On the Essentiality of Techniques to Enlarge Integration Steps in Transient Analysis Against Digitized Excitations

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ABSTRACT

In transient analysis against digitized excitations, and specially, practical seismic analyses, the steps, by which, the strong ground motions are digitized, might be smaller than those recommended for the accuracy of integration. In these cases, in order to consider the total excitation information, the integration steps are to be set as small as the excitation steps. The result is extra computational cost. In the last decade, techniques are proposed, to provide the capability of time integration with larger steps, without disregarding the excitations information. Though the resulting responses are sufficiently accurate, a fundamental question persists. It is on the loss of accuracy, when we simply omit the inter-integration-step excitations. This paper is dedicated to this concern, in the area of seismic analysis. The study is carried out, through theoretical discussions and numerical examples. As the consequence, omission of inter-integration-step excitations may impair the responses, the inaccuracies can be significant, and even, halts of analyses are expectable, in nonlinear analyses. Implementation of techniques for enlargement of integration steps can lead to more efficient and more reliable analyses.

Keywords:
Time integration; Digitized excitations; Ground motion; Step size; Computational cost; Accuracy

1. Introduction

Time history analysis and time integration are of the most powerful tools to study structural behaviors, specifically when the excitations are digitized. For seismic analysis of a structural system, the conventional approach is to discretize the mathematical model, in space [1-2], analyze the semi-discretized model against several earthquake records and put the responses together, according to a seismic code, e.g. [3-6]. A typical semi-discretized model is noted below [1, 2, 7]:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{f}_{\text{int}}(t)=\mathbf{f}(t)$$

$$\text{Initial Conditions}:\left|\begin{array}{l}\mathbf{u}(t=0)=\mathbf{u}_0\\ \dot{\mathbf{u}}(t=0)=\dot{\mathbf{u}}_0\\ \mathbf{f}_{\text{int}}(t=0)=\mathbf{f}_{\text{int}_0}\end{array}\right. \qquad (1)$$

$$\text{Additional Constraints}:\mathbf{Q} \qquad 0\le t<t_{end}$$

In Eqs. (1), t and t_{end} imply the time and the duration of the dynamic behavior; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\text{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}(t), \dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$, denote the unknown vectors of displacement, velocity, and acceleration; $\mathbf{u}_0, \dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$, define the initial status of the model (see also [7]); and $\mathbf{Q}$ represents some restricting conditions, in nonlinear problems, e.g. additional constraints in problems involved in impact or elastic-plastic behavior [8, 9]. In seismic analyses, a main reason, for considering $\mathbf{Q}$, and specifically, $\mathbf{f}_{\text{int}}$, in Eqs. (1), is the hysteretic behavior of many materials, when subjected to severe excitations [7, 10]. As a versatile approach to analyze Eqs. (1) [11], time integration starts, with the selection of time steps or a criterion for adaptive time stepping [12]; and considering the initial conditions, and if needed, applying an starting procedure [13], proceeds, marching along the time axis, and computing the responses, for distinct consecutive time stations [14] (generally, together with nonlinearity iterations, at the detected nonlinearities [15]). The versatility originates in the simplicity of both the approach and the algebraic formulation; and the price of the versatility is the inaccuracy and considerable computational cost of the analyses [14, 16, 17]. Because of the approximate formulation and the step-by-step nature of the computation, the analysis cost and the accuracy of the responses both depend on Δt, that is a measure for uniformly scaling the distances between each two sequential integration stations [17]. Smaller values of Δt generally lead to more accuracy (in view of the essentiality of convergence [18, 19]) and more computational cost; see [14, 20, 21]. Accordingly, efforts towards more efficient analyses is in everyday progress (e.g. [22-24]), and it is conventional, to assign values to Δt, small enough to provide sufficient accuracy, and not smaller [6, 14, 16, 25]. The related main and broadly accepted comment is as stated below [6, 17, 25]:

$$\Delta t \le \mathrm{Min}\left(h, \frac{T}{10}, {}_f\Delta t \right) \qquad (2)$$

(for nonlinear problems, there are comments suggesting the replacement of the '10' in the denominator in the right hand side, with '100' or even '1000' [6, 26]). In Eq. (2), h stands for the largest step size guaranteeing numerical stability and consistency [14], T denotes the smallest dominating period of vibration (that when divided by 10, 100, or 1000, implies a measure to control the accuracy), and ${}_f\Delta t$ is the step size, by which, the digitized excitation is recorded [6, 17, 25, 27]. When the last term in Eq. (2) dominates, i.e. Eq. (2) leads to $\Delta t = {}_f\Delta t$, the analyses suffer from significant computational cost, to consider the excitation information completely. In order to implement the comment stated in Eq. (2), the convention is to simply obtain the step sizes from (for nonlinear problems after replacing the '10's in the denominators with '100' or '1000'):

Obviously, implementation of the second line in Eq. (3) implies partial disregard of the excitation. Several techniques are proposed in the last decade, directly or indirectly relaxing the probable effects of the excitation omission [17, 28, 29, 30]. The most successful technique is seemingly the convergence-based technique proposed in [17, 31, 32]. This technique omits the inter-integration-step excitations (as also addressed in the second line in Eq. (3)), and redefines the excitations at the integration stations, according to a convergence-based formulation. Compared to ordinary integration with excitation steps, implementation of the convergence-based technique, prior to the ordinary analysis, leads to considerably less computational cost, while the excitation information is completely taken into account, and the loss of accuracy is acceptable; see Table (1). Still, the essentiality of any step size enlargement technique is unclear. To say better, we need to check whether, simple omission of inter-integration-step excitations is indeed unreliable and implementation of step enlargement techniques is really needed. This is the main concern in this paper. In more detail, the objective here is to respond to two questions: (1) whether, simple omission of inter-integration-step excitations can be considered reliable, in the sense of leading to sufficiently accurate responses, and (2) whether, compared to ordinary time integration, with steps obtained from the second line in Eq. (3) (considered here as Technique 1), implementation of the technique proposed in [17], followed with ordinary analysis (afterwards addressed as Technique 2) is superior. In Section 2, the ambiguities are studied in brief. In Sections 3 and 4, Techniques 1 and 2 are compared numerically. Some complementary discussions are presented in Section 5, and finally, in Section 6, the paper is concluded, with a brief set of the achievements.

2. Theoretical Discussion

In view of Eqs. (2) and (3), provided we can assign a positive integer to n, satisfying:

$$\Delta t = \begin{cases} \dfrac{1}{n}\,{}_f\Delta t \;\; n: \dfrac{1}{n}\,{}_f\Delta t \le \mathrm{Min}\left(h, \dfrac{T}{10} \right) < \dfrac{1}{n-1}\,{}_f\Delta t, \; n = 2, 3, 4, \ldots \quad \text{when} \quad {}_f\Delta t > \mathrm{Min}\left(h, \dfrac{T}{10} \right) \\ n\,{}_f\Delta t \;\; n: n\,{}_f\Delta t \le \mathrm{Min}\left(h, \dfrac{T}{10} \right) < (n+1)\,{}_f\Delta t, \; n = 1, 2, 3, \ldots \quad \text{when} \quad {}_f\Delta t \le \mathrm{Min}\left(h, \dfrac{T}{10} \right) \end{cases} \qquad (3)$$

Table 1. Experiences on the computational cost reduction technique introduced in [17].

