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Research Note

Bayesian and Frequentist Approaches for the Estimation of the Maximum Expected Earthquake Magnitude in Iran

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ABSTRACT

The maximum earthquake magnitude plays a crucial role in different aspects of seismic hazard and risk assessments. Previous work by Salamat et al. [1] shows the divergence of the confidence interval of the maximum possible earthquake magnitude M_{max} for high levels of confidence 1-α, in different seismotectonic zones of Iran. For this, M_{max} is replaced by the maximum expected earthquake magnitude μ_t that is calculated for different predefined future time intervals T_f. In this work, the frequentist and Bayesian approaches are applied to calculate the upper bound of the confidence interval of μ_t. The frequentist confidence intervals are calculated for the level of confidence 1-α = 95% and 99%, and future time intervals T_f= 30, 50 years. In the Bayesian approach, the posterior distributions of the maximum expected earthquake magnitude are calculated for T_f= 30, 50 years and 90% confidence level. The stationary Poisson process in time and Gutenberg Richter relation are assumed as a statistical model for the magnitude distribution. In order to estimate μ_t in each seismotectonic zone, three different scenarios of M_{max}= 8.5, 9.0, 9.5 are assumed. In order to find the influence of the declustering, all calculations are applied for both original and declustered catalogs. The results show, as long as the length of the time interval is short or moderate, different values of M_{max} have a minor effect on the estimation of the maximum expected earthquake magnitude μ_t.

Keywords:
Pseudo-dynamic approach; Inclined retaining wall; Seismic active earth pressure; Inclined cohesive soil backfill; Soil amplification factor

1. Introduction

Iranian plateau as a part of Alpine Himalayan belt has a high density of active and quaternary faults, which experienced lots of destructive earthquakes [1]. It is located between the Arabian plate in southwest and the Eurasian plate in northeast, which are controlled the tectonic states of the Iranian plateau. The 856 Qumes earthquake (M_w 7.8), the 951 Taleghan earthquake (M_w 7.7), the 1721 Tabriz earthquake (M_w 7.7), the 1945 Balochistan earthquake (M_w 8.1), the 1962 Buin Zahra earthquake (M_w 7.2), the 1968 Dashte-Bayaz (M_w 7.2), the 1976 Muradiye, the 1978 Tabas and the 1909 Silakhor earthquakes (M_w 7.4), the 1990 Manjil and the 1997 Ardekul Qayen earthquakes (M_w 7.3), the 2013 Saravan earthquake (M_w 7.7) are some examples of earthquakes with fatal casualties in the history of the Iranian catalog. Occurrences of many devastating historic and recent earthquakes in different seismotectonic zones of Iran indicate the importance of the seismic hazard and risk analysis in this country. Considering the essential role of the precise estimation of M_{max} in the seismic hazard analysis in

Iran, only a few studies with insufficient accuracy are available. Different point estimators [3-4] for $M_{\max}$ must be considered as useless, because an accurate quantification of uncertainties is not possible in these methods [5].

In the previous study [1], the confidence interval of the maximum possible earthquake magnitude $M_{\max}$ are calculated for different levels of confidence in each seismotectonic zone of Iran. In this work, the method of Holschneider et al. [6] is applied to estimate the maximum possible earthquake magnitude using mostly accepted physical model and instrumental part of the earthquake catalog. Using the doubly truncated Gutenberg-Richter distribution for the magnitude distribution [7], the frequentist confidence interval of $M_{\max}$ are calculated based on the confidence level (1-α). The confidence interval is from $[\mu,\psi(\mu)]$, which μ is the maximum observed magnitude and $\psi(\mu)$ is an upper bound of the confidence interval calculated for different levels of confidence. Based on Holschneider et al. [6], Pisarenko [8] and Pisarenko et al. [9], the maximum earthquake magnitude $M_{\max}$ used in seismic hazard assessment is a questionable quantity, since the limited data from the earthquake catalog cannot use for the estimation of the maximum possible magnitude for all times [3] and causes the divergence of the confidence interval. Zoller et al. [10] suggest using the maximum expected magnitude μ_t in a finite future time interval T_f, and Pisarenko et al. [11] show the same finding in the context of extreme value theory. Because the Bayesian approach delivers a probability distribution, the calculation of the confidence interval becomes straightforward [10]. This study use the frequentist and Bayesian approaches to estimate the maximum expected earthquake magnitude μ_t in each seismotectonic zones of Iran. For this, Iran is subdivided into six seismotectonic regions and all calculations are performed for both original and declustered catalogs of each region. The frequentist confidence intervals are calculated for the level of confidence $1-\alpha = 95\%$ and 99%, and future time intervals $T_f = 30$, 50 years. The Bayesian posterior distributions of the maximum expected earthquake magnitude are calculated for $T_f = 30{,}50$ years and 90% confidence level in each zone. In order to calculate the Bayesian confidence interval, different scenarios of $M_{\max} = 8.5$, 9.0, 9.5 are assumed.

