Research Paper

Finding Reliable Identification Results for Nonlinear SDOF Systems, Based on Sensitivity Analyses

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ABSTRACT

Post-earthquake assessment of buildings is one of the fundamental questions that needs to be answered immediately after a strong seismic event. Taking a building out of service after an earthquake can have a financial impact even greater than the earthquake itself. Up to now, damaged buildings are categorized in three groups, with red, yellow, or green labels, by engineering judgment based on visual screening. To have a more accurate method, the response of the buildings in aftershocks can be focused on new vibration-based system identification methods. But determining the system parameters is still a challenging subject; involving parameters with low identifiability, or correlated parameters can potentially influence the results in model updating problems. In this paper, a sensitivity matrix-based method is introduced to prioritize parameter estimability. The matrix-based process is capable of quickly determining the correlation between different parameters. Moreover, this method provides an explicit criterion for determining the optimal number of identifiable parameters. To indicate the efficiency of the method, a nonlinear Single-Degree of Freedom (SDOF) system has been simulated. Multiple model updating procedures have been carried out on the selected system, using the Unscented Kalman Filter (UKF). The result shows that system identification based on the sensitivity analysis outcome improves the quality of the identification precision. Additionally, this method decreases identification time by 35 percent that this amount can be crucial for updating large-scaled models.

Keywords:
Estimability; Unscented Kalman Filter; Sensitivity analysis; Post-earthquake assessment

1. Introduction

Finding the residual capacity of damaged structures after a strong earthquake is a significant challenge in earthquake engineering. For this goal, it is necessary to have enough information about damaged buildings and seismic hazards in an earthquake-affected area. Visual inspection is currently used to assess structures following a seismic event. Visual inspection provides valuable information about a damaged building; however, it has significant drawbacks such as being subjective, time-consuming, requiring multi-stage inspections and experts, etc. [1]. The Applied Technology Council published an instruction, ATC-20 [2], for post-earthquake safety assessment of buildings at the end of the 1980s, and the new versions of ATC-20 have been released based on the lessons learned from late earthquakes [3-5]. Since the focus of these publications is on what buildings experienced during an earthquake, they do not provide owners with explicit criteria for decision-making regarding the future of buildings. In other words, visual inspection only describes what structures have gone through during a seismic event.

Researchers have gradually taken new steps to respond to this demand, with a lot of progress being made in data collection and system identification. Data extracted from a structure before, during, and after a seismic event can be quite useful. Post-earthquake safety assessment will be achievable by comparing this multi-stage information, which constitutes the separate analysis of each stage, and combining the results with analytical and numerical models. Studies that have been done in this field can be categorized as data-driven, model-driven, and fusion methods. In data-driven methods, safety is evaluated by comparing information gathered from measurements and limited inspection, and by making use of a database of similar buildings that have sustained varying degrees of damage [6]. Another subset of research focuses on model updating via field measurements. This method's outputs are analytical models that can simulate the behavior of buildings under future possible loading scenarios [7-9]. The last category is fusion methods, which determine the amount of damage and employ an updated model and a database. For instance, an updated analytical model estimates the maximum drift experienced during an earthquake; then, according to the database, the decision-making process is carried out [10-11]. As a result, system identification occupies a central position in post-earthquake building safety assessment.

The Kalman filter is a well-established identification method that was originally proposed first for linear systems [12]. The Unscented Kalman Filter (UKF) and the Extended Kalman Filter (EKF) are two well-known nonlinear extensions of the Kalman filter [13-14]. In the Kalman filter, the observability matrix was a basis upon which parameter estimability was determined. According to the definition, a system is observable if the initial state can be uniquely identified from measurements made at each time step. Calculating the observability matrix for a nonlinear system is computationally taxing and quite complex [15]. Parameter estimation is highly dependent on the quality of measurements, i.e. all parameters of a system are not necessarily identifiable by a given measurement. The sensitivity of the vibration measurements to parameter changes can be considered as parameter identifiability [16]. Accordingly, a sensitivity analysis prior to model updating can provide valuable insight. In Structural Engineering, sensitivity analysis is a common way to study the impact of input parameters (e.g., boundary conditions, model parameters, initial conditions, etc.) on the response of the structure [17-22]. Ramanacha et al. [21] showed "identifiability analysis" before updating the model, results in reducing the computational costs and better results.

This study attempts to implement a matrix-based sensitivity analysis right before solving a structural model updating problem. The objectives of this paper are defined as:

- Investigating the effect of wrong model parameter selection in an identification problem;
- To study the relation between estimability and the parameter sensitivity rank;
- Determining the optimum number of model parameters in a model updating problem.

2. Methods

2.1. Problem Statement

The Kalman filter identifies the parameters of a system in a stochastic space using the system's fundamental equation, which is usually a differential equation, and noise-polluted measured responses. The quality of model updating depends on the quality and adequacy of the measurements. A given parameter of a system is identifiable when the measured system response is sensitive to changes in that parameter and is uncorrelated to other unknown parameters of the system. Recognizing these issues in a small linear model is quite easy. In a complex nonlinear system with numerous unknown parameters; however, the problem takes on a whole new character.

In this study, the estimability of the parameters of a nonlinear SDOF system is calculated using the sensitivity matrix-based method [23], and the results have been validated using the Forward Finite Difference (FFD) method. The matrix-based method, beside determining the correlation between system parameters, provides a specific criterion for determining the optimal number of parameters involved in the identification process. *UKF* is used in this paper because of its intrinsic advantage over *EKF* [24] for a nonlinear *SDOF* model updating in four different cases in terms of parameters selection scenarios. The nonlinear *SDOF* model has six main parameters (two of which are correlated) that depict the behavior of the system under a given seismic record. In the first scenario, all of them are assumed as unknowns. In the remaining three scenarios, parameters deemed to have lower identifiability according to the estimability analysis were set aside from the identification process, and the quality of the model updating operation was also studied.

2.2. The Unscented Kalman Filter

For a discrete-time system in the recursive form, the equations of a dynamic system and its measurement that have, respectively, an additive process and measurement noise are:

$$x_{k+1} = f_k(x_k) + w_k \tag{1}$$

$$z_k = h_k(x_k) + v_k \tag{2}$$

where k is the time step; $x_k \in R^n$ is the state vector; $w_k \in R^n$ is the process noise vector; $z_k \in R^n$ is the measurement vector, and $v_k \in R^n$ is the measurement noise vector.