System Analyzed	Cost Reduced When the Loss of Accuracy is Pictorially Negligible (%)	Source
An SDOF System	75	[17]
A 2-DOF Nonlinear System	49.27	[17]
An Eight Story Shear Frame	80	[33]
A Thirty-Story Building	50	[34]
Buildings Subjected to 3-Component Earthquakes	66.7	[35]
A Silo	77.65	[36]
A Water Tank	66.7	[37]
Buildings in Pounding	12.7	[38]
A Bridge with Two Structural Systems in Separate Studies	45-80	[39, 40]
Residential Buildings with Structural Systems Regular in Plan and Height	50-87	[41]
A Power Station	>50	[42]
Space Structures	>50	[43]
Several Bridges of Different Types Also Subjected to Multi-Support Excitation	30-70	[44]
Residential Buildings with Structural Systems Regular in Plan and Irregular in Height	>50	[45]

$$\frac{\mathrm{Min}\left(h,\frac{T}{10}\right)}{\mathrm{Min}\left(h,\frac{T}{10},\ {}_{f}\Delta t\right)}-1<n\le\frac{\mathrm{Min}\left(h,\frac{T}{10}\right)}{\mathrm{Min}\left(h,\frac{T}{10},\ {}_{f}\Delta t\right)}\qquad n=2,3,4,... \tag{4}$$

the two techniques, i.e. Technique 1: simple omission of inter-integration-step excitations, and Technique 2: omission of inter-integration-step excitations and consistently changing the excitations at integration stations [17], can be implemented, prior to ordinary time integration analysis, with steps 2, 3, ...n times larger than those of the excitation; see Figure (1). In Figure (1),

$$\begin{aligned}
&t_i=0: && \tilde{\mathbf{f}}(t_i)=\mathbf{f}(t_i),\\
&0<t_i<t_{end}: && \tilde{\mathbf{f}}(t_i)=\frac{1}{2}\mathbf{f}(t_i)+\frac{1}{4n'}\sum_{k=1}^{n'}\left[\mathbf{f}\left(t_{i+k/n}\right)+\mathbf{f}\left(t_{i-k/n}\right)\right],\\
&t_i=t_{end}: && \tilde{\mathbf{f}}(t_i)=\mathbf{f}(t_i),
\end{aligned} \tag{5}$$

$$\begin{aligned}
&t=\Delta t:\quad n'=n-1\\
&\Delta t<t<t_{end}-\Delta t:\quad n'=\begin{cases}\dfrac{n}{2} & n=2j\quad j\in Z^{+}\\[2mm] \dfrac{n-1}{2} & n=2j+1\quad j\in Z^{+}\end{cases}\\
&t=t_{end}-\Delta t:\quad n'=n-1
\end{aligned} \tag{6}$$

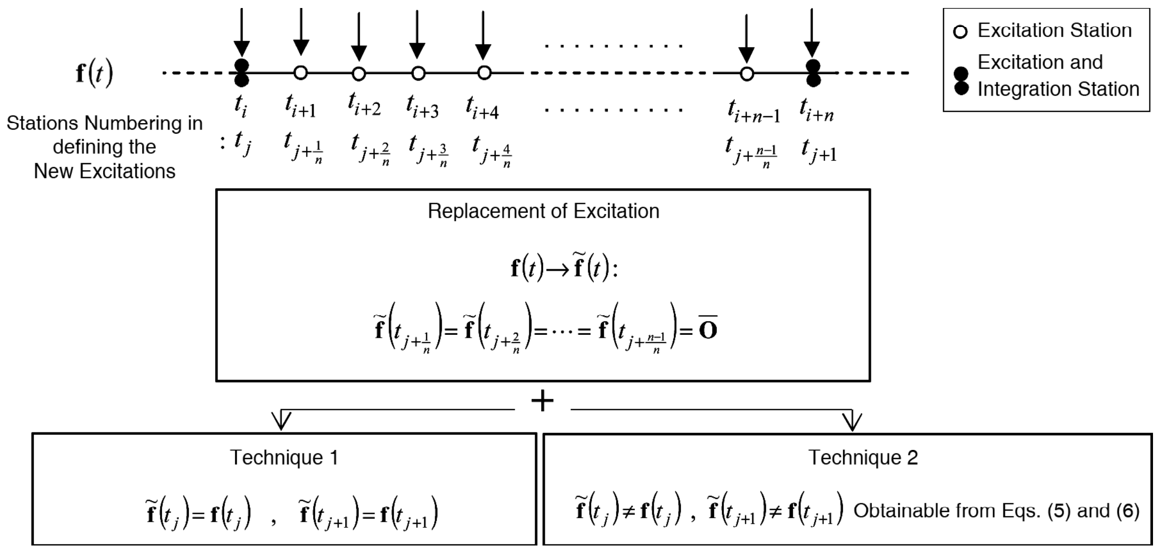


Figure 1. Schematic representation of excitation replacement according to the Techniques 1 and 2.

where, in consistence with Eqs. (3), Δt is obtainable (for both techniques), from:

$$\Delta t = n\ {}_f\Delta t \le \text{Min}\left(h, \frac{T}{10}\right) \qquad (7)$$
$$\Delta t \le t_{end}$$

and n can be computed using Eq. (4) (the second relation in Eq. (7) is essential in computerizing Technique 2). Considering the facilities provided by seismological instrumentation [25, 27], and the fact that ground motions are natural phenomena, the assumptions to be met for implementation of Technique 2 are (see Figure (2) and [17]):

1. The integration steps, $\Delta t_j \quad j = 1,2,\ldots$, are equally sized ($t_j$ implies the j^{th} integration station),

$$\forall\ j \quad \Delta t_j = t_j - t_{j-1} = \Delta t > 0 \qquad (8)$$

2. The excitation steps are equally sized (all sized ${}_f\Delta t$) and embedded by the integration steps (the first time station, i.e. t_0, is a station for both excitation and integration),

$$\exists\ \ n \in Z^+ \qquad \frac{\Delta t}{{}_f\Delta t} = n < \infty \qquad (9)$$

The above assumptions, though are imposed basically on Technique 2, are mostly valid, also in implementation of Technique 1, and hence, do not imply deficiencies for Technique 2. Because of this, the simplicity of Eqs. (5) and (6), and the fact that, practically, the computational cost reductions for linear analyses, associated with either of the two techniques, equal:

$$A_C \le\cong \frac{n-1}{n} 100\ \ (\%) \qquad (10)$$

(for a specific value of n, the computational cost of Technique 1 is less than Technique 2; the difference is however trivial, unless in analyses of systems with few degrees of freedom and for few steps), the adequacy of the two techniques can be studied, from the standpoint of accuracy. In nonlinear analyses the computational cost reductions, when implementing Techniques 1 and 2, can be different, from each other, and also from Eq. (10). Accordingly, both accuracy and computational cost are to be taken into account in the nonlinear studies.

From relations like the convolution or the Duhamel integral [25], it is obvious that omitting some parts of the excitations histories would affect the accuracy of the responses. Nevertheless, since ordinary time integration analyses (either direct or along with modal superposition [16, 25]) lead to approximate responses even for linear problems [14], the effect of the approximation may cancel out the effect of the omission in Technique 1, leading to sufficiently accurate responses. Though this is hardly the case, throughout the integration interval, if we consider the nature of time integration, and specifically the dependence of the computed response on the history of the excitation [14, 46], expressed as:

$$\begin{pmatrix} \mathbf{u} \\ \dot{\mathbf{u}}\Delta t \\ \ddot{\mathbf{u}}\Delta t^2 \end{pmatrix}_j = \mathbf{A}^j \begin{pmatrix} \mathbf{u} \\ \dot{\mathbf{u}}\Delta t \\ \ddot{\mathbf{u}}\Delta t^2 \end{pmatrix}_0 + \sum_{k=1}^{j} \mathbf{A}^{j-k}\mathbf{L}\begin{pmatrix} \mathbf{F}_{k-1} \\ \mathbf{F}_k \end{pmatrix} \qquad (11)$$
$$j = 1, 2, \ldots$$

for linear analyses, provided the cancellation of the two above-mentioned errors occurs at the starting parts of the integration interval, the response, obtained after implementing Technique 1, will have the chance to be sufficiently accurate; see the second term in Eq. (11). In Eq. (11), **A** and **L** respectively stand for the amplification matrix and load operator [14], each right subscript implies the instant under consideration, each top dot denotes once differentiation with respect to time, and **F** represents the excitation vector, theoretically