The reminder of this paper is organized as follow: first the frequentist and Bayesian approaches that are used to calculate the maximum expected earthquake magnitude are explained. Then, the methods are applied to the earthquake catalog of each seismotectonic zone of Iran. Finally, results for each zone are presented in the corresponding Figures and Tables and summarize the findings.

2. Frequentist Approach

Considering $\{m_1,\ldots, m_n\}$ be n distributed magnitudes with a probability density $f(m)$ and cumulative distribution function $F(m)$, which follow a Poisson process with productivity λ. Using the total probability theorem, the probability that all $m_i \le m$ defined as:

$$Pr(\max\{m_i\} \le m) = P(x) = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \exp(-\lambda)\left[F(m)\right]^n = \exp\{-\lambda[1-F(m)]\} \quad (1)$$

$$F_{\beta M}(m) = \frac{\exp(-\beta m_0) - \exp(-\beta m)}{\exp(-\beta m_0) - \exp(-\beta M)}; \quad m_0 \le m \le M \quad (2)$$

Based on the assumption that the events in the past T and future T_f follow the same Poisson process with a constant Poisson rate λ and λ_f respectively, we have $T_f = \frac{\lambda_f}{\lambda} T$.

The probability that the maximum expected earthquake magnitude μ_t in future time T_f is smaller than or equal to m with n events in the catalog during time T is [10]:

$$\Pr(n, \mu \le m/\lambda, \beta) = \frac{\lambda^n}{n!} e^{-\lambda} \exp\left\{ \frac{T_f}{T} \lambda \left[1 - F_\beta(m)\right] \right\} = \frac{\lambda^n}{n!} e^{-\lambda} \exp\left\{ -(T_f/T)\lambda\left[1 - F_\beta(m)\right] \right\} \quad (3)$$

In order to calculate the frequentist confidence interval, the unbounded Gutenberg-Richter distribution $F_\beta(m) = 1 - \exp\left[-\beta(m - m_0)\right]$ is applied that depends only on the Gutenberg-Richter b-value or $\beta = b\, ln(10)$. In this approach, β is estimated according to Aki's formula [12],

$$\beta = \frac{1}{\bar{m} - m_0} \quad (4)$$

The frequentist confidence interval for the level of confidence 1-α, calculated as [10]:

$$\psi_n = m_0 - \frac{1}{\beta}\log\left[\frac{\alpha}{(T_f/T)(n+1)}\right] \tag{5}$$

3. Bayesian Approach

In this approach, we assume the stationary Poisson process in time with unknown productivity $\lambda > 0$ and Gutenberg Richter relation in the magnitude distribution with unknown b-value which is estimated by the Bayes theorem from the data in the past. Inserting prior information about β and λ into a prior distribution $P_0(\beta,\lambda)$, the likelihood function $L(\{m_i\}|\beta,\lambda)$ and the posterior distribution $P(\beta,\lambda/\{m_i\})$ can be calculated:

$$P(\beta,\lambda/\{m_i\}) = L(\{m_i\}|\beta,\lambda)P_0(\beta,\lambda) \tag{6}$$

in which L is the likelihood function

$$L(n,\{m_i\}/\lambda,\beta) = \frac{\lambda^n e^{-\lambda}}{n!}\prod_{i=1}^{n} f_\beta(m) = \\ g_\lambda(n)\times h_\beta(\{m_i\}) \tag{7}$$

Inserting a flat prior for parameters λ and β, Zoller et al. [10] showed that the marginal pdf of μ, for predefined future time intervals T_f, and the posterior pdf of β, are defined as:

$$g_\mu(\mu/\{m_i\},n,\beta,T_f) = \\ \frac{(T_f/T)(n+1)f(\mu|\beta)}{\{1+(T_f/T)[1-F(\mu|\beta)]\}^{n+2}} \tag{8}$$

$$h(\beta|\{m_i\},n) = \\ \frac{\beta^n \exp(-n\beta\overline{m})}{[\exp(-\beta m_0)-\exp(-\beta m_{\max})]^n}; \qquad \beta > 0 \tag{9}$$

where $\overline{m} = \frac{1}{n}\sum_{i=1}^{n} m_i$.

