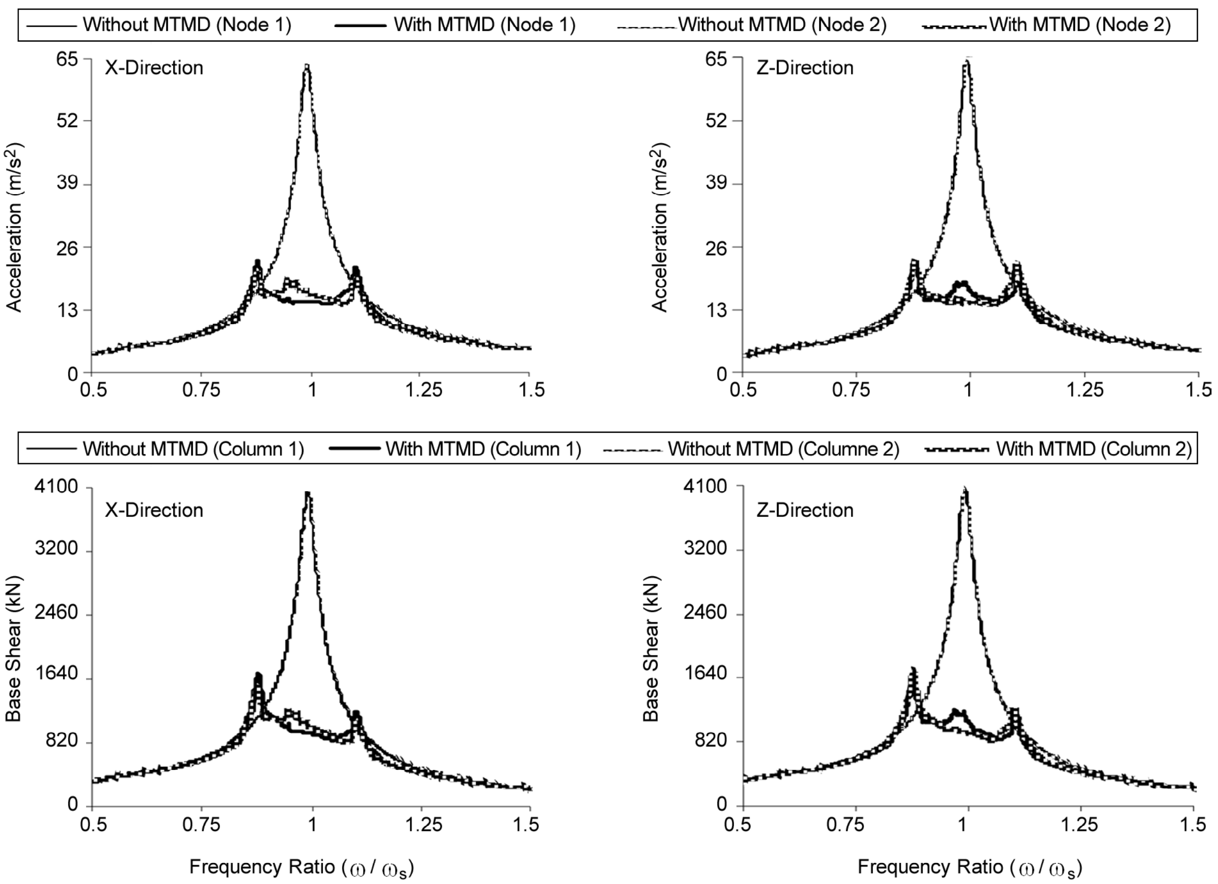

Figure 2b. Variation of acceleration, base shear and bending moment of structure against frequency ratio.