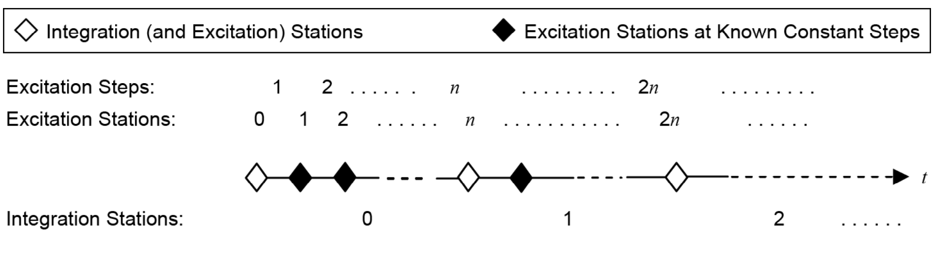


Figure 2. Typical distribution of excitation and integration stations in implementation of Techniques 1 and 2.

slightly different from the earthquake induced excitation **f**. Nevertheless, being dependent on the system, the excitation, and the integration method, appropriate cancellation of the two errors cannot be guaranteed; therefore, Technique 1 is unreliable. Regarding Technique 2, the technique and its formulation are based on convergence and the second order of accuracy of broadly accepted and conventional time integration methods [14, 16, 47], and there is no omission of excitation information in implementation of Technique 2 [17, 32]. Considering these, the past experiences on Technique 2, reported in Table (1), and meanwhile the fact that, based on the definition of error [48], i.e.

$$E = \left\| \mathbf{U} - \mathbf{U}_e \right\| \tag{12}$$

convergence is a main essentiality of approximate computations [18, 19], Technique 2 seems as an adequate alterative for reducing the computational costs. Still, it is reasonable to study the performance further, especially, compared to Technique 1.

The above discussion on reliability has not taken into account nonlinearities, see Eqs. (7), (10), and (11). Nevertheless, considering the complexities of nonlinear analyses (different stress/strain descriptions, updating the status, nonlinearity iterations, etc.) [2, 15, 16], it is reasonable to extend the discussion to nonlinear analyses and do not rely on the accuracies, when implementing Technique 1 in nonlinear analyses. Furthermore, considering nonlinear behaviours as sets of finite or infinite linear behaviours [49], we can expect the better performance of Technique 2 compared to Technique 1, even in nonlinear analyses.

3. Numerical Study

As the first example, consider the simple structural model below:

$$\begin{aligned} &\ddot{u} + 0.1\dot{u} + u = -\ddot{u}_g \\ &u(t=0) = 0.1 \quad \dot{u}(t=0) = -0.005 \\ &\qquad\qquad 0 \le t \le 30 \end{aligned} \tag{13}$$

where, $\ddot{u}_g$ stands for the ground acceleration, introduced in detail, in Figure (3), and g (in Figure (3)) is the constant of gravity. The exact displacement history is as displayed in Figure (4). The approximate response is computed by the

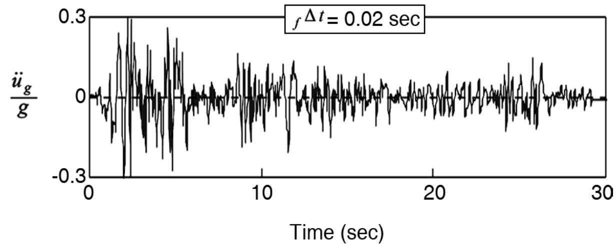


Figure 3. The excitation in the analysis of Eq. (13).

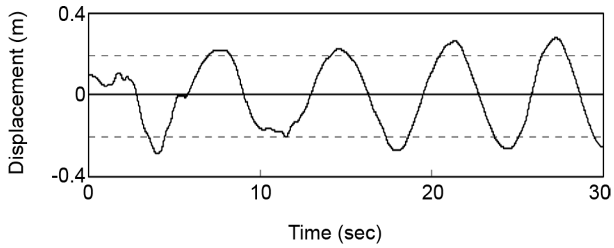


Figure 4. The exact response of Eq. (13).

average acceleration method [50], once ordinarily, considering $\Delta t = {}_f\Delta t$, and then again, after implementing Techniques 1 and 2, and considering $\Delta t = n\, {}_f\Delta t$, where:

$$n = 2, 5, 25 \tag{14}$$

(Equation (4) recommends $n = 14$ (as an upper estimate for n, considering the total integration interval); however, in view of the changes of T throughout the integration interval, and the purpose of this study, several values are assigned to n. The results are reported in Figures (5) and (6). Apparently, compared to the responses obtained, after implementing Technique 1, the responses, obtained, after implementing Technique 2, are closer to the responses computed ordinarily. The study is repeated for other responses, and also in analyses against other strong motions and other time integration methods, and, conceptually, similar results are observed, not reported here for the sake of brevity.

As the second example, attention is paid to the structural system, introduced, in Table (2) and Figure (7). The Houbolt time integration method [13, 51, 52] is considered as the analysis tool. Once again, taking into account Eq. (14) leads to the comparison of Techniques 1 and 2, reported in Figure (8), as a clear evidence for the superiority of Technique 2 to Technique 1 (Eq. (4) recommends $n = 3$).

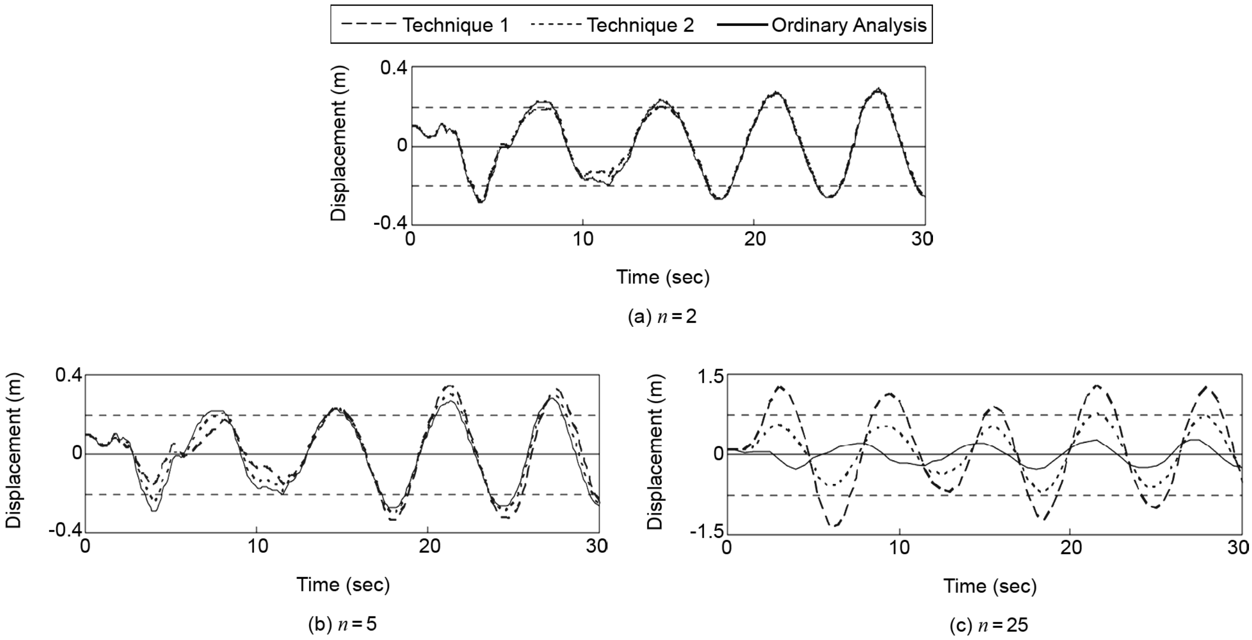


Figure 5. Displacement history in analysis of Eq. (13) by the average acceleration method.