Thus, the Bayesian posterior distribution of μ in a predefined time interval T_f is obtained as [10]:

$$p_\mu(\mu|\{m_i\},n,T_f) = \int_0^\infty g_\mu(\mu/\{m_i\},n,\beta,T_f)h(\beta/\{m_i\},n)\,d\beta \\ \propto \int_0^\infty \beta^{n+1}\frac{\exp(-n\beta\overline{m})\exp(-\beta\mu)}{[\exp(-\beta m_0)-\exp(-\beta m_{max})]^{n+1}\times\{1+\left(\frac{T_f}{T}\right)\left[\frac{\exp(-\beta\mu)-\exp(-\beta m_{max})}{\exp(-\beta m_0)-\exp(-\beta m_{max})}\right]\}^{n+2}}\,d\beta \tag{10}$$

4. Data

In order to apply the method, the earthquake catalogs of different seismotectonic zones of Iran are investigated. Based on the developed maps by Mirzaei et al. [13] and Tavakoli and Ghafory-Ashtiany [14], Iran is subdivided into six seismotectonic zones namely Alborz, Azerbaijan, Central Iran, Zagros, Kopeh Dagh and Makran shown in Figure (1). The earthquake catalog consisting of different national and international data banks is subdivided in six zones. In order to homogenize the catalog, different magnitude scales are converted to the moment magnitude M_w using the conversion relation of Shahvar et al. [15]. The earthquake catalog covers events between 24°-42°N and 44°-64°E from ancient history until 2015 with the magnitude range between $5.5 \le M_w \le 8.1$. In order to fulfill the Poisson assumption, declustering the catalog is considered. Using the method of Gardner and Knopoff [16], the seismicity of the original and declustered catalogs are compare, and the results of the parameters for both catalogs are given in Table (1). It must be noted that although declustering leads to the Poisson

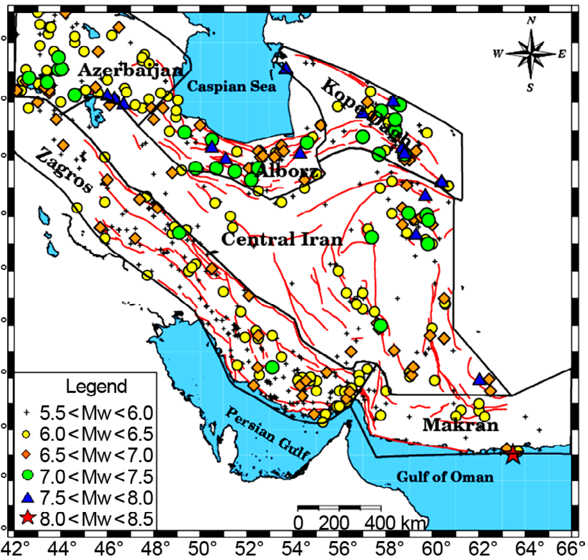


Figure 1. Distribution of earthquakes with the magnitude $5.5 \le M_w \le 8.5$ in different seimotectonic zones of Iran, the solid lines present the location of active faults (map of major active faults of Iran [17] is used).

Table 1. Parameters of the original and declustered catalogs for different seismotectonic zones of Iran (values in parenthesis are for declustered catalogs).

Catalog	m_0	n	$\bar{m}$	m_{max}^{obs}	T
Alborz	5.5	70 (54)	6.20 (6.35)	7.8	1270
Azerbaijan	5.5	105 (86)	6.13 (6.22)	7.7	1586
Central Iran	5.5	151 (110)	6.00 (6.07)	7.6	1279
Zagros	5.5	257 (180)	5.84 (5.90)	7.1	1141
Kopeh Dagh	5.5	59 (51)	6.29 (6.36)	7.6	1248
Makran	5.5	36 (26)	5.92 (5.92)	8.1	94

process, the reduction of the number of events causes inaccurate estimation of b-value. Therefore, in this study, all calculations are applied for both catalogs to find the influence of the declustering on events.

5. Conclusion and Results

Iranian plateau as a part of Alpine Himalayan belt has a high density of active and quaternary faults, which experienced lots of destructive earthquakes [2]. Occurrences of many historic and recent earthquakes in different seismotectonic zones of Iran indicate the importance of the seismic hazard and risk analysis in this region. The estimation of the maximum earthquake magnitude and associated uncertainty plays a crucial role in seismic hazard assessment. A proper treatment of uncertainties is missing in different point estimators used by Kijko [3]. In this work, the confidence interval of the maximum expected earthquake magnitude is calculated to quantify the uncertainty. For this, the frequentist and Bayesian approaches introduced by Zoller et al. [10, 18] are applied to calculate the maximum expected earthquake magnitude in different seismotectonic zones of Iran. The goal of this study is to represent different scenarios of statistical estimates of the maximum expected earthquake magnitude, which is a derived quantity depending on the future time interval and the arbitrary chosen maximum possible earthquake magnitude $M_{\max}$. For this, a Poisson process in time and Gutenberg Richter distribution in the magnitude domain are assumed.