The Kalman filter (*KF*) is created from the time updating step (prediction) and measurement updating step (updating). *KF* is based on the assumption that all noise vectors are uncorrelated with Gaussian distribution having means equal to zero. Originally, the Kalman filter was introduced to estimate the state of a linear system [12]. Following that, the *EKF* (Extended Kalman Filter) was proposed to solve nonlinear problems. The filter is suitable for a wide range of nonlinear systems; however, it has a few drawbacks, including convergence issues, high computational cost, and impaired performance for systems with high levels of nonlinearity [24]. Because *UKF* is based on statistical linearization, unlike *EKF*, it does not require local linearization. The basic idea behind *UKF* is to propagate means and covariances by passing some carefully-selected points (sigma points) through a nonlinear system, rather than the entire statistical function. The means and co-variances are then computed using transformed sigma points. The unscented transform is the core of *UKF*.

Consider a random variable, x, with n dimensions. The mean $\hat{x}_0$ and covariance P_{x0} of x are supposed to propagate through an arbitrary non-linear function f.

$$\hat{x}_0 = E[x] \tag{3}$$

$$P_{x0} = E\left[(x - \hat{x}_0)(x - \hat{x}_0)^T\right] \tag{4}$$

The sigma point matrix χ with $2n + 1$ columns and its related weights are calculated by the following equations:

$$\chi_0 = \hat{x}_0 \tag{5}$$

$$\chi_i = \hat{x} + \left(\sqrt{(n+\lambda)P}\right)_i^T, \; i = 1, \ldots, n \tag{6}$$

$$\chi_{(i+n)} = \hat{x} - \left(\sqrt{(n+\lambda)P}\right)_i^T, \; i = 1, \ldots, n \tag{7}$$

$$W_0^{(m)} = \frac{\lambda}{(n+\lambda)} \tag{8}$$

$$W_0^{(c)} = W_0^{(m)} + (1 - \alpha^2 + \beta) \tag{9}$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)}, \; i = 1, \ldots, 2n \tag{10}$$

where $(\sqrt{(n+\lambda)P})_i$ represents the i^{th} row (or column) of a cubic matrix calculated by Cholesky decomposition. λ is a scale factor $(\alpha^2(n+k) - n)$; α determines the sigma point dispersion around the mean within the range of one and zero. Another scale is k, which is usually considered as zero. β considers pre-knowledge of the distribution of x in the problem (for Gaussian distribution, 2 is the optimum value). $W_i^{(m)}$ and $W_i^{(c)}$ are the mean and covariance weights of the ith point, respectively. Transformed sigma points were obtained after passing sigma points through the nonlinear function. As a result, prior-estimation means vector $\hat{x}_k^-$ and prior-estimation covariance P_k^- were calculated as follows:

$$\hat{x}_k^- = \sum_{i=0}^{2n} W_i^{(m)} f\left((\chi_k^-)_i\right) \quad (11)$$

$$P_k^- = \sum_{i=0}^{2n} W_i^{(c)} \left[f\left((\chi_k^-)_i\right) - \hat{x}_k^- \right] \times \left[f\left((\chi_k^-)_i\right) - \hat{x}_k^- \right]^T + Q_k \quad (12)$$

where Q_k denotes the process noise covariance matrix and represents the uncertainty in the system states. The mean and covariance of measurement are as follows:

$$\hat{z}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \left(h(\chi_k^-)_i \right) \quad (13)$$

$$P_{vv} = \sum_{i=0}^{2n} W_i^{(c)} \left[h(\chi_k^-)_i - \hat{z}_k^- \right] \times \left[h(\chi_k^-)_i - \hat{z}_k^- \right]^T + R_k \quad (14)$$

where $h(\chi_k^-)_i$ is the transformed sigma point in the measurement space; χ is the sigma point matrix; $\hat{z}$ is the mean of measurement; P_{vv} is the measurement covariance matrix; R_k is the measurement noise, and h is the measurement function that maps the sigma points to the measurement space. The cross-correlation between the sigma points in the state and measurement space has been used to calculate the *UKF* prediction error. For this, the cross-correlation matrix and Kaman gain are as follows:

$$P_{xz} = \sum_{i=0}^{2n} W_i^{(c)} \left[f\left((\chi_k^-)_i\right) - \hat{x}_k^- \right] \times \left[h(\chi_k^-)_i - \hat{z}_k^- \right]^T \quad (15)$$

$$K = P_{xz} P_{vv}^{-1} \quad (16)$$

Finally, the posterior mean value $\hat{x}_k$ and the covariance matrix P_k are computed, and the prediction step is finished.

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - \hat{z}_k^-) \quad (17)$$

$$P_k = P_k^- - K_k P_{vv} K_k^T \quad (18)$$

in which z_k is the real measurement. Although the noise covariance matrix in the original formulation of UKF is assumed to be fixed, in this study, the Robbins-Monro method has been used to update the covariance matrix. For more information, the reader please refer to the reference [24].

2.3. Forward Finite Difference Method (FFD)

The forward Finite Difference method (FFD) is a standard method used for sensitivity analysis. Despite its defects, the method is regularly used for its simplicity. In this method, the changes in outputs are investigated by introducing a perturbation and applying it to a given parameter. According to [21], the amount of perturbation could affect the results. The accuracy of results improves as this value is reduced, but further decreases deteriorates the results, which is due to the rounding of the error. This is known as the "perturbation size dilemma" in the literature.

2.4. Sensitivity Matrix-Based Analysis for Prioritizing Parameters Estimability

Yao et al. [23] proposed a sensitivity-based/orthogonal-based method for parameter estimability analysis. Sensitivity analysis is a method for investigating changes in outputs caused by changes in the value of a given system parameter. The matrix elements in the sensitivity matrix-based method are equal to the ratio of output changes to the changes of each system parameter in a given time step. This method can investigate two topics: the first is determining the parameters that cause the most sensitivity in the output, and the second is determining whether there is any correlation between the selected subset of parameters. The steps of the methods are as follows:

1. Calculating the sensitivity matrix elements, Z by Equation (19)
2. Choosing, as the first identifiable parameter, the parameter whose column's elements has the greatest sum of squares value;
3. Marking the preceding column as X_L, where $L \in \{1, \ldots, n_p\}$;
4. $\hat{Z}_L = X_L (X_L^T X_L)^{-1} X_L Z$ is used to calculate matrix $\hat{z}_L$;
5. Calculating the residual matrix R_L where $R_L = Z - \hat{Z}_L$;
6. Selecting, as the next identifiable parameter, the parameter whose column's elements has the greatest value of the summation of squares of R_L;

7. In step 6, augment X_L with the corresponding column of Z of the selected parameter. The augmented matrix is denoted as X_{L+1};
8. Repeating steps 4 to 7 until $L = n_p$.

