

# Preliminary Seismic Evaluation of the Historic Sultaniyeh Dome

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**ABSTRACT:** *This paper presents the results of seismic vulnerability analysis of the historic sultaniyeh dome constructed for the tomb of Uljaytu. This monumental building was constructed about 700 years ago and is now recognized as one of the largest masonry structures in the world. Finite element analysis is used to assess the seismic resistance of the building for three levels of seismic hazard. For each level of seismic hazard, the locations and extent of damage to the building are identified.*

**Keywords:** Historic building; Structure; Earthquake; Masonry; Dome

## 1. Introduction

The Sultaniyeh Dome is the largest dome type building in Iran and before construction of the famous Cathedral of Santa Maria del Fiore in Florence and the aya Sofia mosque in Turkey, it was the largest dome type structure in the world. This monumental building is 50 meter high brick masonry structure with a dome type roof. The roof is a two layer shell structure with a base diameter of 25.5 meters. Andre Godard in Upham Pope and Ackerman [1] has described this monument as "... the skillful, confident work of a great builder, a consummate technician who was at the same time an artist. Here is a dome with a span of 80 feet built solely of bricks, without any buttresses, pinnacles, or shoulders of any kind, which stands simply by virtue of a perfectly conceived and constructed profile". According to Sanpaolesi and Kassai [2], one can not find any two layer shell dome before construction of this building either in the West or in the East. Figure (1) shows a picture of the building. The plan comprises an octagon with a rectangular burial chamber protruding from the southern side. The dome rests on the upper terrace, carried on the interior by the corbels of a thick wall. Minarets rise from the upper terrace at each of the eight corners. There are eight pillars which support the weight of the dome. The cross sectional area of each pillar is approximately 50 square meters. The pillars are placed on corners of the octagonal plan.

The Sultaniyeh Dome has been registered as a

World Heritage site by *UNESCO's* World Heritage Committee. This Committee has so far registered seven sites in Iran as the World Heritage Sites. Tchogha Zanbil, Persepolis, and Naqsh-e-Jahan were registered in 1979 and Takht Soleyman, Pasargade, and Bam Citadel were registered in 2004. The Sultaniyeh Dome was registered in 2005 session of the World Heritage Committee.

## 2. Description of the Analytical Model

Finite Element method is used to study the performance of the main octagonal structure under gravity and seismic loads. In this study, only the main building is analyzed and the burial chamber and minarets are disregarded. Shell elements with both bending and membrane capabilities are used to model the double crust domes and the ribs that connect the outer crust to the inner crust. Structural solid elements are used for modeling the rest of the structure. The underlying soil is modeled by shell elements with elastic foundation stiffness (in-plane stiffness) capability at base of structure.

The mathematical model required a vast number of solid elements in order to accurately model all the opening and arcades within the octagonal structure. Because of the large size of the model and limited hardware capabilities, attempts to perform nonlinear analysis which account for cracking and crushing of masonry material were not fruitful. Alternatively, linear



Figure 1. The mausoleum of Uljaytu at Sultaniyeh.

analyses are performed and during post-processing the stress results are compared with failure envelope of masonry material. With such post-processing, the locations where the structure cracks or crushes are identified. Figure (2) shows an example of a failure surface in principal stress space where  $f_c$  and  $f_t$  are respectively the ultimate compressive and tensile stress in uniaxial loading condition [3].

The building materials consist of brick masonry and lime/gypsum mortar. Based on test results [2], the ultimate tensile stress  $f_t$  and ultimate compressive stress  $f_c$  are respectively taken as  $175\text{ kN/m}^2$  and  $3000\text{ kN/m}^2$ . In the space where principal stresses are compressive, we conservatively ignore the principal stress,  $S_2$ , and use Kupfer et al [4] Formulas in the principal stress state to define the failure surface.

$$\text{Compression failure: } \left( \frac{s_1 + s_3}{f_c} + \frac{s_3}{f_c} \right)^2 + \frac{s_3}{f_c} + \frac{3.65s_3}{f_c} = 0$$

$$\text{Tensile cracking: } \frac{s_1}{f_t} = 1 + \frac{0.8s_3}{f_c}$$

The specific weight of material is  $1.6\text{ ton/m}^3$  and elastic modulus and Poisson's ratio are  $2000\text{ MPa}$  and  $0.15$  respectively. The elastic foundation stiffness, defined as the pressure to produce a unit normal deflection of the foundation, is set equal to  $24000\text{ kN/m}^3$ .