It is sometimes assumed that results are dependent on the magnitude distribution and a tapered power law distribution [19] may lead to different values. Zoller and Holschneider [20] argue that the tail of the assumed distribution is not important, as long as it is not covered by a large amount of data.

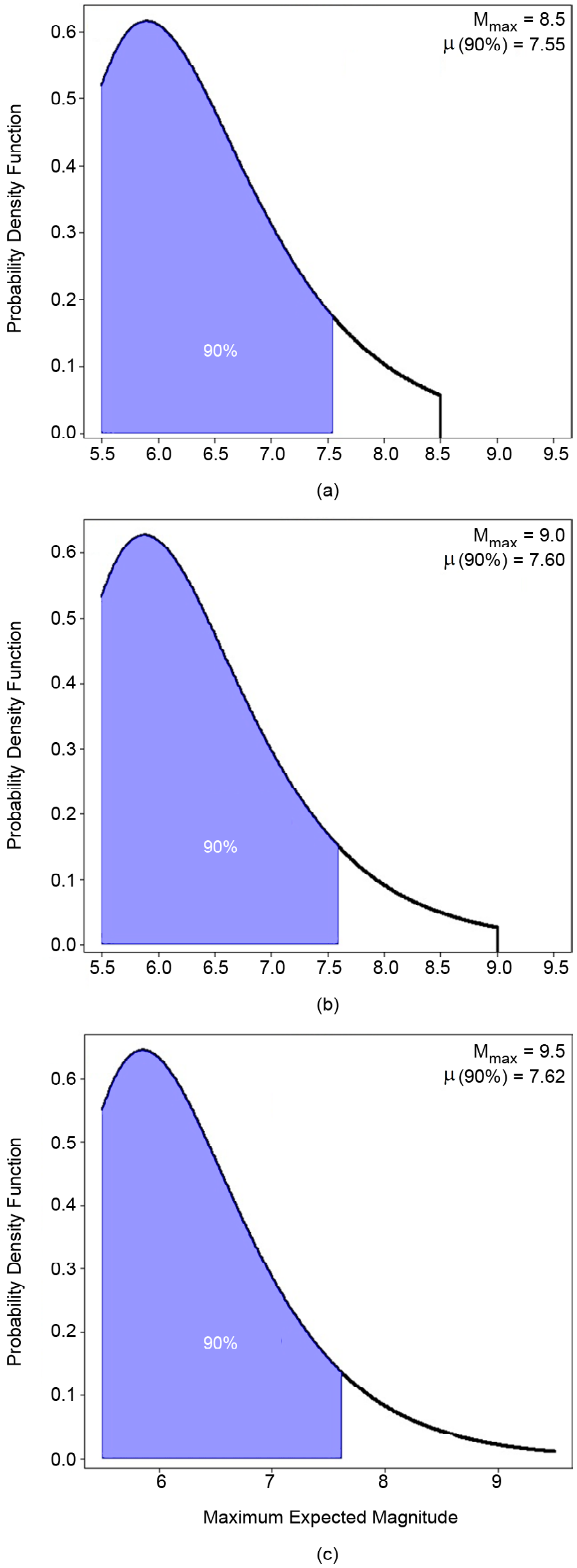
Based on the formula derived in the frequentist approach for the known magnitude distribution, the upper bound of the frequentist confidence intervals are calculated for the original and declustered earthquake catalogs of Iran. Results for time intervals of $T_f = 30$ and 50 years and confidence levels $1-\alpha = 95\%$, 99% are listed in Table (2), results for declustered catalogs are given in parentheses. Since the Equation (5) is an asymptotic approximation $(\alpha \ll 1, \lambda \gg 1)$, the frequentist confidence intervals are not provided for ≈ 1.0.

In the Bayesian approach, the posterior distribution of the maximum magnitude μ_t in a future time interval T_f and the confidence level $1-\alpha$ is calculated. The three choices of the absolute maximum magnitude $M_{\max} = 8.5$, 9.0, 9.5 are given for the calculation of μ_t. Figures (2) to (7) show the

Table 2. The upper bound of the frequentist confidence interval Ψ_n for the original and declustered catalogs of six seismotectonic zones of Iran.