The sensitivity matrix is defined as:

$$Z = \left.\frac{\partial y_i}{\partial \theta_j}\right|_{t=t_k} = \begin{bmatrix} \left.\frac{\partial y_1}{\partial \theta_1}\right|_{t=t_1} & \cdots & \left.\frac{\partial y_1}{\partial \theta_P}\right|_{t=t_1} \\ \vdots & \ddots & \vdots \\ \left.\frac{\partial y_R}{\partial \theta_1}\right|_{t=t_1} & \cdots & \left.\frac{\partial y_R}{\partial \theta_P}\right|_{t=t_1} \\ \left.\frac{\partial y_1}{\partial \theta_1}\right|_{t=t_2} & \cdots & \left.\frac{\partial y_R}{\partial \theta_P}\right|_{t=t_2} \\ \vdots & \ddots & \vdots \\ \left.\frac{\partial y_R}{\partial \theta_1}\right|_{t=t_N} & \cdots & \left.\frac{\partial y_R}{\partial \theta_P}\right|_{t=t_N} \end{bmatrix} = \begin{bmatrix} z_{1,1} & \cdots & z_{P,1} \\ \vdots & \ddots & \vdots \\ z_{1,R\times N} & \cdots & z_{P,R\times N} \end{bmatrix} \quad (19)$$

where y_R is the Rth output of the system, y_P is the Pth parameter, and t_N is the Nth time step. Central finite difference is used to calculate each element of the sensitivity matrix.

$$z_{i,j}(t_k) = \frac{y_i(\theta_j + \Delta\theta_j, t_k) - y_i(\theta_j - \Delta\theta_j, t_k)}{2\Delta\theta_j} \times \theta_j \quad (20)$$

the parameter $\Delta\theta_j$, is the size of the perturbation that can affect the results. Baker et al. [25] proposed the value of $\Delta\theta_j$ to be equal to the square root of the error covariance matrix of θ_j. In the present study, a small ratio (0.001) of θ_j is assumed for $\Delta\theta_j$.

3. Modeling and Analysis

3.1. Case Study: A Nonlinear SDOF System

The goal of this section is to prioritize estimability of the parameter. For this purpose, a nonlinear *SDOF* system is studied in four different cases. Figure (1) depicts the model of the SDOF system. The Bouc-Wen hysteresis model [26] is used for nonlinear behavior of this system, which is a mass-spring-damper.

The differential equation of motion for the above system is defined as follows:

$$m\ddot{x} + c\dot{x} + kr = -m\ddot{x}_g \quad (21)$$

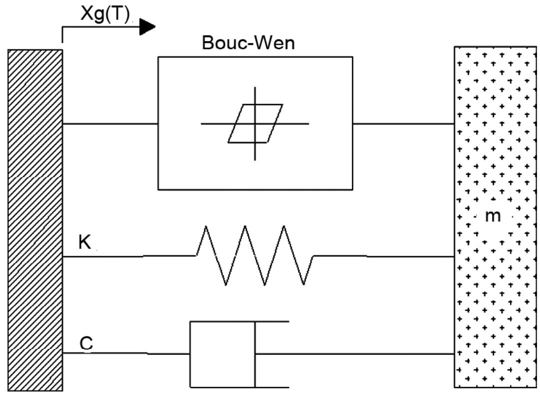

Figure 1. SDOF Model with nonlinearity.

$$\dot{r}(t) = \dot{x} - \beta|\dot{x}|\,|r|^{n-1} r - \gamma(\dot{x})|r|^n \quad (22)$$

where $r(t)$ is the hysteretic displacement vector; and γ, β, and n represent the Bouc-Wen hysteresis parameters [26]. By merging the above equations and transforming them to classical state-space, the following equation is achieved:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{r} \\ \dot{k} \\ \dot{c} \\ \dot{m} \\ \dot{\beta} \\ \dot{\gamma} \\ \dot{n} \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{z_5}{z_6} \times z_2 - \frac{z_4}{z_6} \times z_3 \\ z_2 - z_6 \times |z_2| \times z_3 |z_3|^{z_8 - 1} - z_7 \times z_2 |z_3|^{z_8} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ w - \ddot{x}_g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (24)$$

These equations can be rewritten as follow:

$$\dot{z} = f(z) - \ddot{x}_g + w \quad (25)$$

$$y = h(z) + v \quad (26)$$

where f and h are the functions of the state variables; z is the state vector; y is the observation vector; $\ddot{x}_g$ is the earthquake acceleration; w is the process noise; v is the measurement noise. The vector of state and parameters has the following form:

$$z = \left[x \ \dot{x} \ r \ k \ c \ m \ \beta \ \gamma \ n\right]^T \tag{27}$$

where k is the stiffness of the system; c is the damping; β, γ, and n are the parameters defining the system's nonlinear behavior (i.e., $\theta = \left[k \ c \ m \ \beta \ \gamma \ n\right]^T$). The vector of true values of parameters is assumed as $\theta_T = \left[12 \ 0.5 \ 1 \ 2 \ 1 \ 2\right]^T$. The process error covariance matrix for augmented state and parameters vector is $diag[(10^{-4})^2] \epsilon R^{9\times 9}$, and the measurement error covariance is equal to 5×10^{-3}. The considered SDOF system is analyzed under the El-Centro earthquake with a frequency rate of 50 Hz. Euler's method with a time step of 10^{-4} is used to solve the equation of motion. Following the simulation of the system with the actual values, the system's output (mass displace-ment) is recoded and polluted by white noise with the mentioned statistical properties. P_{x0} and $\hat{x}_0$ are $diag[0.25(\hat{x}_0)^2] \in R^{9\times 9}$ and $\left[0 \ 0 \ 0 \ 0.6\times12 \ 0.3\times0.5 \ 0.8\times1 \ 1.4\times2 \ 0.6\times1 \ 0.8\times2\right]^T$, respectively.

Case 1: System with six unknown parameters. It is assumed that all system parameters are unknown in the first step, and UKF is applied to estimate the value of each of the six parameters. UKF is presented in section ý2.2, and all of its requirements are mentioned there. Estimated values are not constant and will change in each iteration due to the random nature of the imposed measurement noise. Figure (2) to Figure (4) show the results of the identification process. The estimation time histories of three state parameters (i.e., displacement, velocity, and restoring force) are depicted in Figure (2). Regarding the results, the filter can precisely estimate state variables in each time step. Figure (3) represents the estimation time histories of the system parameters, whereas the filter is unsuccessful in converging to the true values. In Figure (4), the restoring force calculated based on the last estimated values of parameters is plotted versus the estimated displacement. The estimated hysteresis loop is not similar enough to the true. It shows that the variables are not chosen properly for system identification.