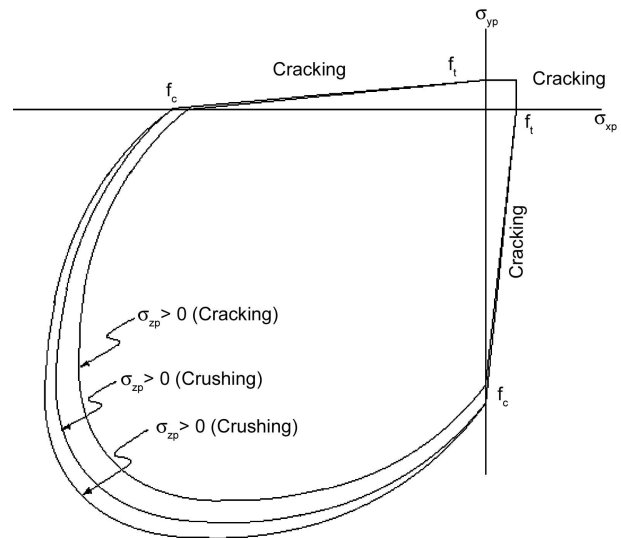


Figure 2. Failure surface in principal stress space [3].

### 3. Gravity Load Analysis

The analysis of the building under gravity load is performed with vertical acceleration of  $9.81\text{ m/s}^2$ . Figure (3) shows the distribution of principal compressive stress in the dome. The maximum compressive stress of  $524\text{ kN/m}^2$  occurs at the base of the dome which is well below the ultimate compressive stress of  $3000\text{ kN/m}^2$ . Figure (4) shows the distribution of vertical compressive stress in the octagonal structure which supports the dome. The maximum compressive stress of  $1009\text{ kN/m}^2$  occurs locally around an opening within the second story. This figure also shows that average compressive stress at the base of the building is about  $500\text{ kN/m}^2$ . This is less than 20% of the ultimate compressive strength of the material indicating a safety margin of more 80% against collapse under gravity load. Thus, it may be concluded that if more than 20% of the supporting pillars

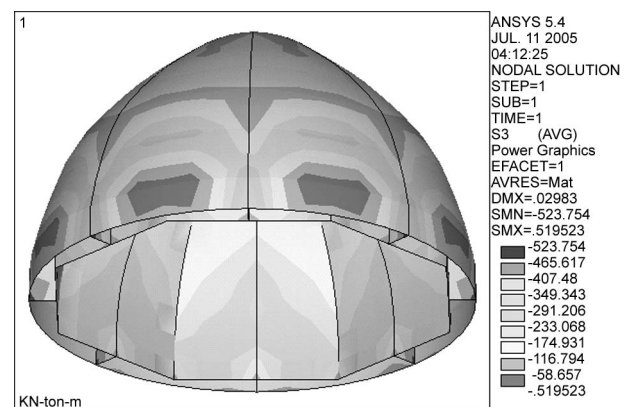


Figure 3. Distribution of principal stress due to gravity load.

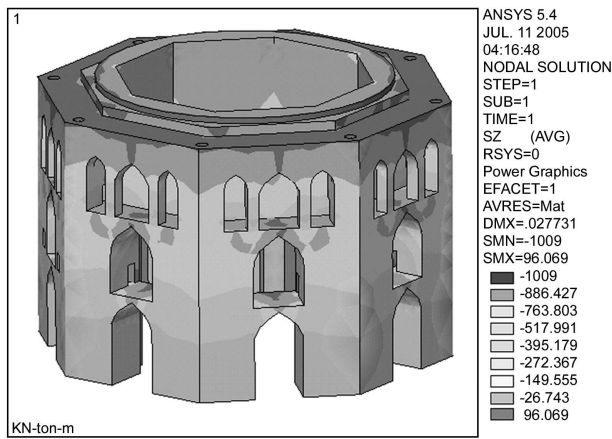


Figure 4. Distribution of vertical compressive stress due to gravity load.

