Technical Note

A Simplified Dynamic Analysis of Symmetric Tube Structures Based on Hamilton's Principle

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ABSTRACT

In this paper, a mathematical model is presented for free vibration analysis of symmetric plan framed tube systems in tall buildings. Then, a closed form solution is derived for obtaining natural frequencies of framed tube structures. The analysis is based on continuum approach in which the framed tube structure is idealized as composed of four equivalent orthotropic plate panels. Therefore, framed tube structure is replaced by an idealized cantilever continuum representing the structural characteristics. Using the analytical method based on the Hamilton's principle and theory of differential equation by considering boundary conditions and normalization of parameters, the governing equation for free vibration of the problem is developed, and the corresponding eigenvalue equation is then derived. A theoretical method of solution is proposed to solve the eigenvalue problem, and a general solution is given to determine the natural frequencies of the framed tube structure. By following the proposed calculation procedure, frequencies of the free vibration are quickly determined. The proposed method for predicting the natural frequencies of framed tube structures is shown to give good agreement with those obtained from computer analysis; thus, the proposed method offers a simple and efficient, yet accurate, means for free vibration analysis of framed tube systems in tall buildings.

Keywords:
Tall building; Framed Tube; Hamilton's principle; Free vibration; Natural frequencies

1. Introduction

For structures of extreme height, it is economical to utilize the exterior shell of the structure to resist lateral loads. Rather than individual elements providing lateral stiffness in a flexural or shear mode, the framed tube incorporates the entire building plan in resisting lateral load. Upon lateral loading, one-half of the perimeter columns will be in tension and the other half in compression. The primary characteristic of a tube is the employment of closely spaced perimeter columns interconnected by deep spandrels, so that the whole building works as a huge vertical cantilever to resist overturning moments [1-2].

Besides its structural efficiency, framed tube buildings leave the interior floor plan relatively free of core bracing and heavy columns, enhancing the net usable floor area thanks to the perimeter framing system resisting the whole lateral load. This system was first used for the Dewitt-Chesmut Apartment Building in Chicago in 1963. It has been used successfully in buildings upwards of 70-80 stories including the 82 storey Standard Oil Building.

Many researchers have studied static and dynamic behaviour of framed tube systems in past decades. Approximate static analysis of framed tube systems based on equivalent orthotropic tubes were presented by Coull and Bose [3-5]; Coull and Ahmed [6];

Ha et al [7]; Foutch and Chang [8]; Takabatake et al [9] and Kwan [10]. They replaced each tube grid with an orthotropic plate with properties that represent the appropriate stiffness of the beams and columns. In the past decades, a considerable amount of research works [11-29] have been done on the dynamic and static analyses of planer and symmetric structures in tall buildings in which static and dynamic characteristics (e.g. stress, deformation, frequency, mode shape, etc.) were calculated.

In this paper, natural frequencies of free vibration of framed tube systems are obtained. Using continuum modeling based on equivalent orthotropic membranes, which presented by Kwan [10] and then applying Hamilton's principle, governing equation of motion and boundary conditions are obtained. Then, calculating non-trivial solution of these equations, a closed form solution is presented that can be used for obtaining natural frequencies of framed tube structures. Through numerical examples, the accuracy of the suggested method is compared with computer analysis results, and the accuracy of the suggested method is demonstrated.

2. Flexural-Shear Vibration of Framed Tube Structures

2.1. Governing Equation of Free Vibration

The framed tube structure consists of a single tube but must be symmetrical about two axes. Additional assumptions or restrictions are given below:

1. All columns and girders must have equal properties on all sides and along the height of structure.
2. Story heights must be uniform.
3. All columns and girders must be of the same material.
4. Bay widths must be uniform horizontally and vertically.
5. All material behaves linear elastically.

The process of the proposed methodology is as follows:

i) Framed tube system is replaced by a continuous cantilever beam with hollow section [10];

ii) According to the replacement beam model, the characteristic stiffness of load-resisting system are applied in Hamilton's principle;

iii) The governing equation of motion and boundary conditions are obtained;

iv) The vibration frequencies of the building is calculated by obtaining non-trivial solution of equations and using separation of variables.

A framed tube structure, as shown in Figure (1), is subjected to lateral loading of intensity $w(x,t)$ along the axis of symmetry. It has uniformly distributed mass $m(x)$, shearing stiffness $GA(x)$, and flexural stiffness $EI(x)$ along the structural height H.