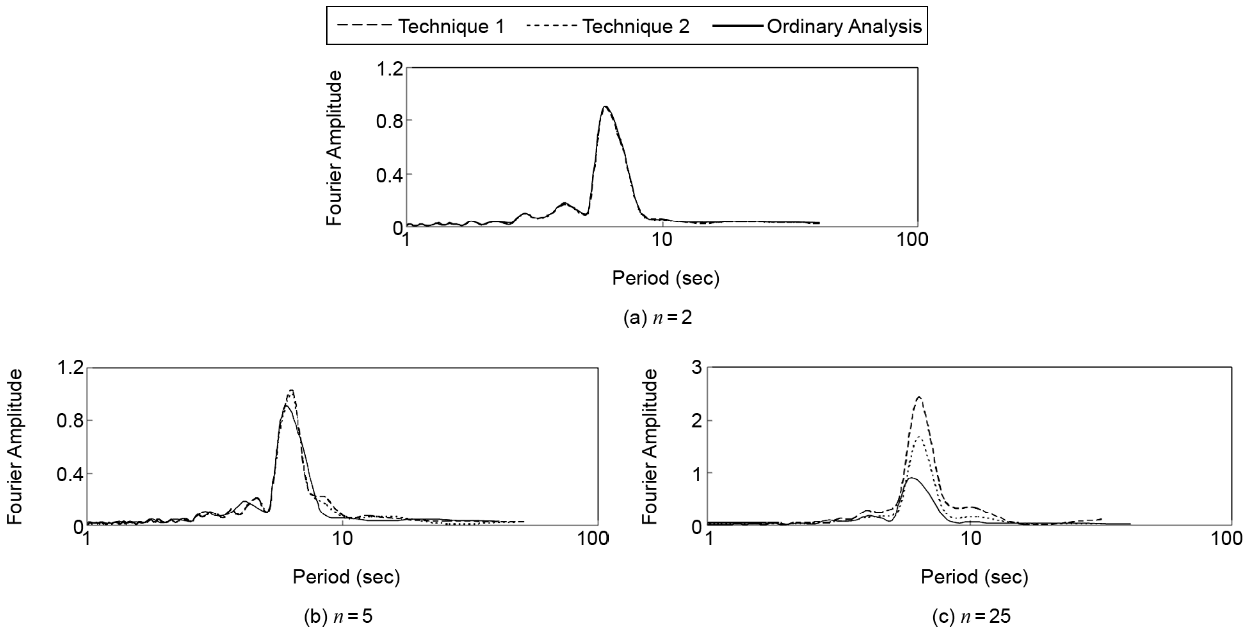


Figure 6. Frequency content of the displacement in analysis of Eq. (13) by the average acceleration method.

Table 2. Complementary information for the structural system introduced in Figure (7).

Floor	1	2	3	4	5	6	7	8
Mass $\times 10^{-3}$ (Kg)	10360	10340	10320	10300	10280	10260	10240	10220
Stiffness$\times 10^{-3}$ (N/m)	8600	8400	8200	7000	6800	6600	6400	6200
Damping (N sec/m)	Negligible (Considered Zero)							

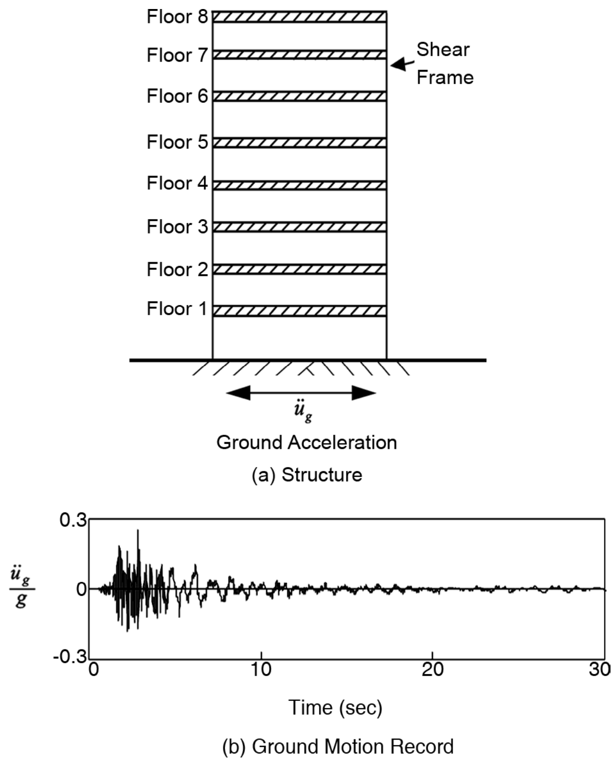


Figure 7. Structural system in the second example.

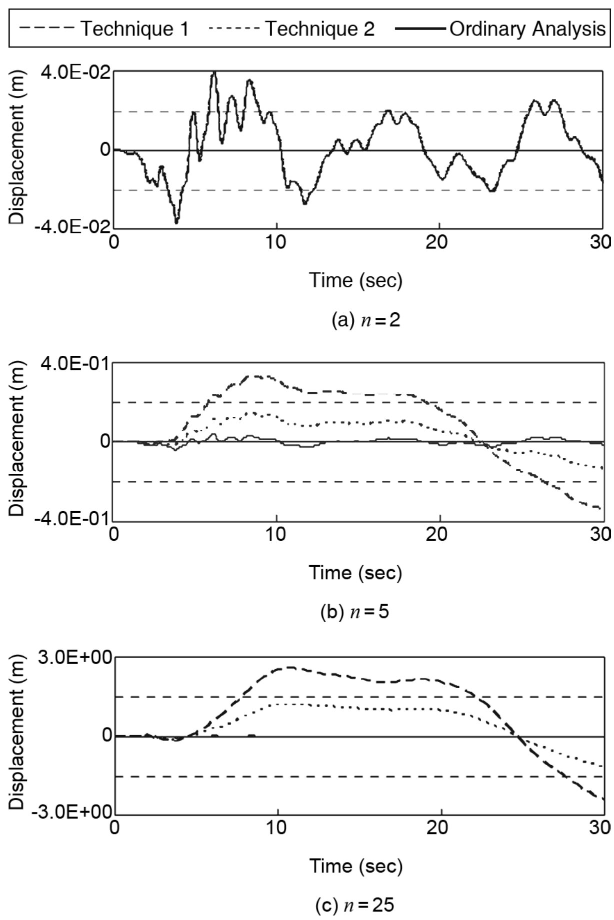


Figure 8. Mid-height displacement history for the system introduced in Figure (7) and Table (2) obtained from Houbolt time integration.

And, as the third example, the structural system defined in Figure (7) and Table (2) is changed to a nonlinear system, by considering a shorter neighboring shear frame, with properties identical to Floors 3-6 of the shear frame in Figure (7a) (see Figure (9)). To consider two severity of nonlinear behavior, the study is carried out twice, for the two excitations displayed in Figures (3) and (7b) (the non-linear beavior is automatically guaranteed when considering the excitation in Figure (7b)). The impacts are all considered elastic (i.e. the restitution factors equal one), and the e in Figure (9) equals 0.2 m. Numerical tests revealing the actual nonlinear behaviours (carried out in view of the super-position principle) are not reported here for the sake of brevity. The exact responses are as typically displayed in Figure (10). Accordingly (and in comparison with Figure (8a)), Figure (10) reveals that, while the first excitation has caused considerable nonlinearity, the nonlinearity induced by the second excitation is of small significance. Four separate sets of analyses, similar to those in the previous example, are carried out, considering the two excitations in Figures (3) and (7b) (as ground accelerations), and the average acceleration and Houbolt methods, for time integration. In view of Figure (10) and Eq. (4) (when considering "100" instead of "10"), $n=2$ is an appropriate selection in implementation of Techniques 1 and 2. Fractional time stepping [53-55] is implemented for nonlinearity iterations, and the effects of nonlinear analysis are