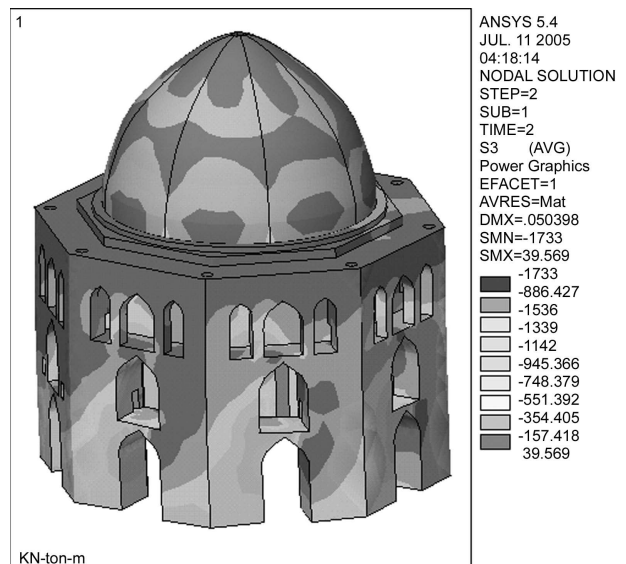


Figure 5. Distribution of principal compressive stress at  $PGA = 0.23g$ .

remain intact after an earthquake, the structure would not collapse under its own weight.

#### 4. Seismic Analysis of the Building

The seismic analyses are performed for three levels of seismic hazard with return periods of 75, 475 and 2500 years using static push-over analysis. The lateral accelerations for these return periods are found respectively to be 0.23g, 0.44g and 0.76g [5-6].

##### 4.1. Seismic Performance for an Event with 75 Years Return Period

The results of seismic analysis for a return period of 75 years and  $PGA = 0.23g$  indicate a maximum drift of 4.0cm at top of the dome. Figure (5) shows the distribution of principal compressive stress,  $S_3$ , in the building. It indicates that the maximum compressive stress,  $S_3 = 1733kN/m^2$ , which occurs locally around an opening within the main pillars at the second story, is well below the ultimate compressive strength of the material.

Figure (6) shows the distribution of principal tensile stress,  $S_1$ , in the building. It indicates that while the tensile stress exceeds the ultimate tensile stress of  $175kN/m^2$  at some locations, it is less than the ultimate tensile stress in major portions of the structure. The maximum tensile stress,  $S_1=1478kN/m^2$ , occurs at the corner of one of the main galleries in the second floor.

Figure (7) shows the in-plane displacement contour of shell elements representing the underlying soil. It indicates that vertical soil deformation increases from 0.98 centimeters on one side of the building to 3.6 centimeters on the opposite side. It should be noted

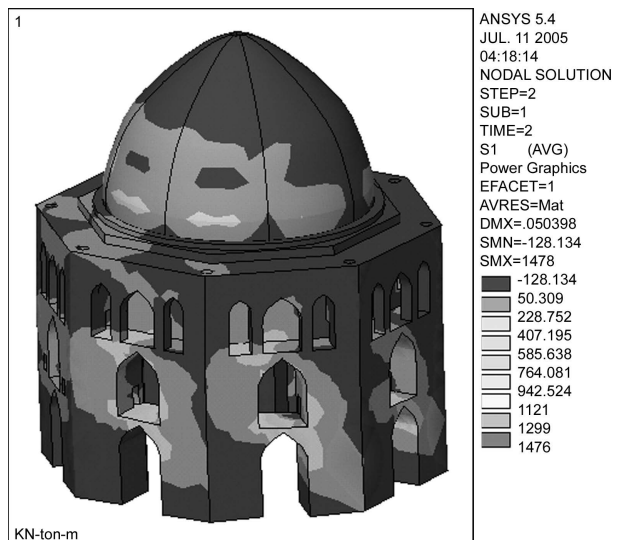


Figure 6. Distribution of principal tensile stress at  $PGA = 0.23g$ .