Using Hamilton's principle, the dynamic characteristics of the structure are governed by the following differential equation of motion [30-31]:

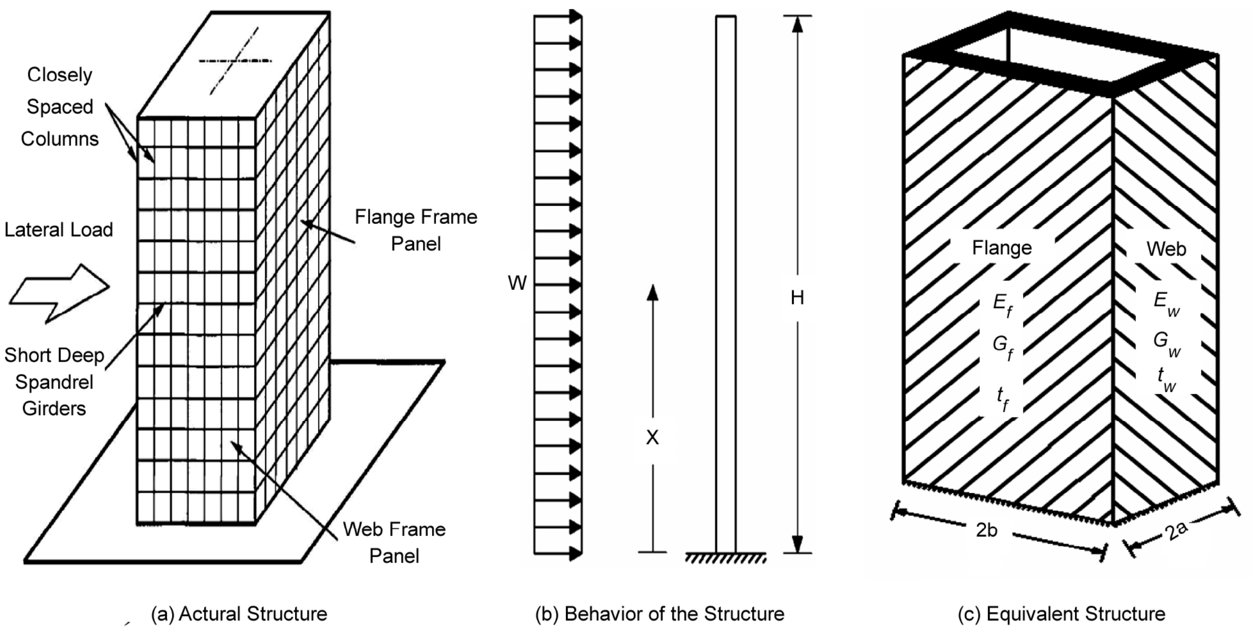

Figure 1. Framed tube system in tall building structures.

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial y^2(x,t)}{\partial x^2}\right)-\frac{\partial}{\partial x}\left(GA(x)\frac{\partial y(x,t)}{\partial x}\right)+ m(x)\frac{\partial y^2(x,t)}{\partial t^2}=w(x,t) \quad (1)$$

where $y(x,t)$ is the deflection at the height $x(0\le x\le H)$ and time t. Since the tall building structure can be considered as a vertical cantilever beam, with zero deflection and rotation at the base, and free (zero moment and shear) at the top, the corresponding boundary conditions are as follows:

$$y=0,\theta=\frac{\partial y}{\partial x}=0 \quad \text{at} \quad x=0$$
$$\frac{\partial^2 y}{\partial x^2}=0, \frac{\partial}{\partial x}\left(EI\frac{\partial^2 y(x,t)}{\partial x^2}\right)-GA\frac{\partial y}{\partial x}=0 \quad \text{at} \quad x=H \quad (2)$$

The governing equation for free vibration is obtained by letting $w(x,t)=0$ in Eq. (1).

2.2. Eigenvalue Problem

Since the motion in free vibration at any point is simple harmonic, and deflected shape is independent of time, y can be written as follows:

$$y(x,t)=y(x) \quad sin(\omega t) \quad (3)$$

where $y(x)$ and ω are mode shape and natural frequency, respectively. Substituting Eq. (3) into Eq. (1) and considering free vibration $w(x,t)=0$ and carried out necessary differential give:

$$\frac{\partial^2}{\partial \eta^2}\left(EI(\eta)\frac{\partial^2 y(\eta)}{\partial \eta^2}\right)-H^2\frac{\partial}{\partial \eta}\left(GA(\eta)\frac{\partial y(\eta)}{\partial \eta}\right)- \omega^2 H^4 m(\eta)\,y(\eta)=0 \quad (4)$$

with boundary conditions:

$$y=\frac{\partial y}{\partial \eta}=0 \quad \text{at} \quad \eta=0$$
$$\frac{\partial^2 y}{\partial \eta^2}=\frac{\partial}{\partial \eta}\left(EI\frac{\partial^2 y}{\partial \eta^2}\right)-H\;GA\frac{\partial y}{\partial \eta}=0 \quad \text{at} \quad \eta=1 \quad (5)$$

where $0\le(\eta=x/H)\le 1$ is the non-dimensional height coordinate. The dynamic analysis of framed tube structure has now been reduced to determining the values of parameter ω^2, for which homogenous linear differential equation (Eq. (4)) has nontrivial solutions $y(\eta)$ satisfying homogenous boundary conditions, i.e. the eigenvalue problems.