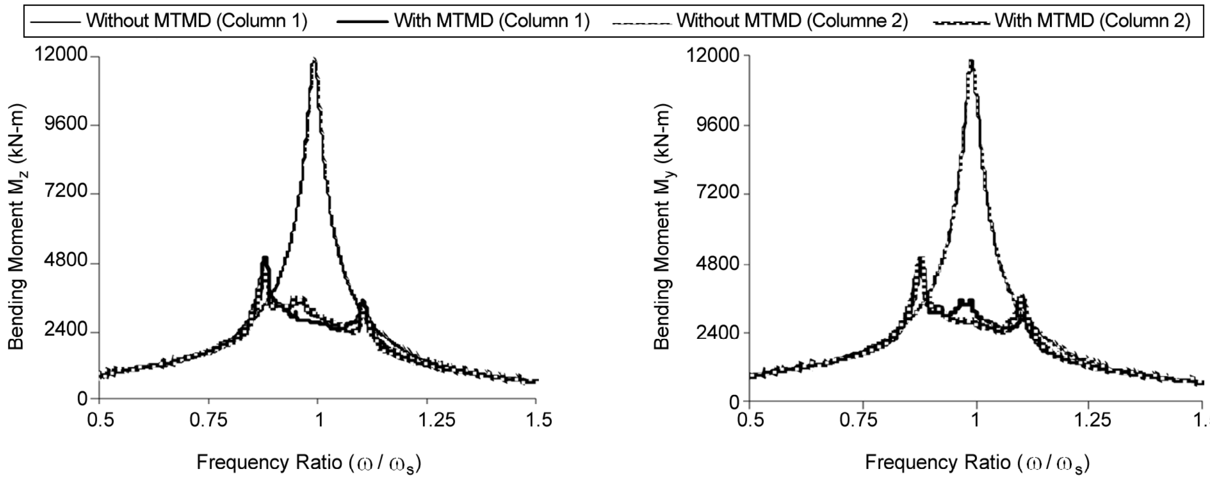

Figure 2b. (Continue)

Hence the *MTMD* can be used effectively to control the acceleration, base shear and bending moment of a structure. This also indicates that the increase in mass due to *MTMD* will not affect the stability of the structure.

4. Parametric Study

The effectiveness of *MTMD* in suppressing the dynamic response of a four-storey symmetrical space frame structure shown in Figure (1a) is analyzed under the important parametric variations. The parameters include: total number of *MTMD* (n), damping ratio of *MTMD* (ζ_T), tuning frequency ratio ($f = f_1 = f_2$), frequency range parameter of *MTMD* (β) and structural damping (ζ_S). Mass ratio is taken as 1% [5]. The structure is subjected to harmonic ground excitation (in x-direction) equal to $a_0 sin(\omega t)$ (where a_0 is equal to 20% of acceleration due to gravity and ω is the excitation frequency).

The response quantity of interest is the *RMS* horizontal displacement of structure. The area under the maximum horizontal displacement curve yields the mean square response (*RMS*). In order to study the effectiveness of *MTMD*, it is convenient to express the response in terms of the ratio rather than plotting their values. For this purpose, the response ratio is defined as:

$$Response\ ratio = \frac{RMS\ horizontal\ displacement\ of\ structure\ with\ MTMD}{RMS\ horizontal\ displacement\ of\ corresponding\ structure\ without\ MTMD}$$

The response ratio is an index of the performance of *MTMD*. The ratio being less than unity implies that the *RMS* horizontal displacement of the structure with *MTMD* has been reduced in comparison to the response without *MTMD*.

4.1. Effect of Total Number of MTMD

Figure (3) shows the variation of response ratio against the total number of *MTMD* for β = 0.1, 0.2, 0.25, 0.3 and 0.4. The damping ratio of the structure is taken as 2% of critical. The damping ratio of each *TMD* is taken as 1% and the tuning frequency ratio is in unity for all cases. The response ratio for the structure studied is less than unity, indicates that the single *TMD* and *MTMD* are effective in reducing the response of the structure. As the total number of *MTMD* increases the effectiveness of *MTMD* in suppressing dynamic response of the structure increases. Further, it can also be seen from the Figure (3) that the *MTMD* is more effective than single *TMD*. However, by increase in the number of *TMD* beyond a certain value (n = 10), the effectiveness of *MTMD* remains almost invariant.

4.2. Effect of Damping Ratio of MTMD

In Figure (4), the variation of response ratio is plotted against the damping ratio of *MTMD* for $n = 1, 4, 8$ and 12. The frequency ratio (f) and frequency range parameter (β) of the *MTMD* are taken as 1 and 0.2, respectively. The damping ratio of the structure is taken as 2% of critical. At lower damping ($\zeta_T < 0.07$), *MTMD* is more effective in controlling the response than a single *TMD*. Further, the *MTMD* is significantly less sensitive than the single *TMD* at low value of damping ratio. However, at higher damping, the response ratio increases with the increase in damping and at node 1, the response ratio is nearly the same for single *TMD* and *MTMD* whereas at node 2, the response ratio for *MTMD* is more than that of single *TMD*. This implies that at higher damping, the relative advantage of *MTMD* (compared to single *TMD*) decreases. The optimum value of damping in *MTMD* is sufficiently lower than that for a single *TMD*.