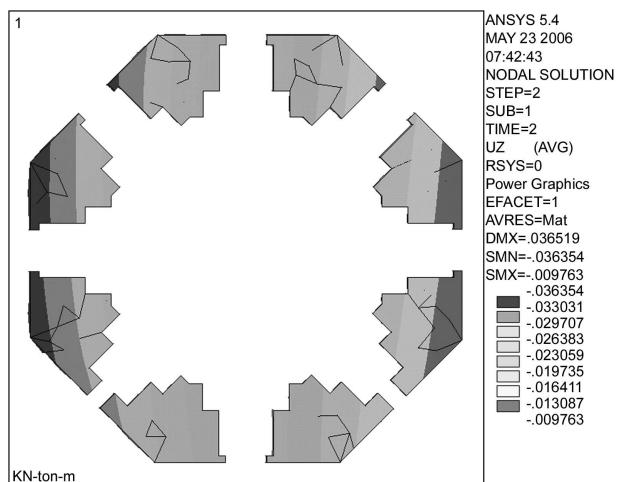


Figure 7. Soil deformation contour at  $PGA = 0.23g$ .

that the soil deformation under gravity load is almost uniform at around 2.2 centimeters.

Figures (8) through (11) compare the results of the analysis with cracking zones of the failure surface shown in Figure (2). The locations with light color shades indicate cracking of the material. These figures indicate that some portions of the main structure, the dome, and the exterior walls in the second story crack, but the main portion of the structure remains intact at this seismic level. Cracking

is more extensive within the two exterior walls on the second story parallel to the direction of the seismic load.

The above results indicate that for an earthquake with return period of 75 years, the structure remains intact, while some cracking occurs in the dome, the main supporting structure, and the exterior walls on the second story. Cracking of the two exterior walls parallel to the direction of seismic load would be more severe than other locations.

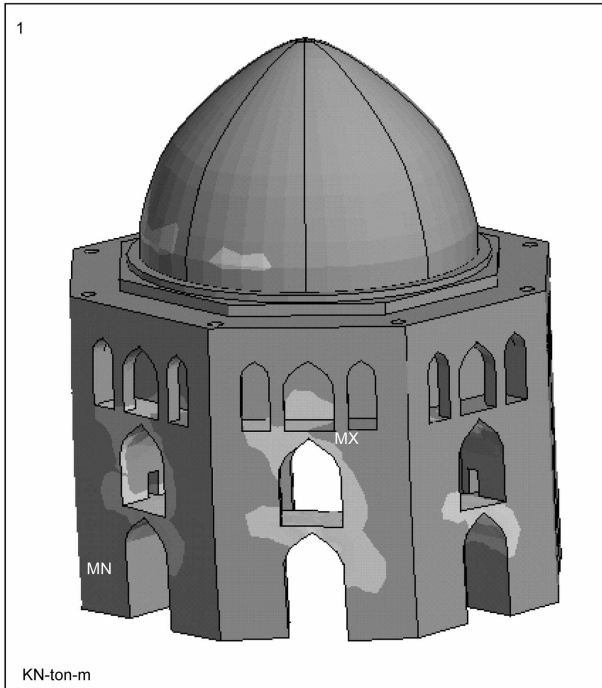


Figure 8. Cracking of the main structure and the dome.

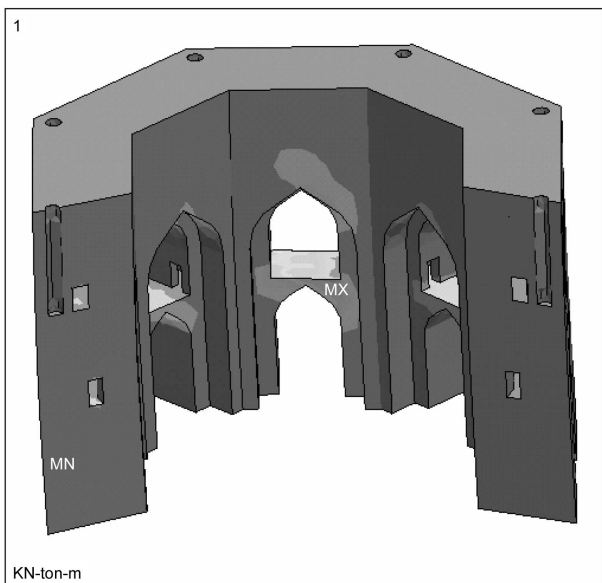


Figure 9. Cracking at a cross section of the main structure.