2.3. Analytical Solution of Uniform Framed Tube Structures

When values of m, EI and GA are constant for a uniform framed tube structure, Eq. (4) can be written as follows:

$$y^{IV}(\eta)-\xi^2 y''(\eta)-\gamma^2\omega^2 y(\eta)=0 \quad (6)$$

with boundary conditions:

$$y=y'=0 \quad \text{at} \quad \eta=0$$
$$y''=y'''-\xi^2 y'=0 \quad \text{at} \quad \eta=1 \quad (7)$$

where the structural characteristics parameters are as follows:

$$\xi^2=\frac{GA}{EI}H^2$$
$$\gamma^2=\frac{m}{EI}H^4 \quad (8)$$

The values of ξ are γ relative to the structural height, distributed mass, shear stiffness and flexural stiffness over height. They are the relative shear and flexural stiffness parameter, respectively. Letting:

$$y(\eta)=ce^{\kappa\eta} \quad (9)$$

and substituting into Eq. (6) leads to:

$$\kappa^4-\xi^2\kappa^2-\gamma^2\omega^2=0 \quad (10)$$

Its solution is:

$$\kappa^2=\frac{\xi^2}{2}\pm\sqrt{\gamma^2\omega^2+\frac{\xi^4}{4}}$$
$$\kappa_{1,2}=\pm\Re_1,\ \kappa_{3,4}=\pm i\Re_1, \quad (11)$$

where

$$\Re_1=\sqrt{\sqrt{\gamma^2\omega^2+\frac{\xi^4}{4}}+\frac{\xi^2}{2}}$$
$$\Re_2=\sqrt{\sqrt{\gamma^2\omega^2+\frac{\xi^4}{4}}-\frac{\xi^2}{2}} \quad (12)$$

Therefore, the solution $y(\eta)$ of Eq. (10) and its derivatives are as follows:

$$\begin{Bmatrix} y(\eta) \\ y'(\eta) \\ y''(\eta) \\ y'''(\eta)-\xi^2 y'(\eta) \end{Bmatrix} = f(\eta,\omega)\begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} \tag{13}$$

where

$$f(\eta,\omega)=\begin{bmatrix} Q_1(\eta) & Q_2(\eta) & Q_3(\eta) & Q_4(\eta) \\ \Re_1 Q_2(\eta) & \Re_1 Q_1(\eta) & -\Re_2 Q_4(\eta) & \Re_2 Q_3(\eta) \\ \Re_1^2 Q_1(\eta) & \Re_1^2 Q_2(\eta) & -\Re_2^2 Q_3(\eta) & -\Re_2^2 Q_4(\eta) \\ \Re_1 \Re_2^2 Q_2(\eta) & \Re_1 \Re_2^2 Q_1(\eta) & \Re_2 \Re_1^2 Q_4(\eta) & -\Re_2 \Re_1^2 Q_3(\eta) \end{bmatrix} \tag{14}$$

where

$$\begin{aligned} Q_1 &= \cosh(\Re_1 \eta) \\ Q_2 &= \sinh(\Re_1 \eta) \\ Q_3 &= \cos(\Re_2 \eta) \\ Q_4 &= \sin(\Re_2 \eta) \end{aligned} \tag{15}$$

Replacing all the boundary conditions in Eq. (13) gives:

$$\begin{Bmatrix} y(0) \\ y'(0) \\ y''(1) \\ y'''(1)-\xi^2 y'(1) \end{Bmatrix} = f(\omega)\begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{16}$$

where

$$f(\omega)=\begin{bmatrix} Q_1(0) & Q_2(0) & Q_3(0) & Q_4(0) \\ \Re_1 Q_2(0) & \Re_1 Q_1(0) & -\Re_2 Q_4(0) & \Re_2 Q_3(0) \\ \Re_1^2 Q_1(1) & \Re_1^2 Q_2(1) & -\Re_2^2 Q_3(1) & -\Re_2^2 Q_4(1) \\ \Re_1 \Re_2^2 Q_2(1) & \Re_1 \Re_2^2 Q_1(1) & \Re_2 \Re_1^2 Q_4(1) & -\Re_2 \Re_1^2 Q_3(1) \end{bmatrix} \tag{17}$$

then

$$C=-A;\; D=\frac{\Re_1}{\Re_2}B;\; B=-\frac{\Re_1^2 Q_1(1)+\Re_2^2 Q_3(1)}{\Re_1^2 Q_2(1)+\gamma\omega Q_4(1)}A \tag{18}$$

and a nontrivial solution for Eq. (16) can be obtained if the determinant $f(\omega)$ of coefficients is zero, i.e.