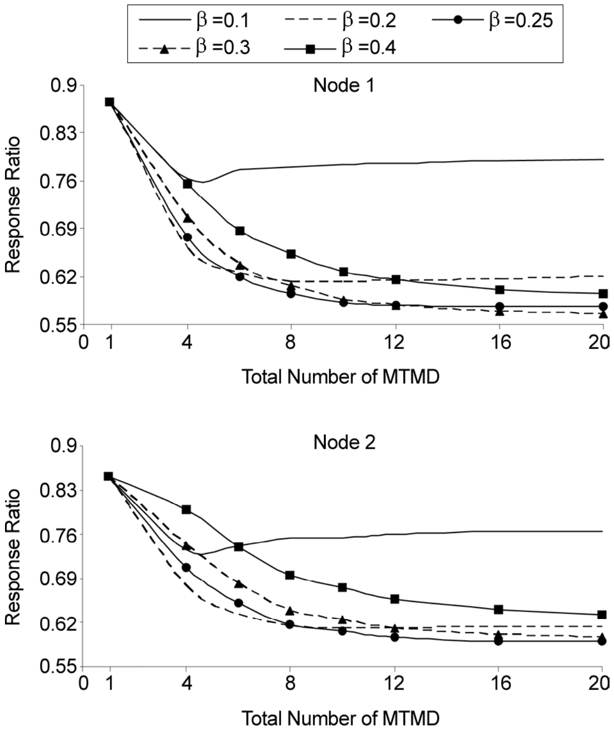

Figure 3. Variation of response ratio against total number of MTMD.

4.3. Effect of Frequency Range Parameter

In Figure (5), the variation of response ratio is plotted against the frequency range parameter (β) of the *MTMD* for damping ratio of *MTMD* equal to 1%, 2% and 5%. The damping ratio of the structure is taken as 2% of critical. The total number of *MTMD* is taken as 8 and the tuning frequency ratio is equal to unity. Figure (5) shows that the frequency range parameter significantly influences the effectiveness of *MTMD*. There exists an optimum value of frequency range parameter which provides maximum effectiveness of *MTMD* for a given damping ratio.

4.4. Effect of Tuning Frequency Ratio

Figure (6), shows the variation of response ratio with the tuning frequency ratio (*f*) for total number of *MTMD* equal to 1, 4, 8 and 12. The damping ratio of the structure is taken as 2% of critical. The frequency range parameter of the *MTMD* is taken as 0.2. The damping ratio of *MTMD* is taken as 1%. Figure (6) shows that there exists an optimum value of tuning frequency ratio at which the response

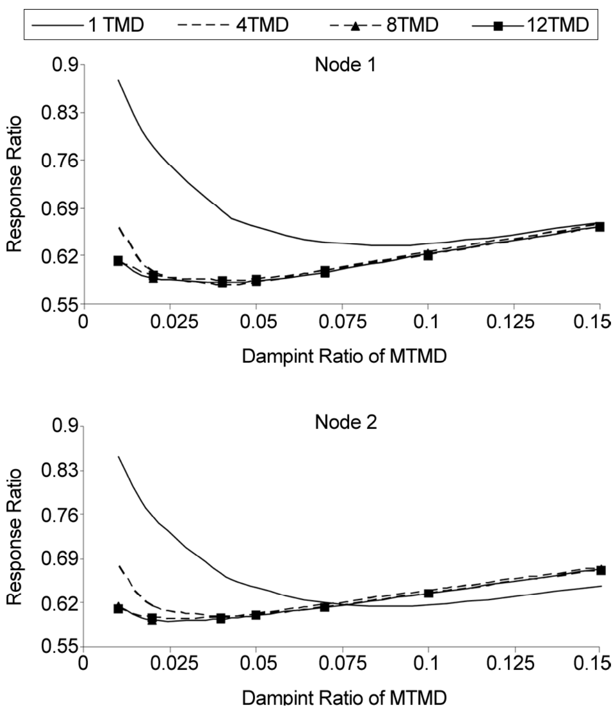

Figure 4. Variation of response ratio against damping ratio of MTMD.