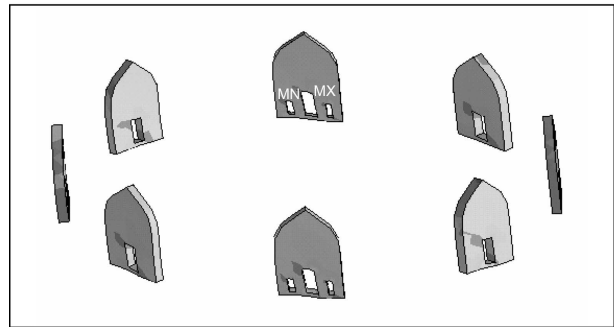


Figure 10. Cracking of the exterior walls on the second story.

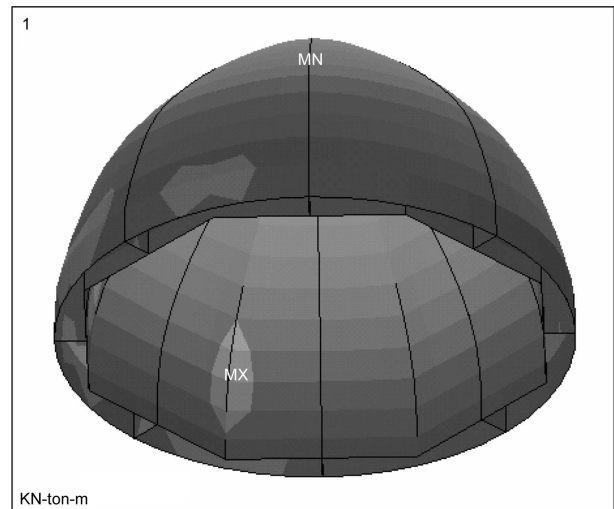


Figure 11. Cracking of the dome structure.

#### 4.2. Seismic Performance for an Event with 475 Years Return Period

The results of seismic analysis for a return period of 475 years and  $PGA = 0.44g$  indicate a maximum drift of 7.6 cm at top of the dome. Figure (12) shows the distribution of principal compressive stress in the main building. The maximum compressive stress,  $S_3 = 2813kN/m^2$ , occurs locally around an opening within the main pillars at the second story. This stress is about 6% below the crushing strength of

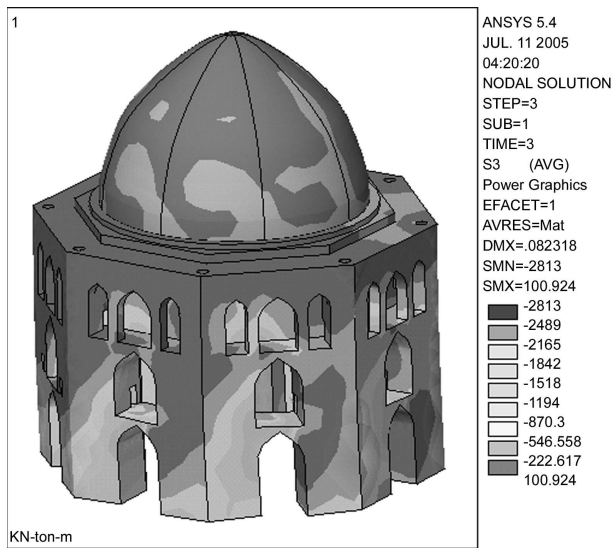


Figure 12. Distribution of principal compressive stress at  $PGA=0.44g$ .

the material. The maximum compressive stress at the base of the main columns is about  $1100kN/m^2$  which is less than 40% of the crushing strength of the material.

Figure (13) shows the distribution of principal tensile stress,  $S_1$ , in the building. The maximum tensile stress,  $S_1 = 2751kN/m^2$ , occurs at the corner of one of the main galleries in the second floor. The cracking zone of the structure is similar to those shown in Figures (8) to (11). However due to higher tensile stresses, the crack widths are expected to be larger than the previous case.

The preceding results indicate that for an earthquake with a return period of 475 years the cracking would be more extensive than the previous case.