Parameter estimability analysis for the nonlinear SDOF system is performed using a code developed in MATLAB. For this purpose, the sensitivity matrix

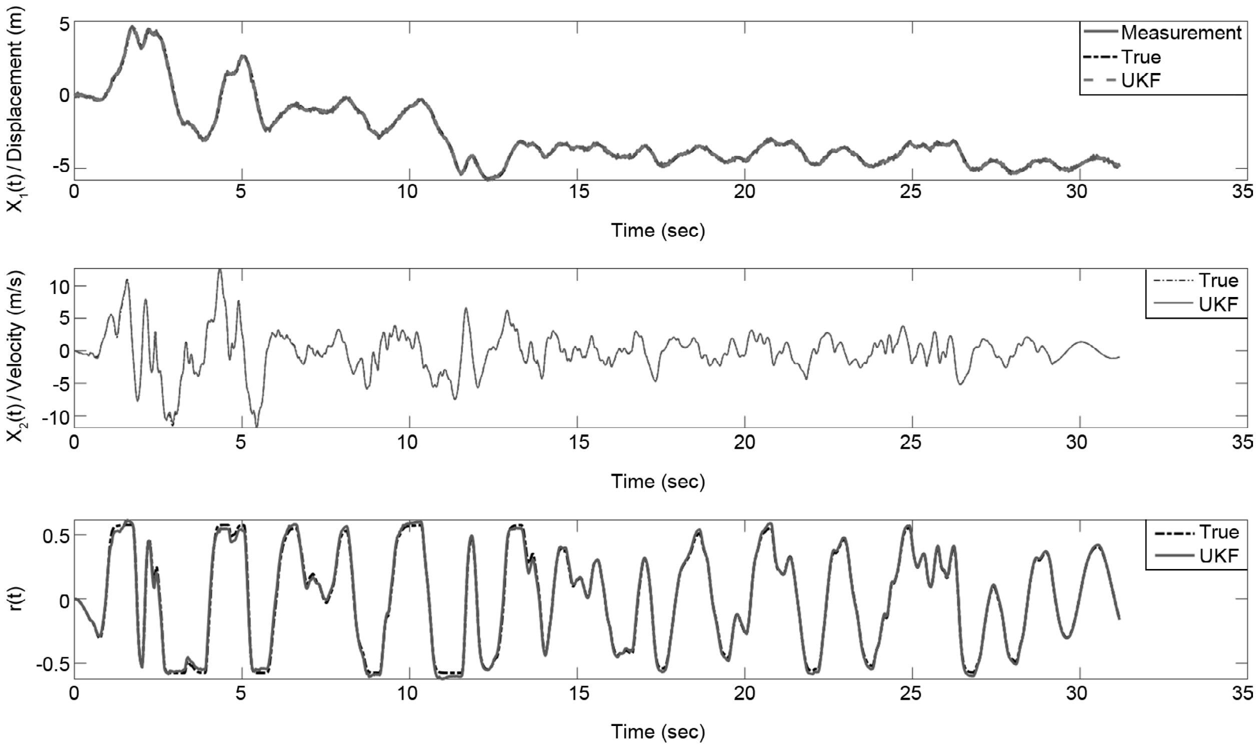

Figure 2. States estimation of the nonlinear SDOF: mass displacement response; mass velocity response; and the restoring force (ordered from top to bottom).

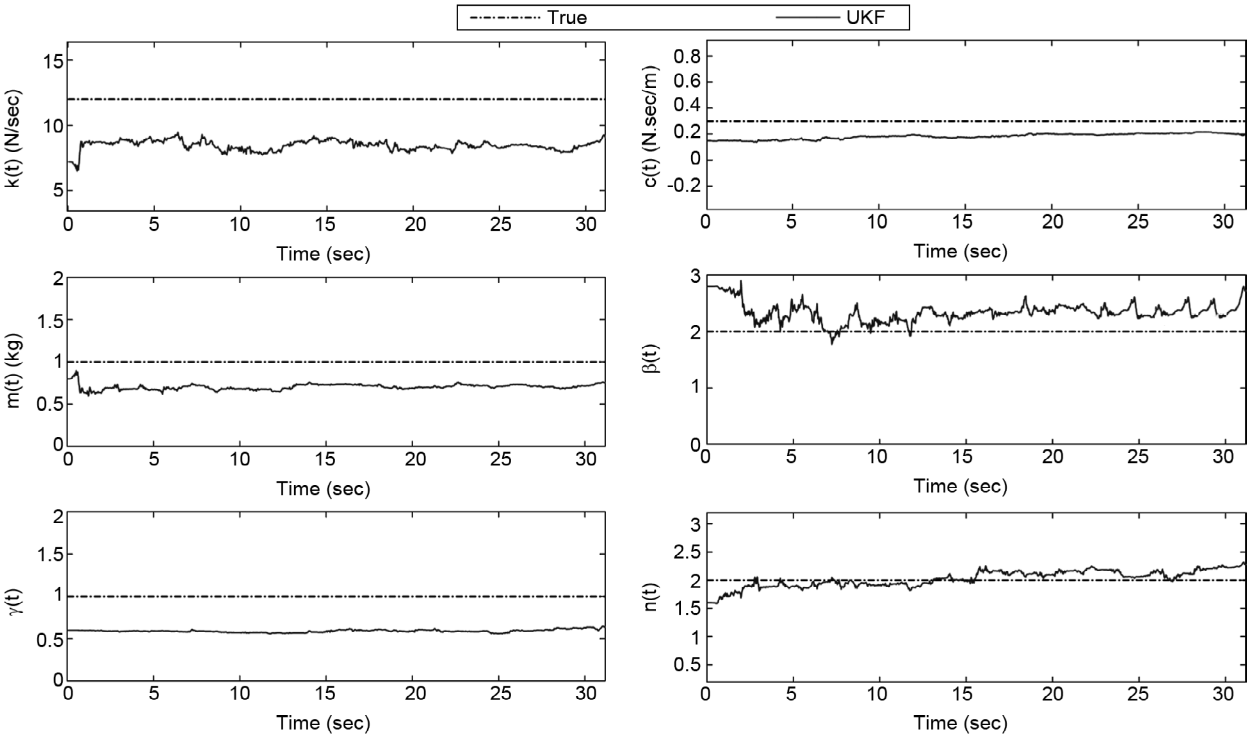

Figure 3. The system parameter estimation time-histories..