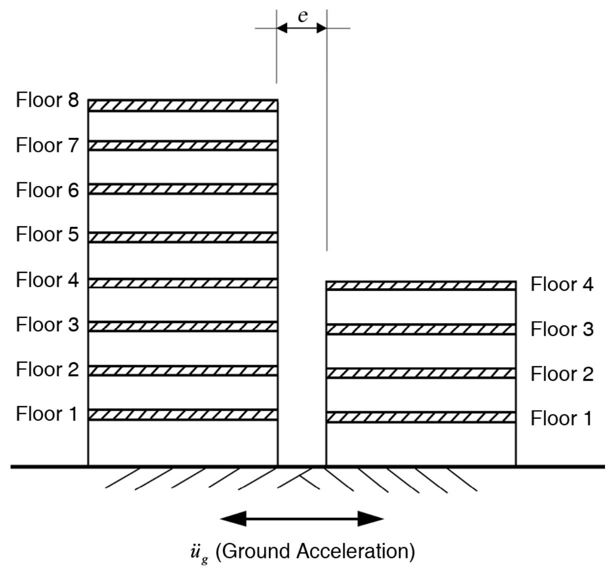
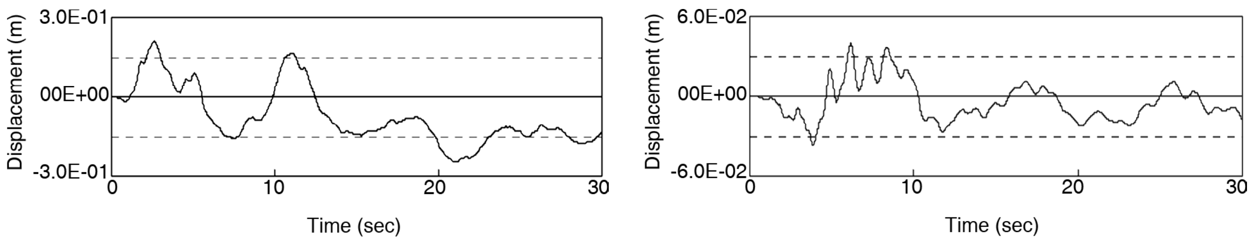


Figure 9. A nonlinear version of the structural system introduced in Figure (7) and Table (2).



(a) Against the Strong Motion in Figure (3)

(b) Against the Strong Motion in Figure (7b)

Figure 10. Exact mid-height displacement history for the taller building in Figure (9).

minimized, by assigning very small values to the non-linearity tolerances and continuing the non-linearity iterations the most possible [15, 38, 49]. The results of the comparison between Techniques 1 and 2 are reported in Figure (11) and Table (3). Similar to Figures (5), (6), and (8), in Figure (11), compared to the responses obtained from Technique 1, the responses obtained from Technique 2 are closer to the responses obtained from ordinary analysis (specifically apparent in Figure (11c)). This, together with the fact that the com-putational costs corresponding to the two techniques, reported in Table (3), are close, implies the superiority of Technique 2 [17] to Technique 1.

4. Real Examples

4.1. Introduction

In the previous sections, we could show that simple omission of inter-integration-step excitations

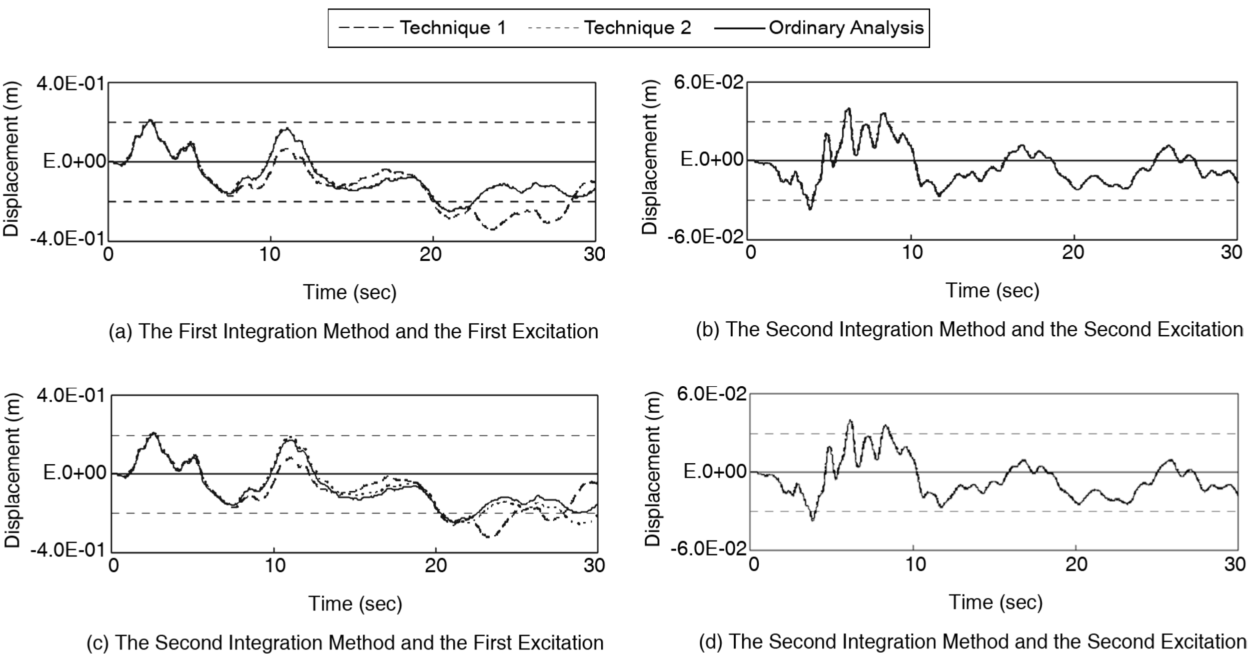


Figure 11. Mid-height displacement history for the taller building in Figure (9), considering n = 2.

Table 3. Number of integration steps in arriving at the responses reported in Figure (11).

Figure	Ordinary Analysis	Technique 1	Technique 2
11(a)	1611	815	841
11(b)	3054	1562	1552
11(c)	1613	804	852
11(d)	3062	1559	1569

(Technique 1) is not reliable, and Technique 2 [17] can be a better alternative. In order to examine this claim in real engineering problems, and make a practical idea about the unreliability and the amount of the probable inaccuracies, in this section, attention is paid to the sloshing phenomenon in water tanks [56, 57]. Four models are taken into consideration; two experimental- and two real-sized; see Figures (12) and (13). In the analysis of the fluid, for the water level, the space discretization is carried out, by finite volume hexahedron elements, in an Eulerian formulation [2, 16], and the time integration is carried out with the backward Euler method [58]. The nonlinearities are modelled with the volume fraction method (based on two phases of air and water in each finite volume), considering the nonlinearity tolerance equal to 10^{-5} [15, 57]. As apparent in Figures (12) and (13), the models possess two horizontal axes of symmetry. Furthermore, in Figures (12a) and (12b), the lengths parallel to the baffles do not influence the behaviours and the analyses, provided being more than a minimum to maintain the possibility of sloshing; and the earthquake records are implemented in the direction of the axes of symmetry perpendicular to the baffles in Figure (12) and along the lengthier axes of symmetry in Figure (13). Accordingly, the analyses are carried out two dimensional, with 16120, 16890, 39878, and 47244 elements, respectively, corresponding to Figures (12a), (12b), (13a), and (13b).