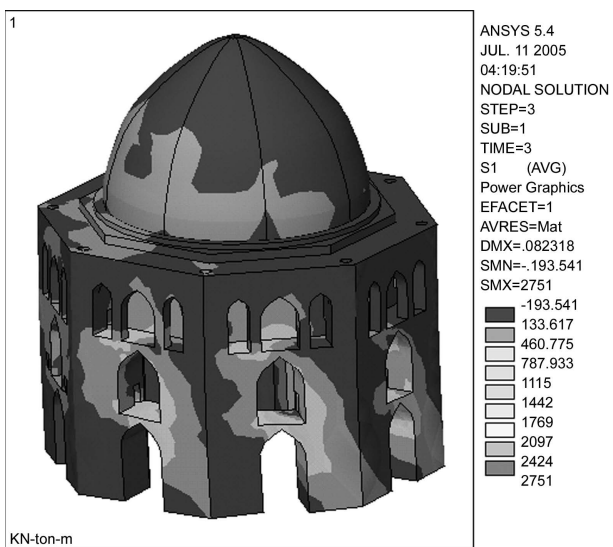


Figure 13. Distribution of principal tensile stress at  $PGA=0.44g$ .

However, a major portion of the octagonal supporting structure remains intact while the maximum elastic compressive stress at the base of one of the main columns reaches to about 40% of the crushing strength of the material. Due to cracking of other portions of the structure, this stress is expected to increase significantly and some crushing of the main column may occur. For short and normal duration earthquakes, the intact portion of the columns are expected to be able to prevent collapse of the structure because only about 20% of column area is required to carry the weight of the structure after the earthquake. However, for an earthquake with long duration, the damage may be so extensive that part of the structure may collapse at this earthquake level.

Figures (14) and (15) show the soil deformation and pressure contours for this level of seismic hazard. With a maximum elastic deformation of 4.8 centimeters the soil pressure under the two most

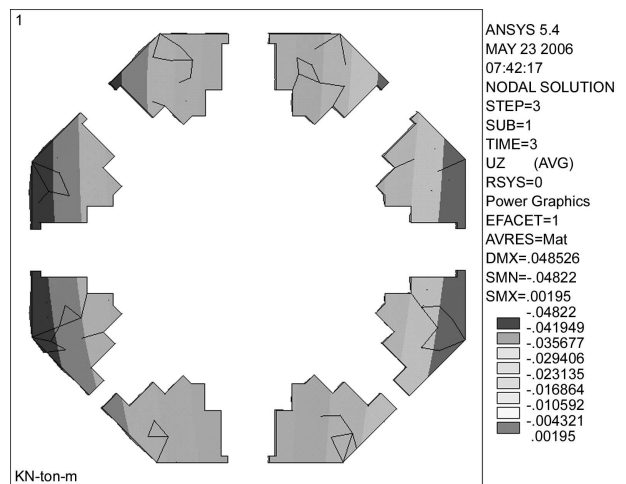


Figure 14. Soil deformation contour at  $PGA=0.44g$ .

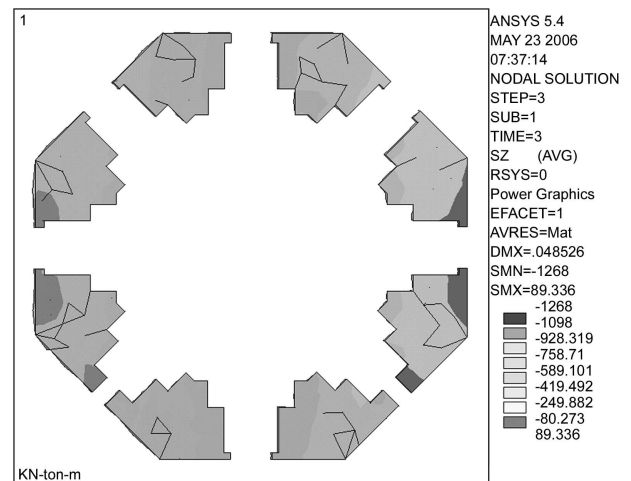


Figure 15. Soil pressure contour at  $PGA=0.44g$ .

highly stressed pillars varies between 0.75 to 1.27Mpa. At such stress level, the soil is expected to undergo appreciable permanent deformation. Therefore, the structure is expected to tilt somewhat parallel to direction of seismic load.

### 4.3. Seismic Performance for an Event with 2500 Years Return Period

The results of seismic analysis for a return period of 2500 years and  $PGA = 0.76g$  indicate a maximum drift of 13.2cm at top of the dome. Figure (16) shows the distribution of principal compressive stress in the main building. The maximum compressive stress,  $S_3 = 4818kN/m^2$ , occurs within one of the main arches supporting the second floor. This stress exceeds the ultimate compressive strength of the material and indicates crushing at that location. The principal compressive stress within some other main supporting arches also exceeds the crushing strength of material. Crushing zone is expected to expand significantly in an earthquake due to stress redistribution caused by cracking and crushing of material.