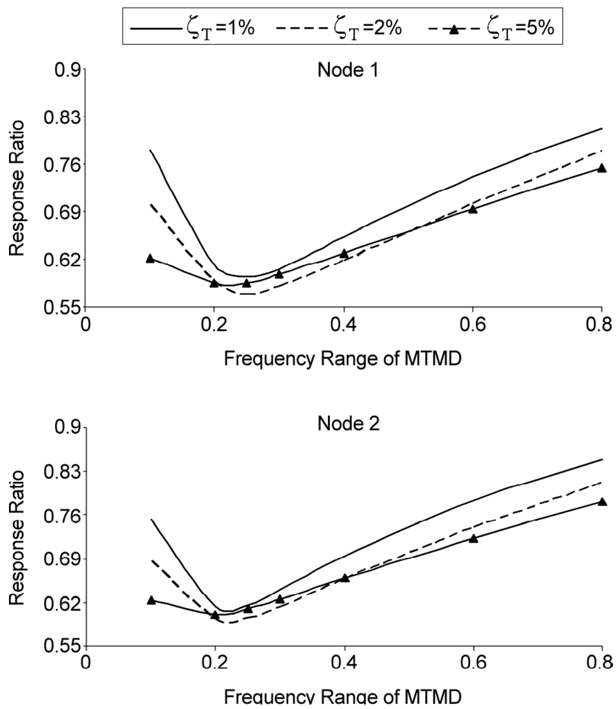

Figure 5. Variation of response ratio against frequency range of MTMD.

of the structure becomes minimum for both single *TMD* and *MTMD*. The optimum value of the tuning frequency ratio occurs in the vicinity of unity.

4.5. Effect of Structural Damping

In Figure (7), the variation of response ratio is plotted against the tuning frequency ratio (f) for damping ratio of structure equal to 2%, 3% and 5%. The total number of *MTMD* is taken as 8, the frequency range parameter of the *MTMD* is taken as 0.2 and the damping ratio of *MTMD* is taken as 1%. The response ratio increases with an increase in structural damping. This indicates that an increase in the damping ratio of the structure decreases the effectiveness of *MTMD*.

5. Dynamic Response of a Multi-Storey Asymmetrical Space Frame Structure with and without MTMD

The response of a multi-storey asymmetrical space frame structure with and without *MTMD* subjected to bi-directional (x and z directions) harmonic ground excitation equal to $a_0 sin(\omega t)$ (where a_0 is equal to 20% of acceleration due to gravity and ω is the excitation frequency) and seismic ground motions are studied. The two earthquake ground motions considered for the study are Mexico and El Centro earthquakes. The material and geometric properties of the uncontrolled asymmetrical four storey space frame structure considered for the study are shown in Table (1). The natural frequency of lateral vibration of the structure corresponding to the dominant mode in x and z directions are ω_{s1}= 8.376*rad/sec* and ω_{s2} = 8.737*rad/sec* respectively. The damping ratio of structure is taken as 2% of critical for all modes, damping ratio of *MTMD* is taken as 1% of critical, mass ratio is taken as 1%, the total number of *MTMD* is taken as 10, the frequency range parameter of the *MTMD* is taken as 0.2 and tuning frequency ratio (f= f_1 = f_2) is taken as unity. Figure (8a) and (8b) shows the variation of

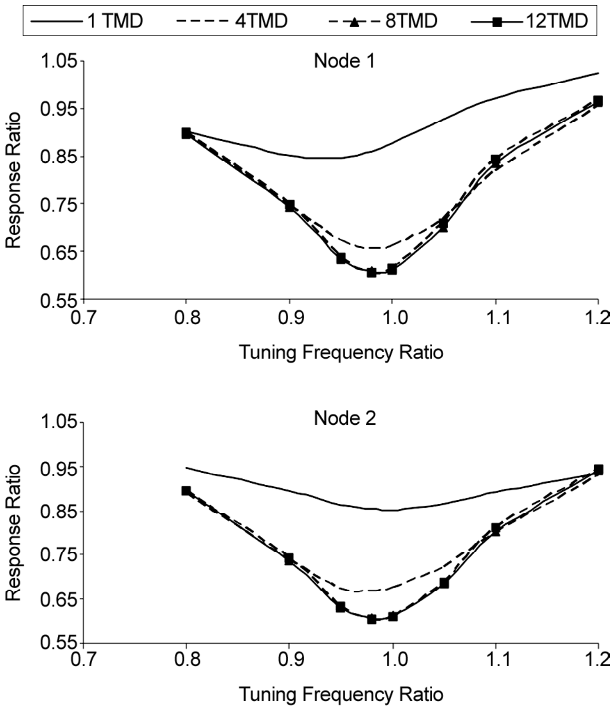

Figure 6. Variation of response ratio against tuning frequency ratio for total number of MTMD.