4.2. Experimental Sized Models

In view of the explanations, presented in Section 4.1, and the importance of baffles in controlling earthquake-induced sloshing in water tanks [56, 59], the systems in Figures (12a) and (12b) are taken into account in this section. The tanks in Figures (12a) and (12b) are respectively with one and two baffles. Techniques 1 and 2 are implemented, in the analyses, considering n = 3, 4, 9, 15. (The value of n recommended by Eq. (4), after replacing

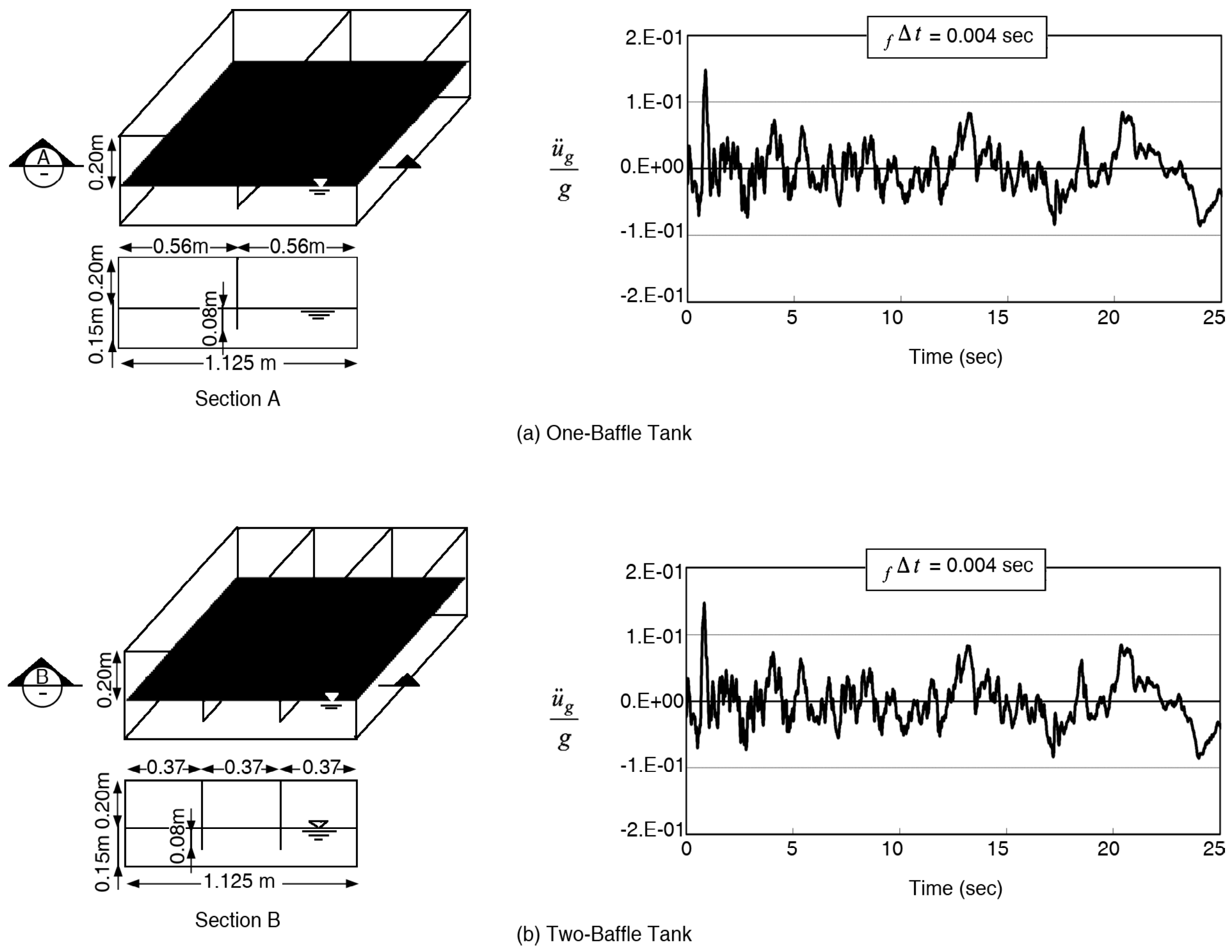
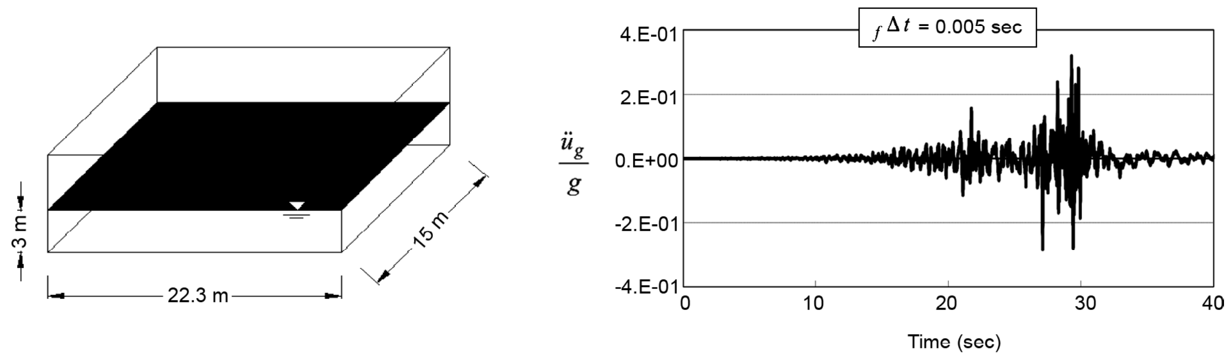
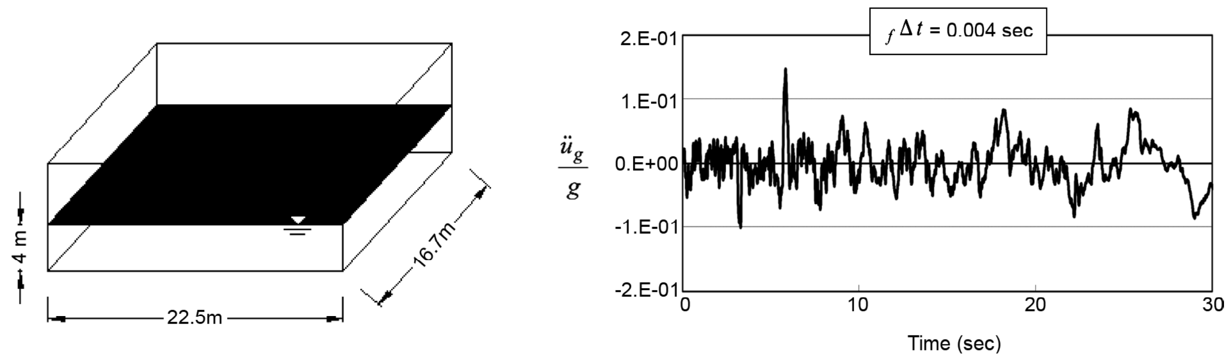


Figure 12. Experimental sized models involved in sloshing.

the "10" with "100", is eleven.) The results are reported in Figures (14) and (15). Apparently, Technique 1 is not reliable and Technique 2 leads to more accurate responses. Specifically, it is worth noting that, as displayed in Figures (14 c), (14 d), and (15 d), in implementation of Technique 1, the analyses can be halted, because of the failure of nonlinearity iterations, while, implementation of Technique 2 can lead to sufficiently accurate responses.

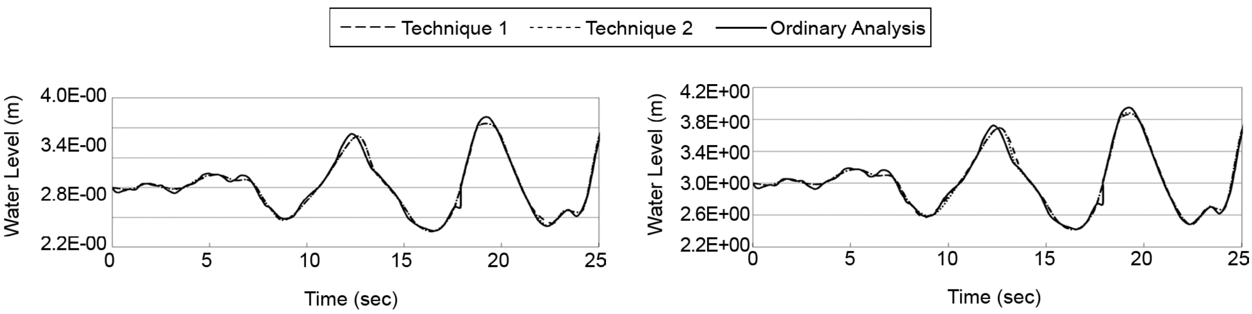


(a) Subjected to an Earthquake with Moderate Frequency Content



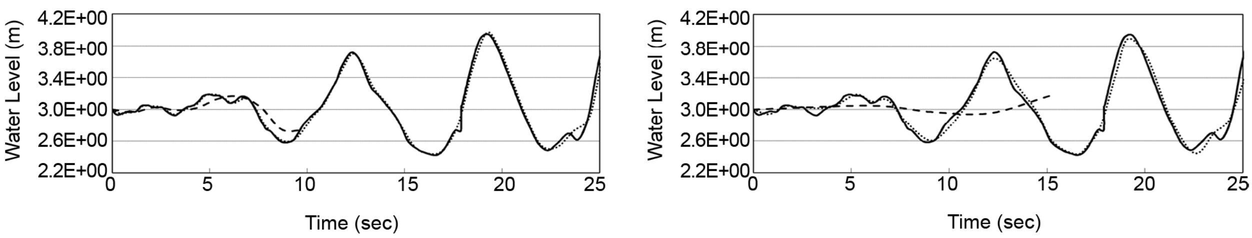
(b) Subjected to an Earthquake with High Frequency Content

Figure 13. Real sized models involved in sloshing.