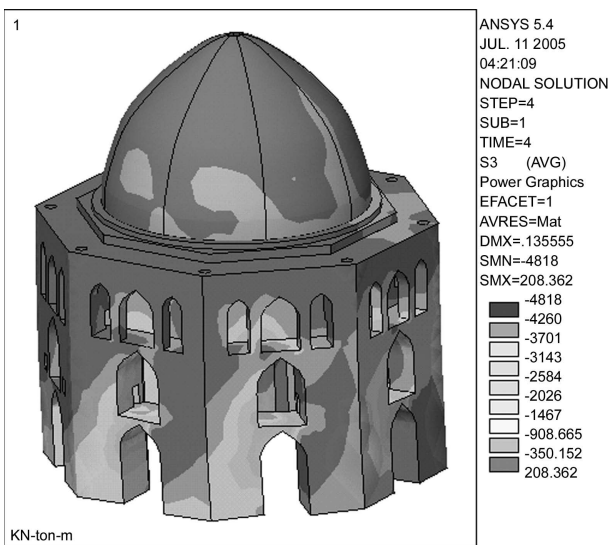


Figure 16. Distribution of principal compressive stress at  $PGA=0.76g$ .

Figure (17) shows the distribution of principal tensile stress,  $S_1$ , in the building. The maximum tensile stress,  $S_1 = 4694kN/m^2$ , again occurs at the corner of one of the main galleries in the second floor.

The preceding results indicate that for an earthquake with a return period of 2500 years, the elastic compressive stresses within some of main supporting arches exceed the crushing strength of the material. Due to severe cracking of other portions of the structure, the compressive stress is expected to

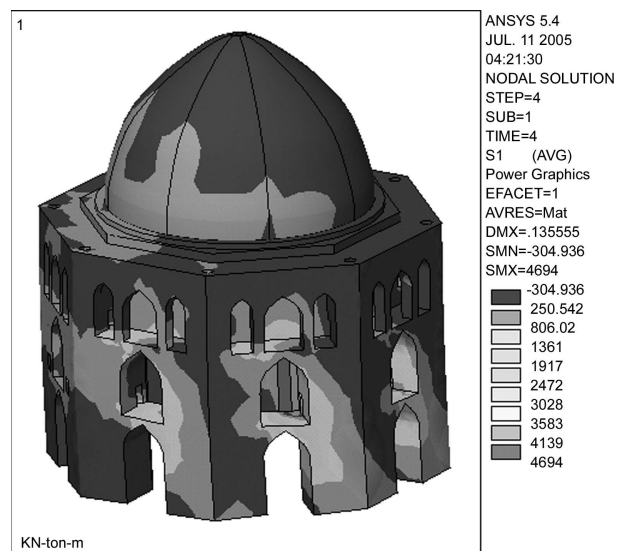


Figure 17. Distribution of principal tensile stress at  $PGA=0.76g$ .

increase significantly and crushing of major portions of the structure may occur. The damage being so extensive, would probably cause collapse of the structure at this earthquake level.

## 5. Conclusions

Elastic seismic analyses were performed for three levels of seismic hazard with return periods of 75, 475 and 2500 years. The results of analyses indicate that for an earthquake with return period of 75 years the structures remain intact while cracking occurs in some parts of the structure. For an earthquake with a return period of 475 years, the cracking intensifies specially within the octagonal structure and the exterior walls on the second story. However, a major portion of the octagonal structure remains intact. For short and normal duration earthquakes, the intact portion of the main pillars are expected to be able to prevent the collapse of the structure but for an earthquake with long duration the damage may be so extensive that part of the structure may collapse. Due to excessive soil pressure under some of the pillars, permanent soil deformation and tilting of the structure is anticipated at this level of earthquake. For an earthquake with a return period of 2500 years, the damage will probably be so extensive that would cause the collapse of the structure.

It should be emphasized that the conclusions are based on linear elastic analyses. The structure would have a nonlinear response as the masonry material cracks and crushes locally during an earthquake. Therefore, the conclusions are only indicative and further studies which properly include the effect of material nonlinearities are needed.

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