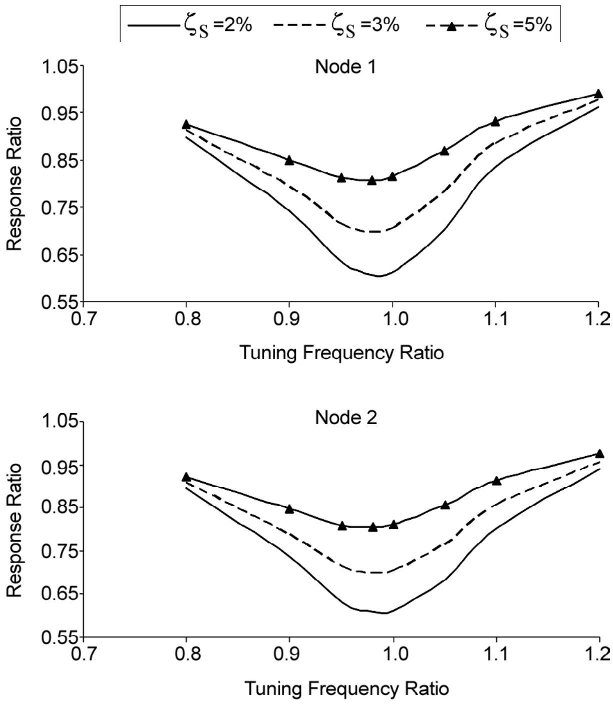

Figure 7. Variation of response ratio against tuning frequency ratio for various values of structural damping.

Table 1. Material and geometric properties of asymmetrical four-storey space frame structure.

Time Period of the Structure (sec)	Mass on Each Beam (kN-sec^2/m^2)	b (m)	d (m)	H (m)	Size of Beam (m)	Size of Column 1, 5, 9, 13, 4, 8, 12, 16 (m)	Size of Column 2, 6, 10, 14, 3, 7, 11, 15 (m)	Modulus of Elasticity (kN/m^2)
0.75	2.9	6	6	3.3	0.3 × 0.6	0.4 × 0.4	0.6 × 0.6	2.24 × 10^7