Zone	Time Interval T_f (Years)	$1-\alpha = 95\%$	$1-\alpha = 99\%$
Alborz	30	7.34 (7.44)	8.19 (8.4)
	50	7.61 (7.74)	8.44 (8.7)
Azerbaijan	30	7.28 (7.35)	8.06 (8.20)
	50	7.53 (7.62)	8.31 (8.47)
Central Iran	30	7.21 (7.25)	7.86 (7.96)
	50	7.42 (7.47)	8.07 (8.19)
Zagros	30	7.02 (7.03)	7.52 (7.57)
	50	7.18 (7.21)	7.68 (7.75)
Kopeh Dagh	30	7.44 (7.50)	8.37 (8.49)
	50	7.74 (7.82)	8.67 (8.82)
Makran	30	7.56 (7.44)	8.17 (8.05)
	50	7.75 (7.63)	8.36 (8.24)

result for different scenarios of M_{max} and T_f = 30. The different shapes correspond to different choices of the absolute maximum magnitude M_{max}: M_{max} = 9.5 allows for arbitrary large event whereas

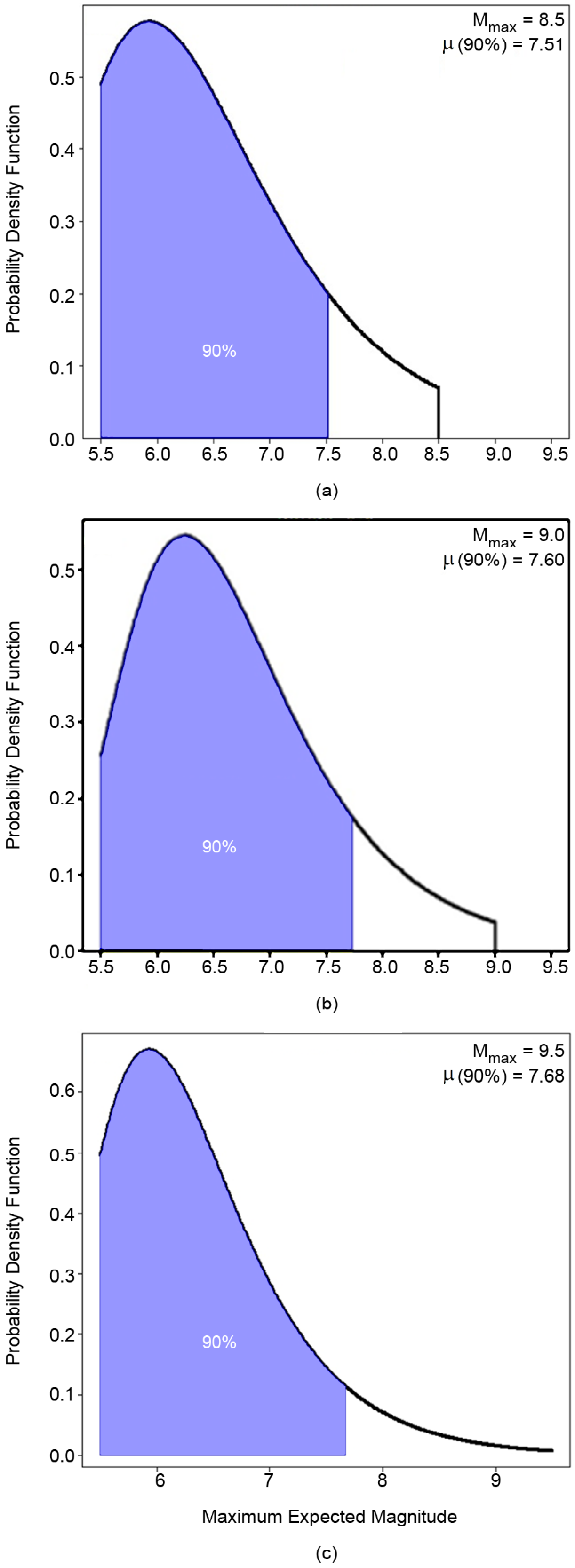


(a)

(b)

(c)

Figure 2. Bayesian posterior density function of μ_t and T_f =30 years for the Alborz seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.



(a)