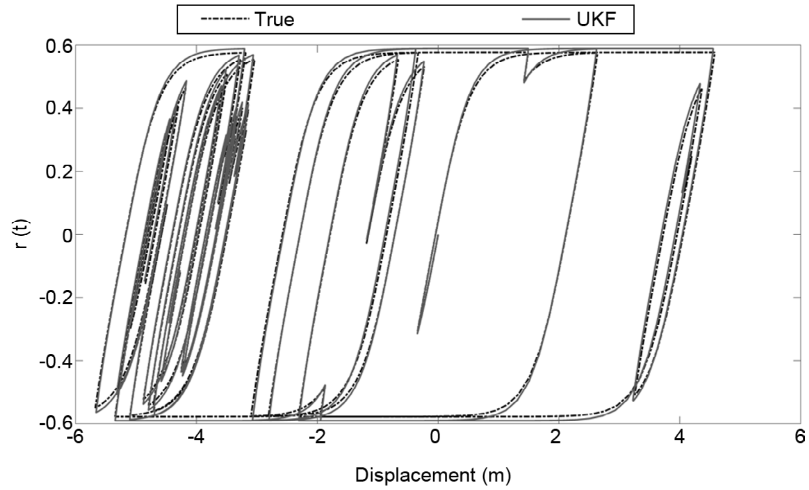

Figure 4. Comparing estimated and true hysteresis loops.

is calculated according to Equation (19); the column corresponding to the mass has the greatest sum of squares; therefore, it is the most estimable parameter. By following steps (3) to (8) other system parameters are prioritized. The result is shown in Table (1). Parameter estimability was also calculated via FFD to validate the sensitivity matrix-based method results.

Table 1. Parameter estimability ranked by sensitivity matrix-based analysis of the nonlinear SDOF.

Parameter	m	n	c	β	γ	k
Priority	1	2	3	4	5	6

The sensitivity based-matrix can determine the linear relation between the given parameters. The mass, *m*, of the *SDOF* has the highest rank, while stiffness, k, has the lowest (Table 1). Following the results of each iteration of the sensitivity matrix-based analysis (step 4 to 7), it was discovered that the stiffness and mass of the system are two correlated parameters (Table 2). As shown in Figure (3) and Figure (4), the presence of both of these parameters in the identification process results in a significant error. In other words, when there are some correlated parameters in an identification problem, the filter cannot converge

Table 2. Parameter estimability ranked by sensitivity matrix-based analysis of the nonlinear SDOF.

Parameter	k	c	m	β	γ	n
1st Iteration	8.787E+03	2.109E+03	1.868E+04	1.393E+03	1.418E+02	2.454E+03
2nd Iteration	1.812E+02	1.812E+02	1.496E-27	4.028E+02	1.592E+00	1.078E+03
3rd Iteration	9.910E+01	9.911E+01	3.411E-27	1.395E+01	1.093E+00	3.614E-28
4th Iteration	7.280E-07	1.780E-26	2.355E-25	7.491E+00	4.382E-01	1.552E-26
5th Iteration	7.243E-07	5.881E-25	5.366E-24	4.892E-25	7.358E-02	8.917E-25
6th Iteration	5.585E-07	4.836E-23	4.967E-22	3.551E-23	3.861E-24	5.552E-23

to the true values of parameters. The results of *FFD* are presented in Figure (5).

The *FFD* ranking, as shown in Figure (5), is identical to the sensitivity matrix-based method; however, the position of the stiffness parameter is different, and stiffness has the second rank. The linear relationship between mass and stiffness is not recognizable, as expected in the FFD method.

In the following, instead of noise randomness, identification outputs are obtained by averaging over a 100-Monte-Carlo simulation. As shown in Figure (6), the mean of identification results is not satisfying.

Case 2: System with five unknown parameters. As pointed above, mass and stiffness are two correlated parameters. Since the mass parameter has the lowest inherent uncertainty among other parameters correlated to stiffness, an accurate estimation of that can be reached through engineering knowledge and is therefore excluded from the identification process.

Therefore, in Case 2, the mass of the system is assumed as its true value; the other conditions and processes are the same as case 1. As shown in Figure (7), parameter identification is more satisfactory than it was in case no. 1 (Figure 6). Almost three of the top-ranked parameters can be found by the *UKF*, but two of them cannot be identified yet.

Two other cases are studied as follows. In these cases, the influence of elimination or participation of parameters with low estimability on the quality of the identification procedure is evaluated. In every case, by eliminating each parameter, the designated value is identical to the initial guess in case 1.

Case 3: System with four unknown parameters. Here, the parameter with the lowest estimability rank, i.e., γ, is taken out of the identification process.

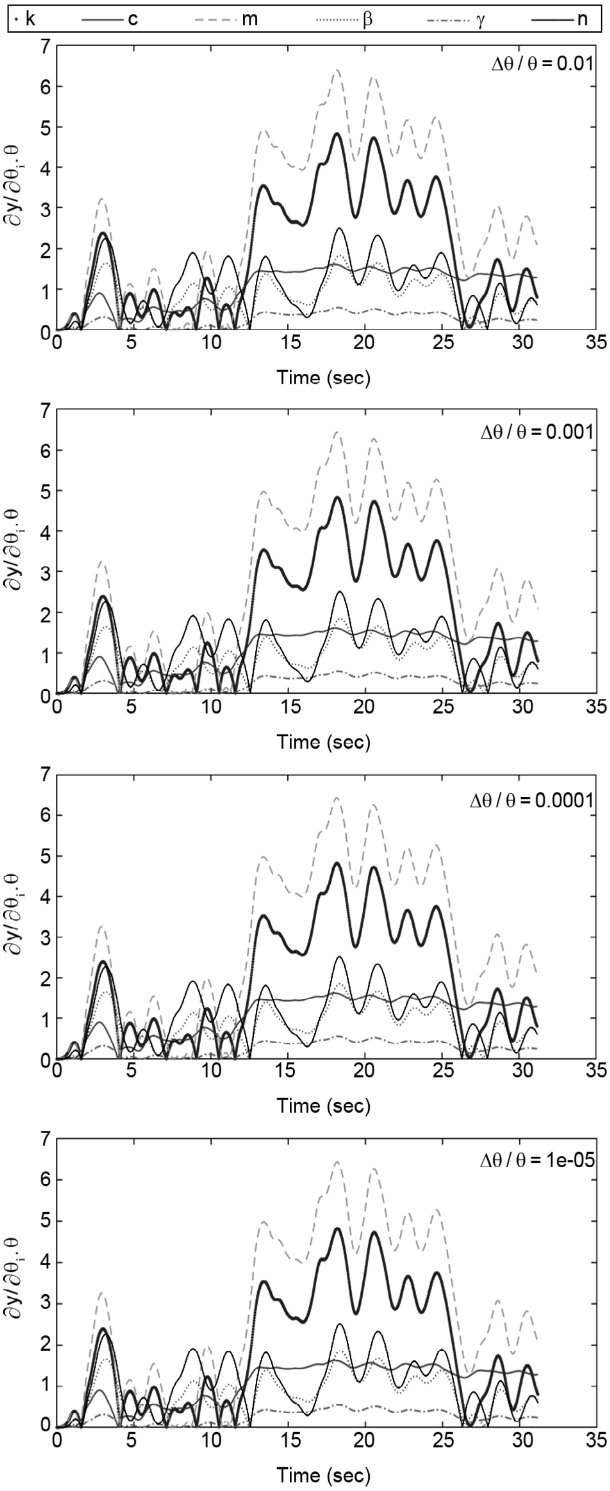

Figure 5. Relative importance of nonlinear system parameters (based on FFD).