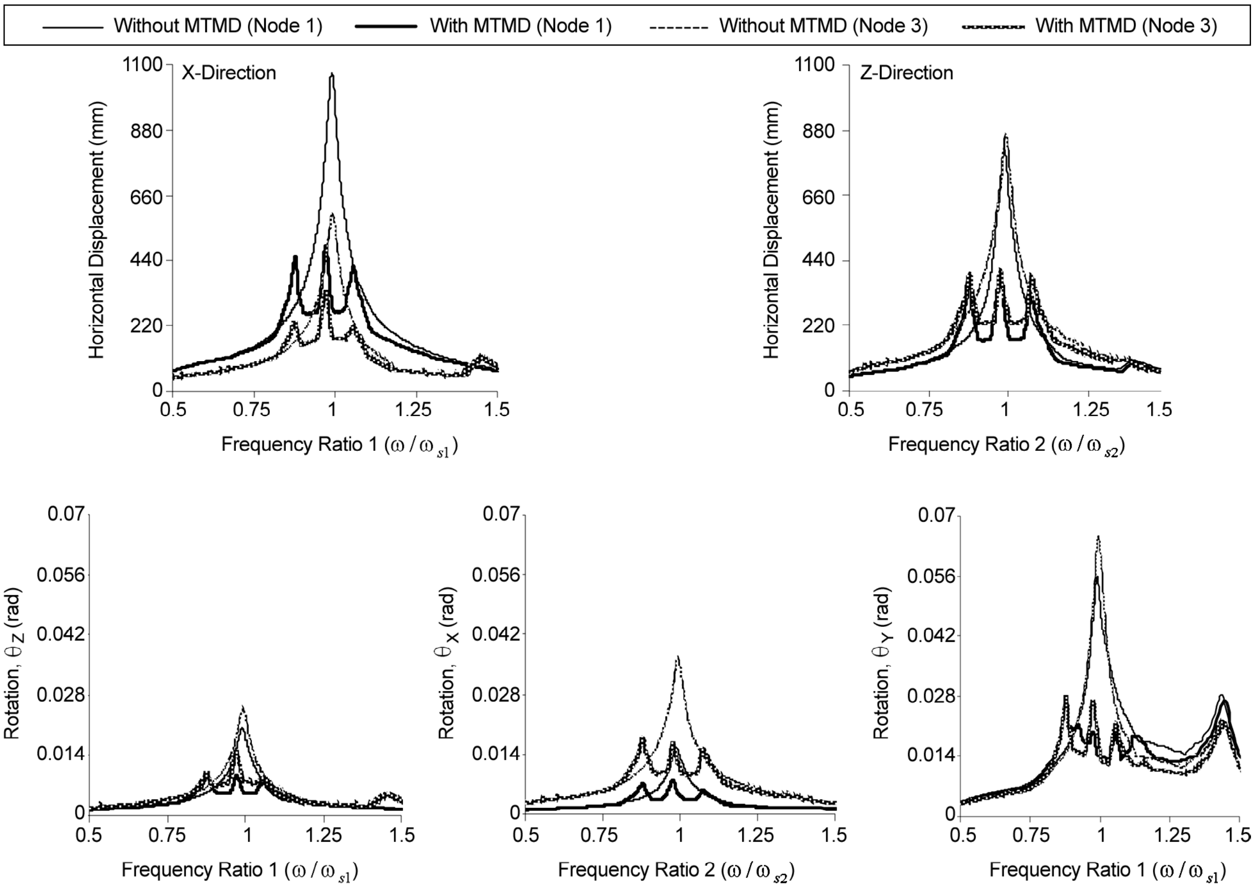

Figure 8a. Variation of translational and rotational responses of asymmetrical structure against frequency ratio.

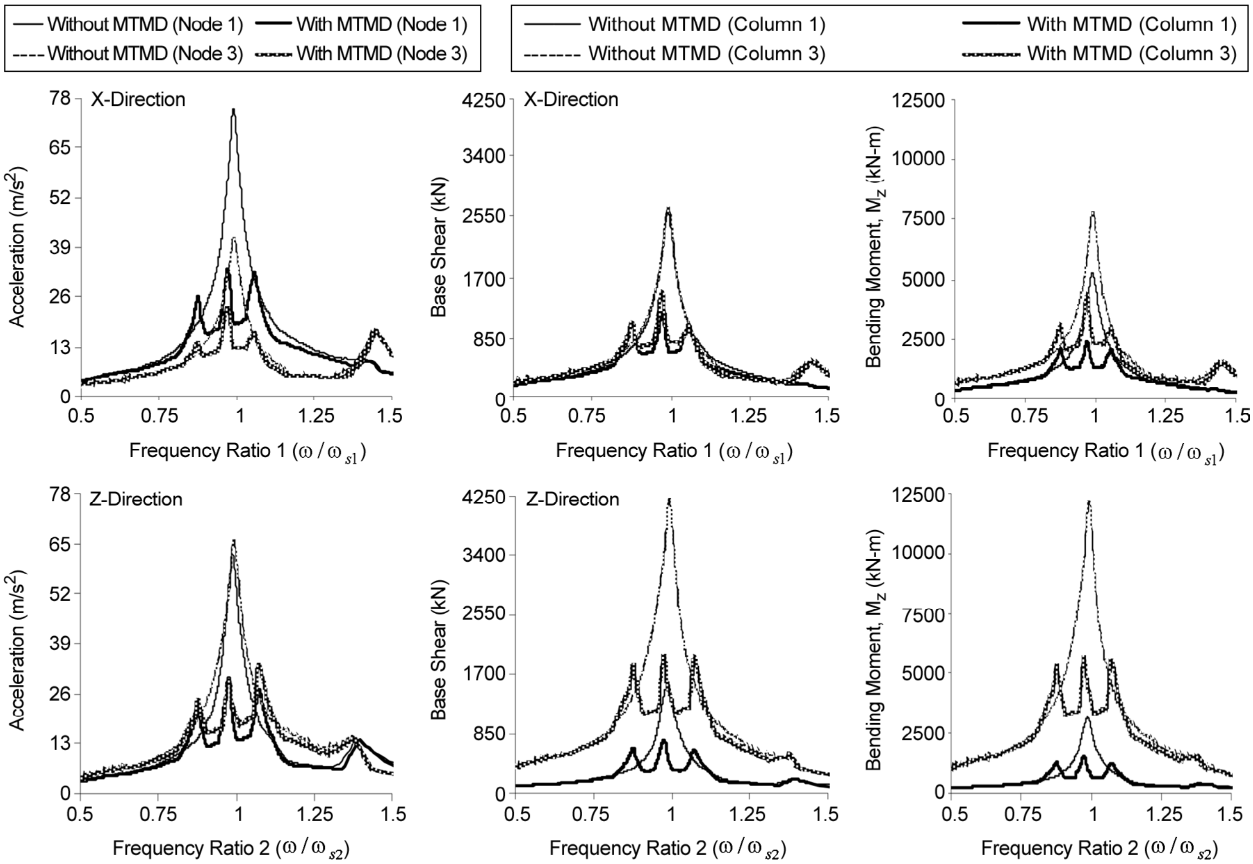

Figure 8b. Variation of acceleration, base shear and bending moment of asymmetrical structure against frequency ratio.

maximum response against the frequency ratio 1 (ω/ω_{s1}) and frequency ratio 2 (ω/ω_{s2}) for a structure with and without *MTMD* subjected to harmonic ground excitation. The response of structure with *MTMD* is found to be less in comparison to the corresponding response without *MTMD*, implying that the *MTMD* can be used effectively to suppress the acceleration, base shear, bending moment, translational and rotational responses of the structure.