(a) n = 3

(b) n = 4



(c) n = 9

(d) n = 15

Figure 14. Responses histories computed for the model in Figure (12a).

4.3. Real Sized Models

4.3.1. The Model in Figure (13a)

The study presented in Section 4.2 is repeated, for the models in Figure (13a), considering:

$$n = 2,5,15,30 \tag{15}$$

The results reported in Figure (16) reveal the better accuracy provided by Technique 2.

4.3.2. The Model in Figure (13b)

In this section, the study reported in Sections 4.2 and 4.3.1 is repeated for the model in Figure (13b), considering:

$$n = 50,120,200 \tag{16}$$

The resulting time histories are displayed in Figure (17), all in conceptual agreement with the previous observations; specially, it is notable that, in Figure (17c), the analysis using Technique 1 is halted at the starting steps.

4.4. A Review on the Observations

With attention to Figures (14) to (17), in real

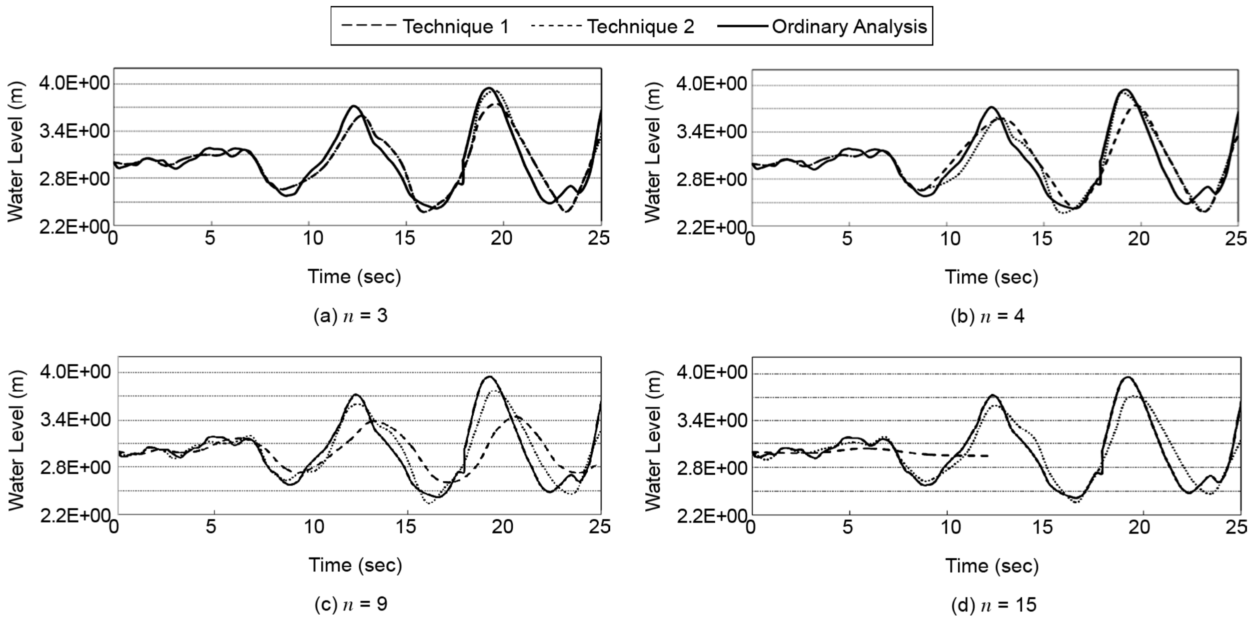


Figure 15. Responses histories computed for the model in Figure (12b).

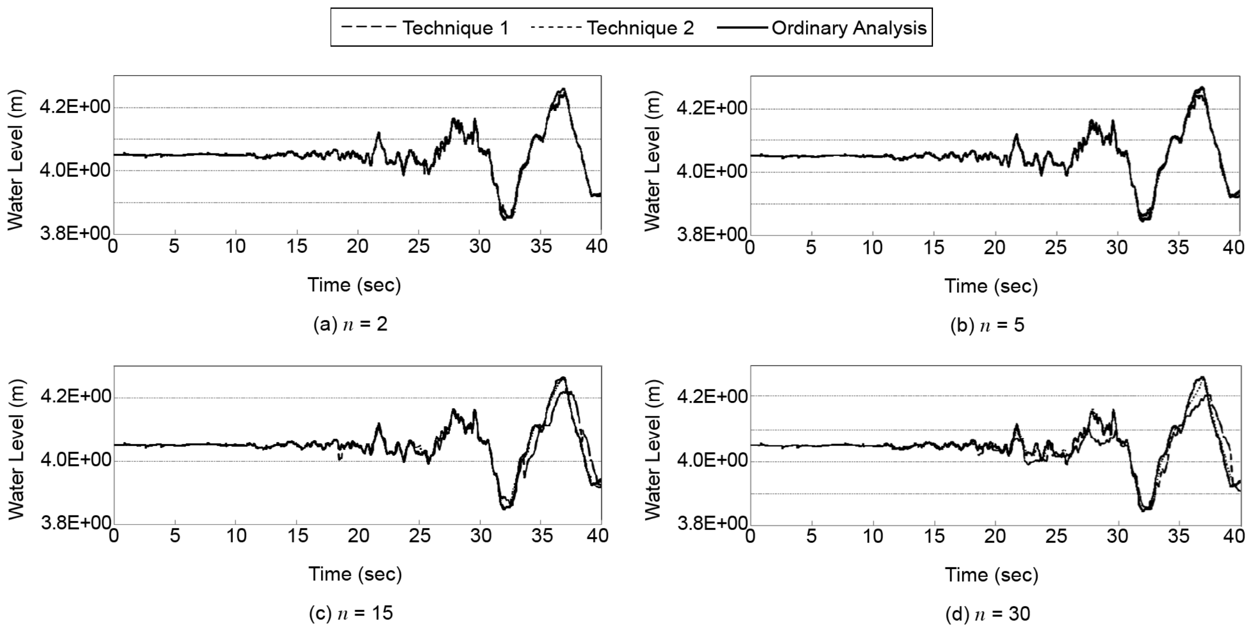


Figure 16. Responses histories computed for the model in Figure (13a).