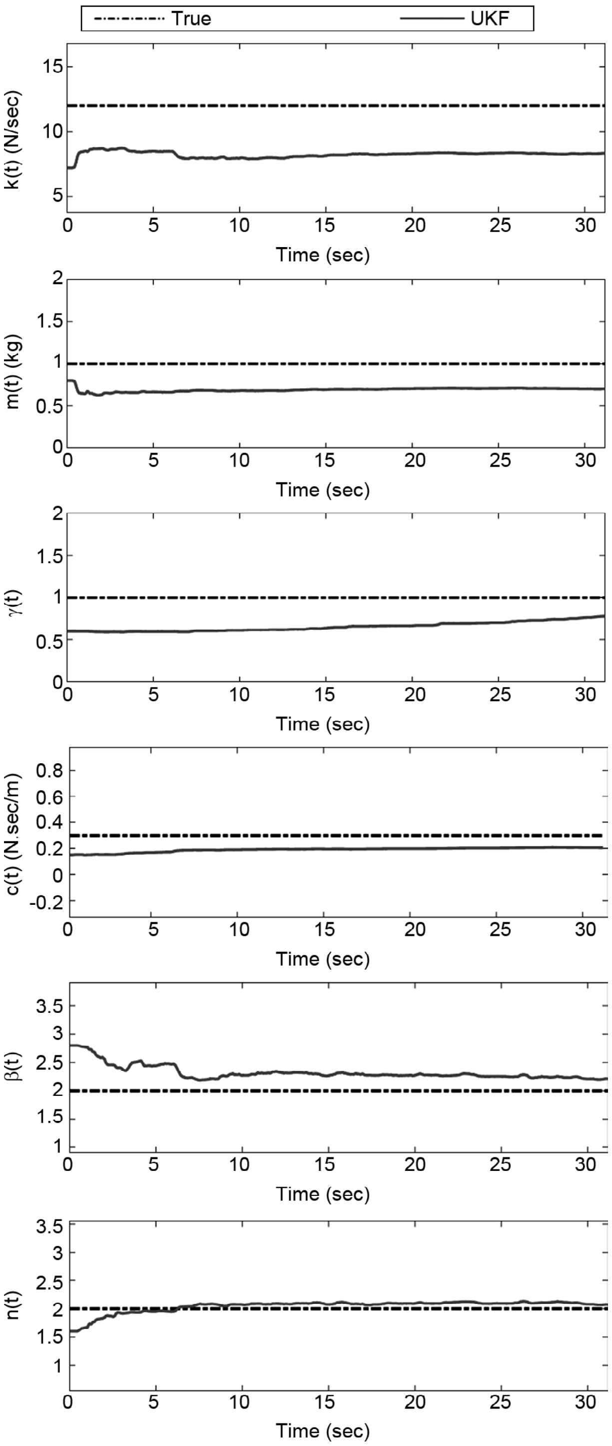

Figure 6. Mean of the identification results of 100 Monte-Carlo simulations for case 1.

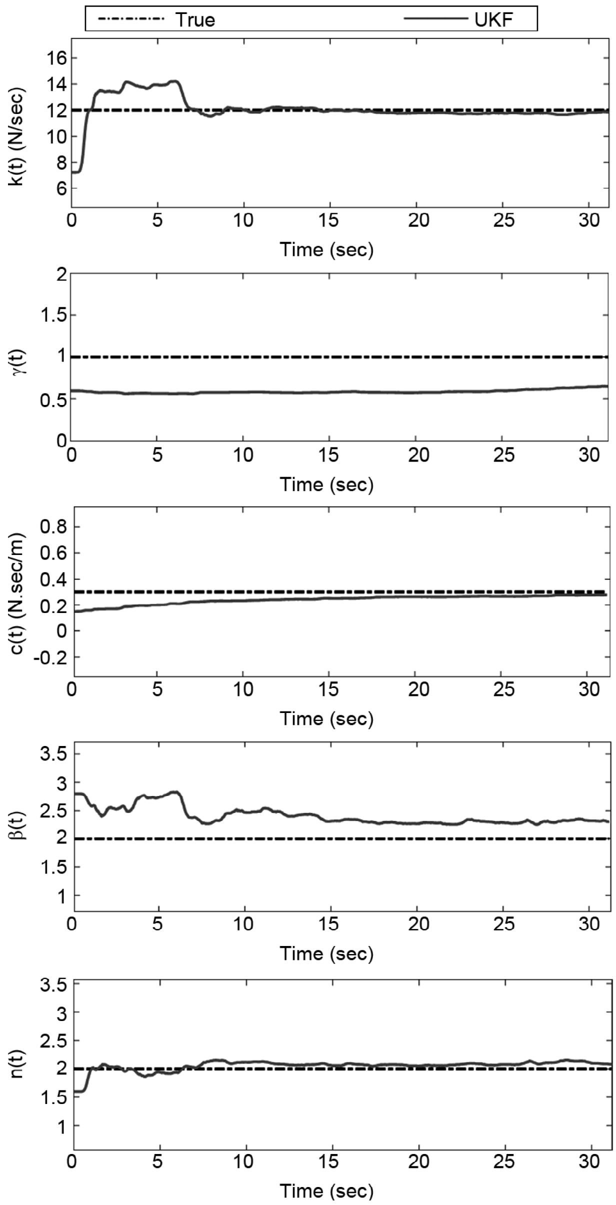

Figure 7. Mean of the identification results of 100 Monte-Carlo simulations for case.

In terms of the preceding point, its value is equal to 60 percent of the actual value. The results show that removing this parameter from the identification process has no effect on the quality of the estimation of other parameters (Figure 8).

Case 4: System with three unknown parameters. In this step, it is assumed that the parameter, β, in addition to γ, are taken out of the identification process. However, incorrect values of these parameters have been given to the filter. It should be noted that removing both of these parameters from the identification has no discernible effect on the outcome (Figure 9). Therefore, it can be concluded that, in an identification problem, parameters with low estimability cannot affect the results.