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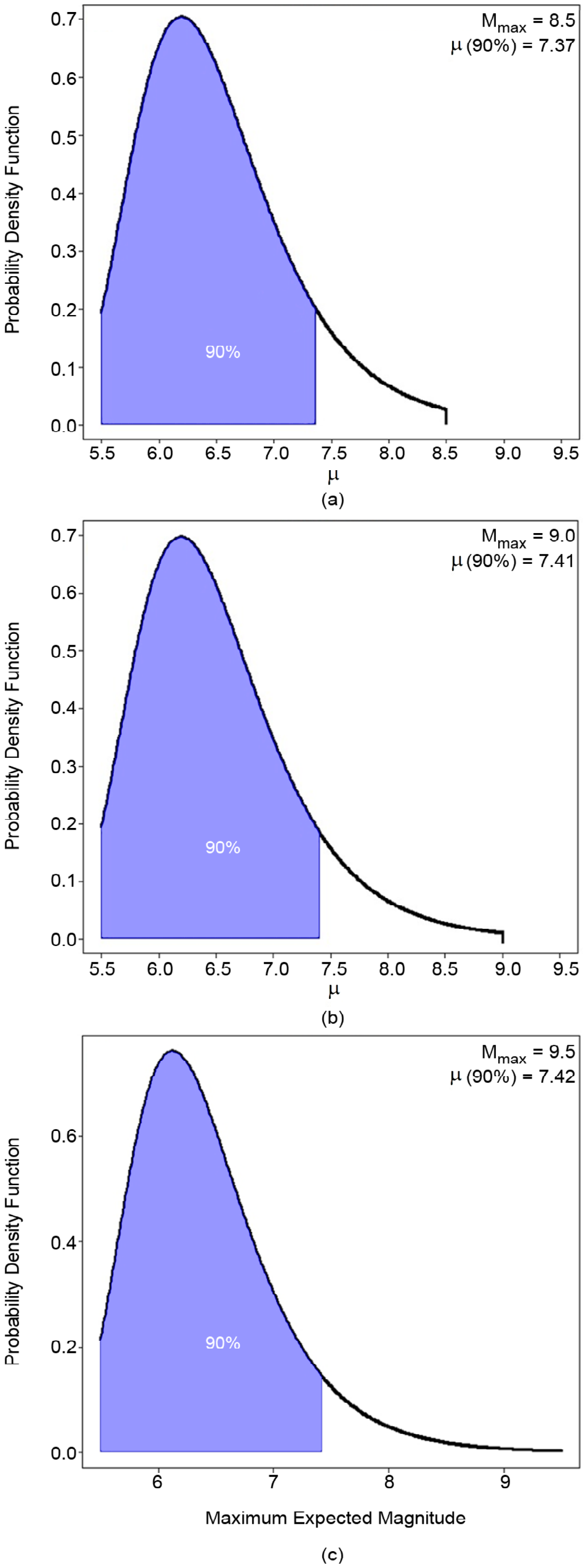
Figure 3. Bayesian posterior density function of μ_t and T_f =30 years for the Azerbaijan seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.

other options are based on the estimated maximum possible earthquake magnitude from catalog (M_{max}= 8.5, M_{max}= 9.0). The upper bounds of the confidence interval of the maximum expected earthquake magnitude μ_t are shaded for the 90% confidence interval.



(a)

(b)

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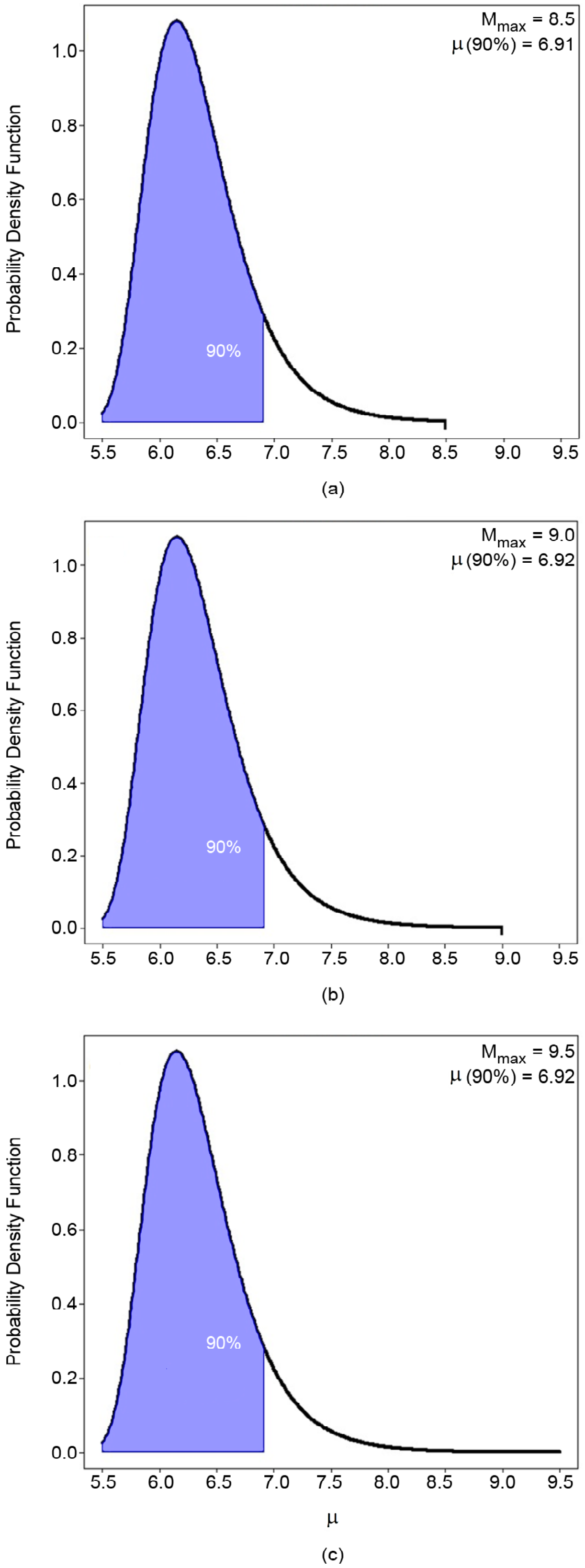
Figure 4. Bayesian posterior density function of μ_t and T_f =30 years for the Central Iran seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.