$$|f(\omega)|=\begin{vmatrix} Q_1(0) & Q_2(0) & Q_3(0) & Q_4(0) \\ \Re_1 Q_2(0) & \Re_1 Q_1(0) & -\Re_2 Q_4(0) & \Re_2 Q_3(0) \\ \Re_1^2 Q_1(1) & \Re_1^2 Q_2(1) & -\Re_2^2 Q_3(1) & -\Re_2^2 Q_4(1) \\ \Re_1 \Re_2^2 Q_2(1) & \Re_1 \Re_2^2 Q_1(1) & \Re_2 \Re_1^2 Q_4(1) & -\Re_2 \Re_1^2 Q_3(1) \end{vmatrix}=0 \tag{19}$$

from which the frequency equation can be obtained as follows:

$$1+\left(1+\frac{\xi^4}{2\gamma^2\omega^2}\right)Q_1(1)Q_3(1)+\frac{\xi^2}{2\gamma\omega}Q_2(1)Q_4(1)=0 \tag{20}$$

3. Numerical Examples

In this section, numerical examples are presented to demonstrate the accuracy and robustness of the approximate calculation. Two 40 and 60 storey reinforced concrete tall buildings are used to investigate free vibration analysis of a framed tube system. A simplified floor plan of the building is shown in Figure (2). All the beams and columns have a cross-section 0.8×0.8 m. The properties of the approximate model are summarized in Tables (1) and (2) (The method of approximate equivalent modeling is described in [10]).

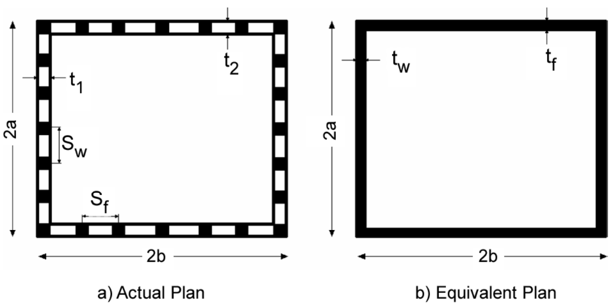

Figure 2. Framed tube's plan.

Table 1. Specifications of the 40 and 60 storey buildings for the test cases.

Plan Dimensions		Height of Storey	Space of Spans	
2a (m)	***2b*** (m)	***H*** (m)	s_w (m)	s_f (m)
30	35	3	2.5	2.5

Table 2. Properties of actual and equivalent structures.

Actual Structure		**Equivalent Membranes**			**Equivalent Membranes**		
Material		Web			Flange		
E (GPa)	***G*** (GPa)	E_w (GPa)	G_w (GPa)	t_w (m)	E_f (GPa)	G_f (GPa)	t_f (m)
20	8	20	1.726	0.256	20	1.726	0.256

Table 3. Comparison of the results between SAP2000 and the proposed approximate method form analysis of 40 and 60 storey buildings with framed tube system.

Number of Stories	**Number of Modes**	ξ	γ	**ω *(rad/sec)* (Proposed Method)**	**ω *(rad/sec)* (SAP2000)**	**% of Error in ω**
40	***1***	1.94	2.78	1.98	1.92	3
40	***2***	1.94	2.78	6.58	6.33	4
60	***1***	2.91	6.26	1.09	1.17	7
60	***2***	2.91	6.26	4.39	3.98	10

Using Eq. (20), the natural frequencies are calculated and compared with those obtained from SAP2000 (Advanced 14.0.0, Computers and Structures, Berkeley, California, USA.) free vibration analysis as shown in Table (3). The proposed approximate method estimates the natural frequencies by 10%. The proposed method shows a good understanding of structural behaviour, easy to use, yet reasonably accurate and suitable for quick evaluations during the preliminary design stages, which requires less time.

The main sources of errors between the proposed approximate method and the finite element method are as follows:

- ❖ All closely spaced perimeter columns tied at each floor level by deep spandrel beams are considered to form a tubular structure;
- ❖ Equivalencing the elastic properties of the framed tube such as *G*.

4. Conclusions

In this paper, a numerical solution of eigenvalues for free vibration analysis of framed tube system in tall buildings by using Hamilton's principle is presented. In comparison with finite element method, it shows that the advantages of availability and reliability of the proposed method can conveniently be taken for obtaining accurate and reliable natural frequencies of tall buildings with framed tube system. The presented closed form solution can compute the number of natural frequencies of the structure, but other numerical methods except finite element methods cannot, because the computations become correspondingly more complicated for the latter if high frequencies are needed. The presented method can save much computational efforts and time than the finite element method.

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