Table (3) shows the mean of the identified parameters for the four cases. There is a significant jump in the estimation precision from Cases 1 to Case 2; bringing the mass out of the estimation process, raised the accuracy. Based on the results of other cases, dropping two parameters with the lowest rank in the identifiability analyses (β and γ), has almost no effect on the results. Table (4) summarizes the standard deviation (STD) of the

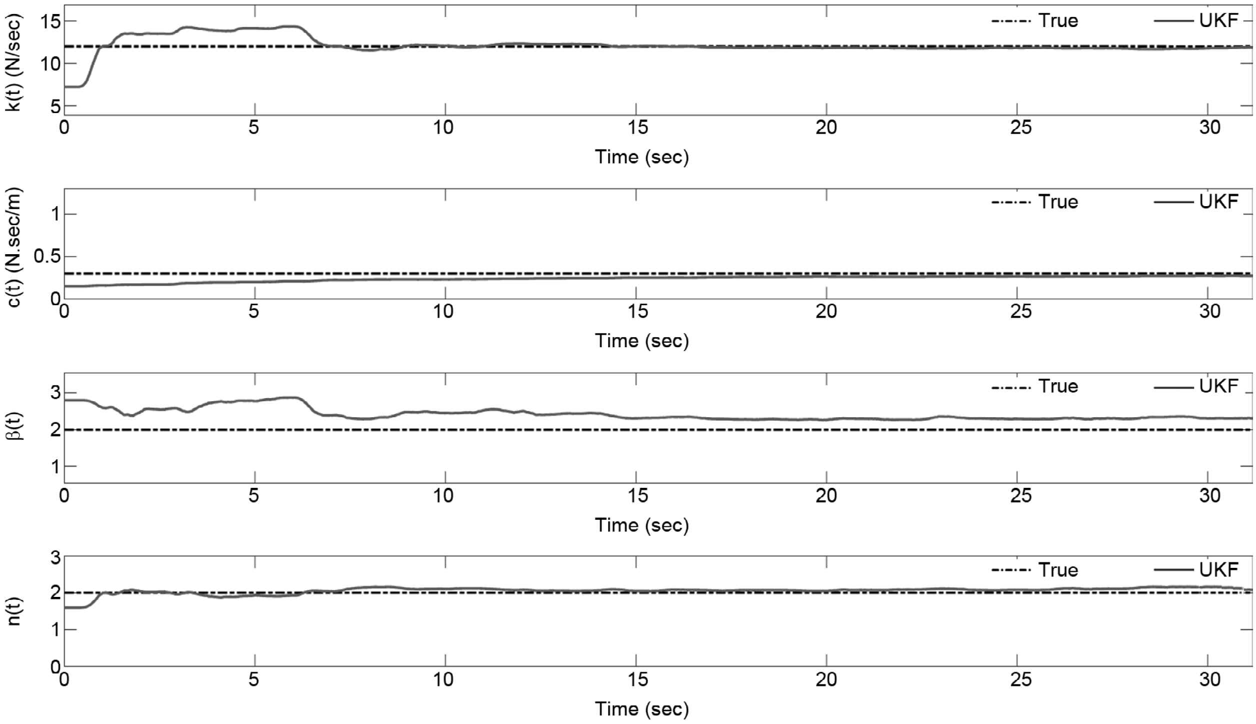

Figure 8. Mean of the identification results of 100 Monte-Carlo simulations for case 3.

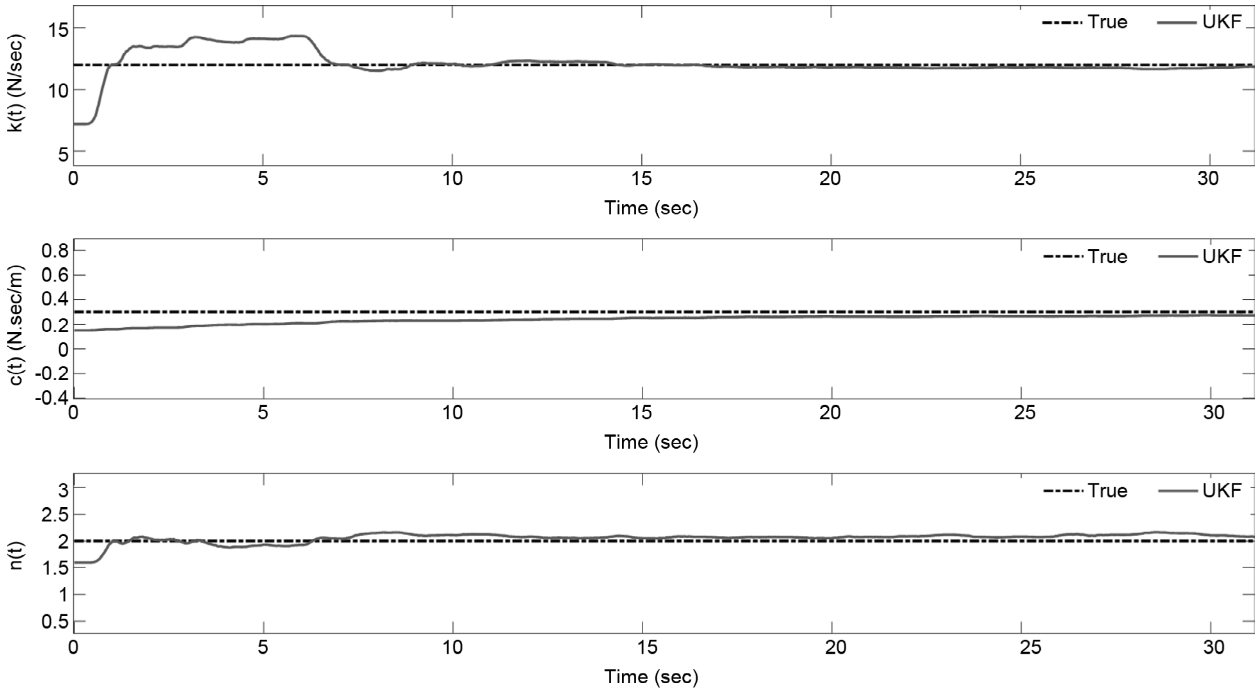

Figure 9. Mean of the identification results of 100 Monte-Carlo simulations for case no. 4.

identified parameters; the STDs are decreased as the number of system parameters is reduced. This shows that the estimation is more reliable when the number of estimating parameters is limited to the estimable parameters. The Schmidt-Kalman filter [27] is proposed to compensate for the reduced uncertainties caused by excluding parameters with low estimability. This, however, is beyond the scope of this paper. Figure (10) depicts the variations of the estimation error for the three most identifiable parameters in all cases. In addition to the El-Centro earthquake, system identification has been done using four other earthquake records. As shown in Figure (10), the results of other ground motions follow the same pattern. As a result, dropping parameters with low estimability does not interfere with the identification process.

This section's results can also be used to establish a threshold of identifiable parameters. The point will be a sensitivity matrix cutoff value that halts the prioritizing algorithm. The aim of this article is not to put forth an amount for this threshold. Furthermore, as presented in Table (5), the identification time of 15% is decreased just by

Table 3. Mean of the identified value for the system parameters in case 1-4.