The ground acceleration records used for the numerical simulations are:

i) Mexico earthquake (Galeta de campos station, 1985),
ii) El Centro earthquake (Imperial Valley, 1940) and
iii) Northridge earthquake (Newhall, 1994).

The peak ground accelerations (*PGA*) of these earthquakes are 1.4068 m/s^2, 3.13 m/s^2, and 5.72 m/s^2 respectively. Figure (9) shows the variation of horizontal displacement at node 1 (*x*-dir.), rotation at node 1 (θ_Y), base shear in column 1 (*x*-dir.) and bending moment in column 1 (M_Z) against the time for a structure with and without *MTMD* subjected to Mexico earthquake ground motion. The peak response values of a structure subjected to Mexico, El Centro and Northridge earthquake ground motions are shown in Table (2). The response of structure with *MTMD* is found to be less in comparison to the corresponding response without *MTMD*, implying that the *MTMD* can be used effectively to suppress the seismic response of a structure.

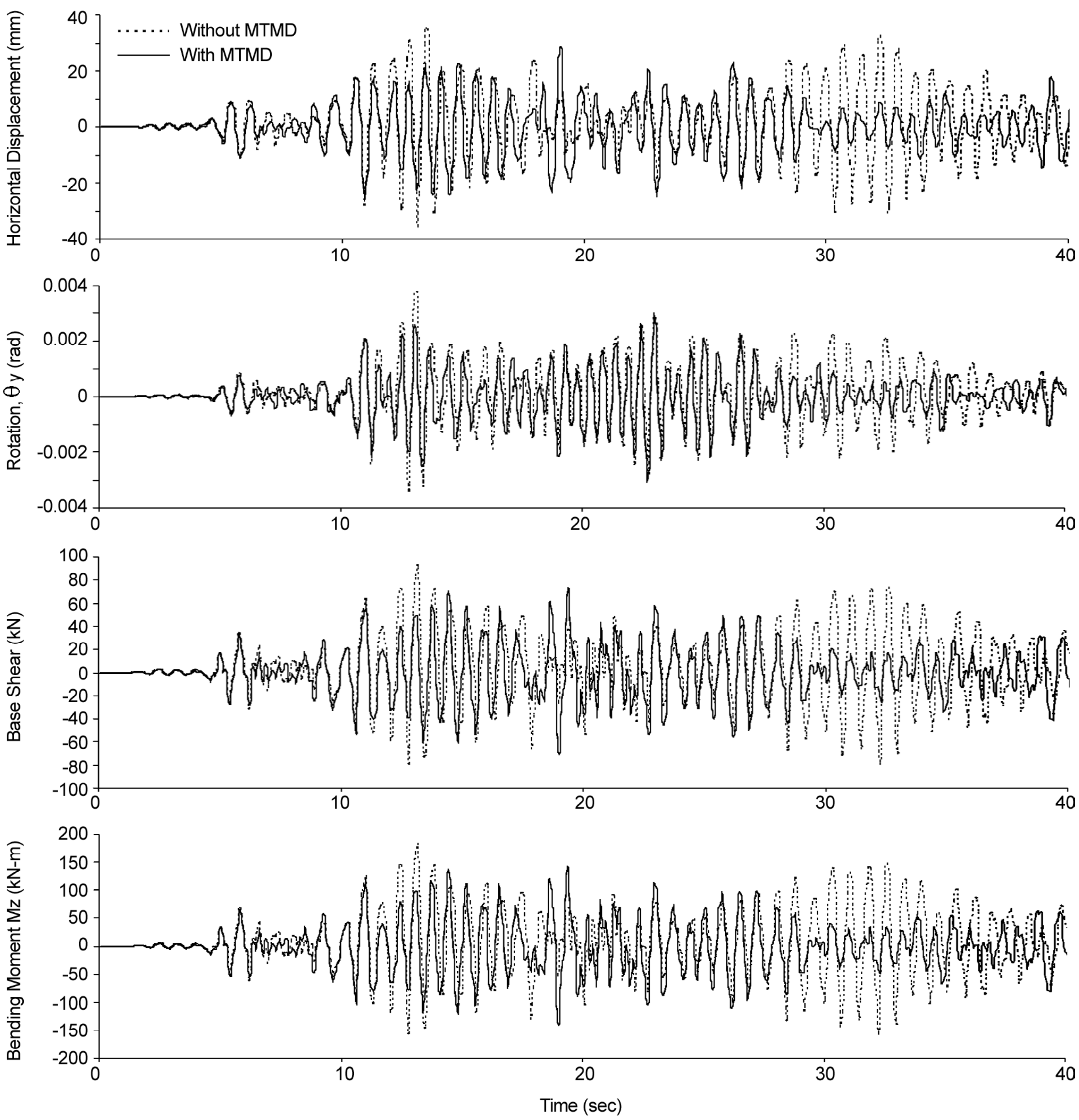

Figure 9. Response of asymmetrical structure subjected to Mexico earthquake ground motion.

Table 2. Peak response values of an asymmetrical structure subjected to real Earthquake ground motion.

			Mexico Earthquake		El Centro Earthquake		Northridge Earthquake	
			Without MTMD	With MTMD	Without MTMD	With MTMD	Without MTMD	With MTMD
Horizontal Displacement (mm)	Node 1	x -Direction	35.853	28.628	129.8603	118.66	197.436	169.647
	Node 3		20.106	16.9384	63.6014	58.75	125.594	103.03
	Node 1	z-Direction	33.5133	22.1092	92.966	81.159	134.476	108.028
	Node 3		41.189	27.723	118.95	108.207	180.33	156.78
Rotation (rad)	Node 1	θ_X	0.000744	0.000483	0.001773	0.00164	0.00297	0.00262
		θ_Y	0.003799	0.003067	0.011173	0.01027	0.01811	0.0171
		θ_Z	0.000778	0.000683	0.00255	0.00233	0.00521	0.00484
	Node 3	θ_X	0.00195	0.001283	0.005176	0.004721	0.00842	0.00685
		θ_Y	0.003097	0.00268	0.009344	0.007711	0.01658	0.0148
		θ_Z	0.000956	0.000759	0.002817	0.00261	0.00574	0.0051
Acceleration (m/s^2)	Node 1	x-Direction	3.422	2.8102	11.595	10.472	23.563	20.189
	Node 3		2.036	1.784	6.60196	5.997	10.94	9.606