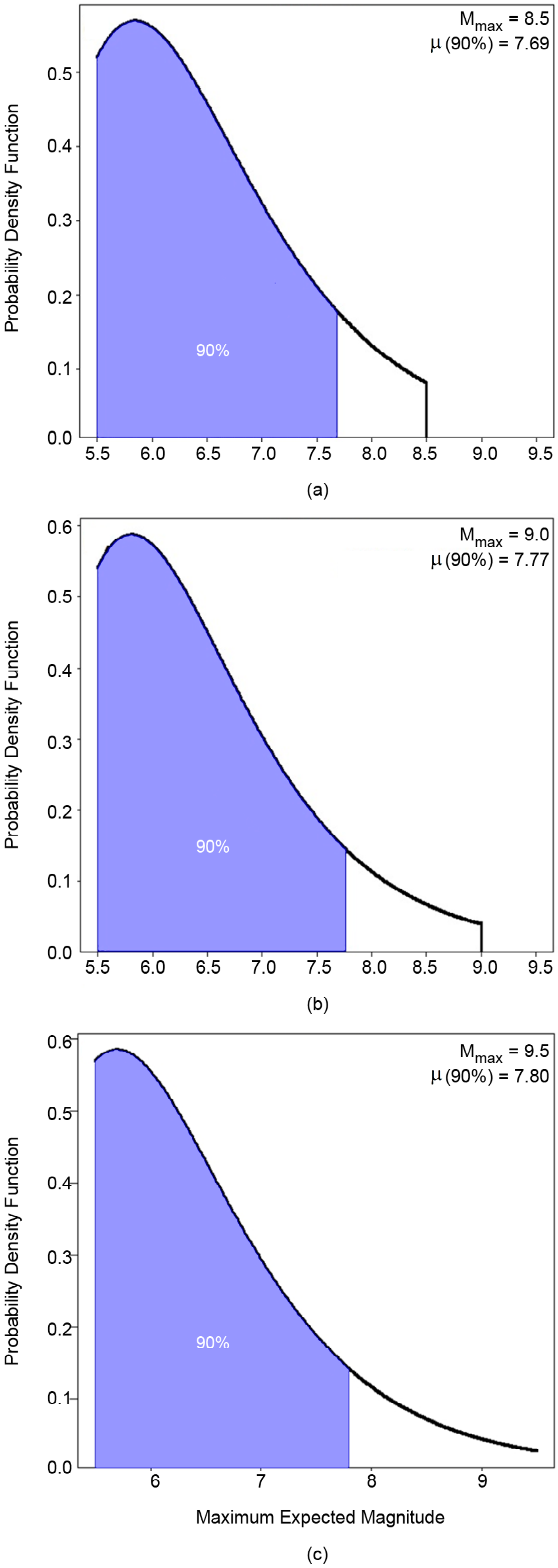


(a)

(b)

(c)

Figure 5. Bayesian posterior density function of μ_t and T_f =30 years for the Zagros seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.