Parameter	True Value	Case 1	Case 2	Case 3	Case 4
		Mean of Final Estimation			
K	12	8.35	11.877	11.88	11.81
C	0.3	0.21	0.277	0.276	0.285
M	1	0.7	N/C	N/C	N/C
β	2	2.21	2.276	2.31	N/C
γ	1	0.78	0.642	N/C	N/C
n	2	2.08	2.067	2.07	2.15

*N/C; Not Calculated.

Table 4. Standard deviation of the identified value for the system parameters in cases 1-4.

Parameter	True Value	Case 1	Case 2	Case 3	Case 4
		STD of Final Estimation			
K	12	10.19	3.488	3.17	3.05
C	0.3	0.262	0.273	0.27	0.26
M	1	0.86	N/C	N/C	N/C
β	2	2.79	3.876	3.81	N/C
γ	1	1.38	1.444	N/C	N/C
n	2	1.69	2.189	2.17	0.9

*N/C; Not Calculated.

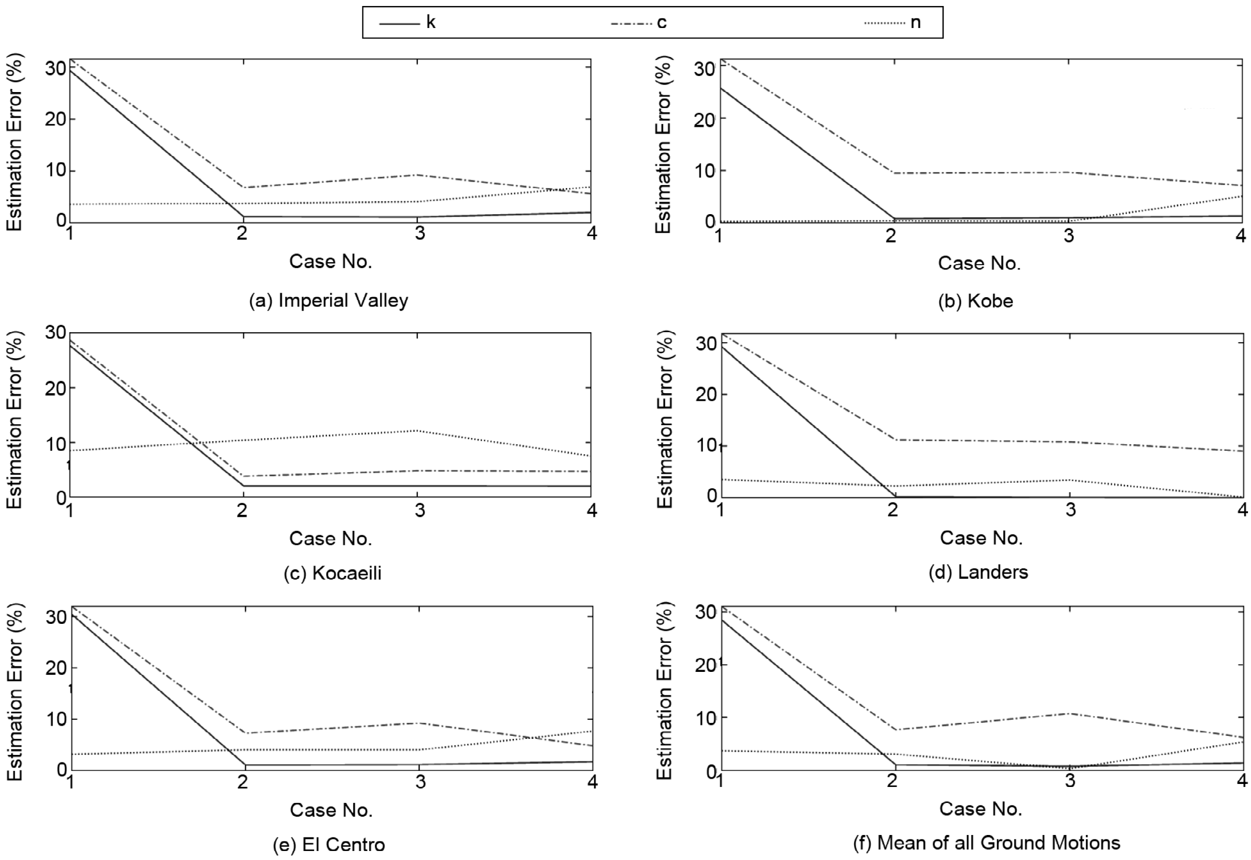

Figure 10. Variation of estimation error for three most estimable parameters in all cases (the results obtained by 100 Monte-Carlo simulations).

Table 5. The identification time spent for four cases.

Case	1	2	3	4
Time Spent (sec.)	259.6	219.096	203.76	167.14

excluding the mass parameter from the identification process. In the same way, reducing the number of identifiable parameters decreases the identification time in the third and fourth cases by 21% and 34%, respectively.

4. Conclusion

Structural safety assessment is the main issue after a strong earthquake. Model updating by field measurements is one of the promising methods for post-earthquake assessment in the aftermath of a seismic event. Identifying the number of parameters based on measurements is an important part of model updating that can reduce calculation costs and errors. This paper describes a method for prioritizing the parameters of structural systems that can detect linear correlations between the parameters. The method was applied to a nonlinear *SDOF*. The *SDOF* system was studied in four different scenarios. In the first one, the filter could not converge to the truth when all system parameters involved in the identification problem. However, the matrix-based sensitivity method illustrated the linear relation between mass and stiffness parameters. Hence, in the second scenario, according to engineering wisdom, the mass parameter is assumed to be known. Finally, in the two last scenarios, omitting the parameters with low estimability was studied. The following conclusions have been achieved:

Estimability and sensitivity are inter-related concepts. In other words, parameters with greater sensitivity are more easily identified.

Selected parameters that are to be updated by measurements must be uncorrelated. Matrix-based sensitivity analysis clearly reveals the correlation between system parameters in a straightforward manner; however, this is an ambiguous process in traditional methods, such as *FFD*.

In the identification process, involving parameters with an estimability magnitude less than a given bound exerts no discernible influence on the identification results. Although leaving such parameters out of the analysis reduces identification time without sacrificing identified mean precision, it also reduces the final identified variances. As stated, the uncertainty of identified parameters is incorrectly reduced by a small amount, which could be taken into account by methods such as *SUKF*. In summary and based on the results, in identification problems by Kalman Filter method, it is proposed to focus just on some most estimable parameters and forget about others. It is planned to have an experimental test in the future, to verify the method, where the lack of knowledge about the measurement noise and model uncertainties are real and probably effective.

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