	Node 1	z-Direction	3.3506	2.5231	8.266	7.82	15.383	13.99
	Node 3		4.083	3.193	11.814	11.028	16.749	15.419
Base shear (kN)	Column 1	x-Direction	93.441	75.5384	324.374	288.609	512.503	473.1
	Column 3		94.935	88.554	313.352	290.13	554.688	474.807
	Column 1	z-Direction	68.385	45.974	179.99	161.06	273.413	238.73
	Column 3		172.929	119.258	450.965	405.99	694.86	625.07
Bending Moment (kN-m)	Column 1	M_Z	184.1524	141.978	643.681	589.98	1007.15	926.6
	Column 3		272.132	242.7597	885.042	823.1	1572.9	1344.3
	Column 1	M_Y	135.991	91.045	359.197	321.126	544.45	470.44
	Column 3		495.671	334.45	1306.43	1170.172	1988.6	1730.7

6. Conclusions

The performance of *MTMD* for controlling the dynamic response of a multi-storey symmetrical and asymmetrical space frame structures having six degrees of freedom at each node is investigated. The responses of the structure with *MTMD* are compared with those of the same structure without *MTMD*. The effect of important parameters on the effectiveness of the *MTMD* is also studied. The results of the study lead to the following conclusions:

- *MTMD* can be used effectively to suppress the acceleration, base shear, bending moment, translational and rotational responses of the symmetrical and asymmetrical structure.
- As the total number of *MTMD* increases the effectiveness of *MTMD* in suppressing dynamic response of the structure increases. However, with an increase in the number of *TMD* beyond a certain value ($n = 10$), the effectiveness of *MTMD* remains almost invariant.
- The optimum value of damping in *MTMD* is sufficiently lower than that for a single *TMD*.
- There exists an optimum value of frequency range parameter which provides maximum effectiveness of *MTMD* for a given damping ratio.
- The optimum value of the tuning frequency ratio occurs in the vicinity of unity.
- Increase in the damping ratio of the structure decreases the *MTMD* effectiveness.
- The effectiveness of *MTMD* is dependent on the frequency characteristics of ground motion.

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