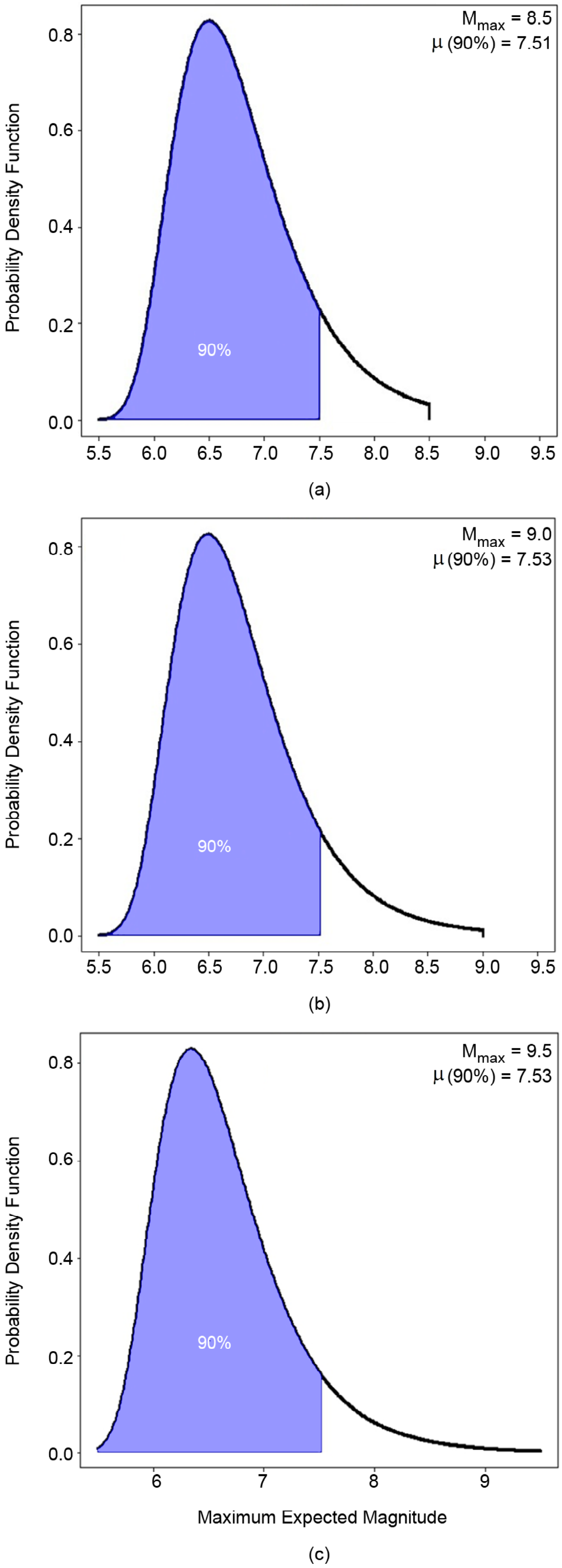


(a)

(b)

(c)

Figure 6. Bayesian posterior density function of μ_t and T_f =30 years for the Kopeh Dagh seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.



(a)

(b)

(c)

Figure 7. Bayesian posterior density function of μ_t and T_f =30 years for the Makran seismotectonic zone and three choices of M_{max}: (a) M_{max} = 8.5, (b) M_{max} = 9.0, and (c) M_{max} = 9.5. The shaded area corresponds to the 90% confidence interval.

Tables (3) to (8) show the result of 90% confidence interval for three scenarios of M_{max} in different seismotectonic zones of Iran, results for declustered catalogs are given in parenthesis. The results show that different values of M_{max} has a minor effect on the maximum expected earthquake magnitude μ_t. The results for $T_f = 30$ and $1 - \alpha = 90\%$ are between 7.55-7.62, 7.51-7.68, 7.37-7.42, 6.91-6.92, 7.69-7.80, 7.51-7.53 in the original catalogs of Alborz, Azerbaijan, Central Iran, Zagros, Kopeh Dagh and Makran seismotectonic zones. The same results for $T_f = 50$ and $1 - \alpha = 90\%$ are 7.74-7.86, 7.71-7.91,7.59-7.68,7.10-7.10,7.86-8.06,7.71-7.77 respectively. The results for declustered catalogs show that declustering has a minor influence on the calculation of μ_t. The aim of this study is to quantify the existing uncertainties in the most accurate way. It should be noted that the selection of the time interval depends on the specific type of applications, which could be defined as a time scale of decades to 1 million years.

Table 3. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Alborz seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	7.55 (7.78)	7.60 (7.87)	7.62 (7.92)
50	7.74 (7.93)	7.82 (8.07)	7.86 (8.14)

Table 4. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Azerbaijan seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	7.51 (7.63)	7.60 (7.72)	7.68 (7.78)
50	7.71 (7.81)	7.83 (7.95)	7.91 (8.06)

Table 5. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Central Iran seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	7.37 (7.30)	7.41 (7.37)	7.42 (7.39)
50	7.59 (7.45)	7.65 (7.56)	7.68 (7.59)

Table 6. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Zagros seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	6.91 (7.02)	6.92 (7.03)	6.92 (7.03)
50	7.10 (7.22)	7.10 (7.23)	7.10 (7.24)

Table 7. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Kopeh Dagh seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	7.69 (7.79)	7.77 (7.88)	7.80 (7.94)
50	7.86 (7.93)	7.97 (8.07)	8.06 (8.16)

Table 8. Upper bound of the Bayesian confidence interval of μ_t within a time interval of 30 and 50 years and confidence level $1 - \alpha = 90\%$ in the Makran seismotectonic zone.

T_f	M_{max}		
	8.5	9.0	9.5
30	7.51 (7.39)	7.53 (7.41)	7.53 (7.41)
50	7.71 (7.61)	7.75 (7.64)	7.77 (7.64)

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