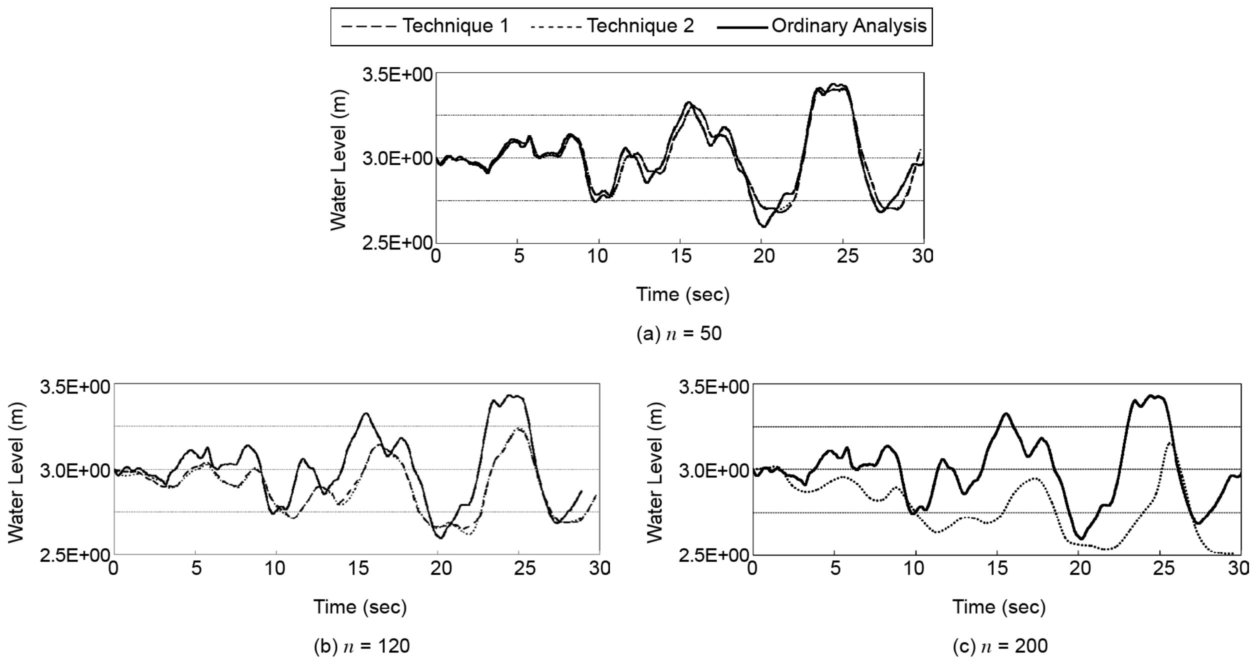


Figure 17. Responses histories computed for the model in Figure (13b).

seismic analyses (generally nonlinear [3-6, 25, 59-61]), the responses obtained after implementation of Technique 1 are unreliable, and compared to the responses obtained after implementation of Technique 2 can be less accurate. In view of the philosophy, design factors, and the details stated in seismic codes [3, 4, 5, 61], the loss of accuracy when implementing Technique 1 might be negligible and unimportant, e.g. cases reported in Figures (14b), (14c), (15b), and (16). The loss might be practically considerable, e.g. cases reported in Figure (15c), or might even prevent a response throughout the transient interval; see Figures (14d) and (15d). Halt of analyses in the starting steps is also probable; see Figure (17c). Accordingly, Technique 1 is unreliable for implementation in real seismic analyses, and Technique 2 can be an adequate alternative for enlargement of integration steps and reducing the computational costs, when ${}_f\Delta t$ considerably dominates the right hand side of Eq. (2). Still, in view of rare cases like that reported in Figure (17b), where the results of the two techniques coincide and are considerably different from the results of ordinary analyses, further study on the accuracy associated with Technique 2 is essential. (Figure (17b) does not necessarily imply an evidence for the weakness of Technique 2; see the explanation in Section 5.) Finally, it is worth noting that, as apparent in Figures (14) to (17), implementation of Technique 2 in transient analyses, of systems discretized in space by finite volume method can be successful; this is reported for the first time in this paper.

5. Discussion

Three issues are discussed in this section:

1) Why, in the numerical study, the responses are compared with the responses of ordinary analyses, not the exact responses?
2) Considering the role of n in Techniques 1 and 2, and the role of the least dominant period of the response, T, in determination of n, how should the value of n be set, practically, prior to the analysis?
3) How would be the future of techniques directly or indirectly effectual in more efficient time integration analysis against digitized excitations?

Regarding the first question, Eqs. (2) to (4), which are the basis of the discussion in this paper, all consider $\frac{T}{10}, \frac{T}{100}$, or $\frac{T}{1000}$, as the step sizes, providing sufficient accuracy for ordinary analyses. This implies the smallness of the difference between the response of the exact and ordinary analyses, at least, from a structural engineering point of view (for an exceptional case, see [45]). Because of this, the objective of this paper, and the fact that the exact responses are not available for nonlinear problems and are not simply available for linear problems, the authors have considered the ordinarily computed responses as sufficiently adequate bases for the study of reliability.

The second question, i.e. the ambiguity in determining T and n, in real analyses, can be responded, considering the fact that the benefit rate of an appropriate selection of n decreases, as n increases; see Eq. (10) and the last row in Table (4). This implies that, by assigning a value to n smaller than the exact value, determined based on the least dominant period of the response (Eq. (4)), we may still considerably reduce the computational cost, while expecting more accuracy. Therefore, in assigning an appropriate value to n, it is sufficient to make a reliable lower estimation for T (when ${}_f\Delta t$ dominates the right hand side of Eq. (2), smaller values of T lead to smaller values of n). This is successfully examined, for mid-rise residential buildings, e.g. see [41]). A secondary question may however arise. It is how to reliably estimate a lower bound for T, to later arrive at a lower estimation for n. As a response, it is correct that ambiguities regarding T persist, even when, instead of determining T, we try to determine a lower bound for T. Nevertheless, regardless of whether the excitation is continuous or digitized, the ambiguities originate in the conventional broadly accepted integration-step-size-selection comments, e.g. Eq. (2) [6, 16, 17, 25, 26], where, for continuous excitations ${}_f\Delta t \to \infty$. Because of this, control of errors based on repetition of time integration analyses is recommended after time integration analyses [20, 25, 58]. Consequently, in implementation of Techniques 1 or 2, a final control, with smaller steps and unchanging value of n, would reveal whether the obtained accuracy is sufficient [62]. As a final explanation, Eq. (4) is based on Eq. (2), which is not a theoretically rigorous inequality. Therefore, it is reasonable to use the values of n obtained from Eq. (4), not as values to be assigned to n, but merely as upper-bounds for the values to be assigned to n.

Regarding the third question, the amount of the study carried out in the past decade, directly or indirectly effectual in reducing the computational costs, e.g. see [17, 28, 29-45, 63-65], is an implication of the current need to more efficient methods. Moreover, in view of the every-day progress in the instrumentation of digitization, e.g. see [27], we can reasonably expect the decrease of excitations steps sizes and specifically the sizes of ground motions steps, at least, for the mid-term future. Furthermore, transient analyses, and accordingly integration analyses, are in rapid progress [11, 22, 24, 66-72], and hence, we can expect continuous improvements towards more accurate time integration methods. Therefore, the difference between the sizes of excitation steps and the sizes recommended for integration steps is in increase, in view of Eq. (4) leading to higher values of n and the need to techniques for providing the capability of more efficient analysis against digitized excitations. Consequently, more relevant research is essential.

Regarding the latter, an ambiguity persists. It is whether, with attention to the everyday growing capacity of computational facilities and hard wares, the difference between efficiencies will be meaningful in the coming decades and long term. If the size and complexity of structural systems and analyses were unchanging or in gradual increase, surely, there would come a day that, not only the efficiency, but also, the computational cost, would be trivial issues. However, our imaginations are unlimited, and the structural systems, the structural material, and the analyses methods, are in continuous advancement with accelerating rates. These entail larger and more complicated structural models to be analyzed every day. The consequence is the essentiality of further attention to analyses efficiency, and specially, to techniques facilitating more efficient sufficiently accurate analyses against digitized excitation, e.g. earthquake induced excitations.

Table 4. Changes of the cost reduction in implementation of Techniques 1 or 2 in linear analyses (%).

n	2	3	4	5	6	7	8	9	10	…
A_C	50	67	75	80	84	86	88	89	90	…
$\frac{\partial A_C}{\partial n}$	50	16	9	5	4	2	2	1	1	…

6. Conclusion

In transient analysis against digitized excitations, recorded at steps smaller than those recommended for the accuracy, the conventional approach not to disregard the excitations information is to time integrate with steps equal to the steps of the excitations. Nevertheless, because of the computational costs, it is also conventional, to scale up the integration steps sizes, by a positive integer, and omit the inter-integration-step excitations. In this paper, the above-mentioned computational cost reduction technique and a more theoretical, convergence-based and successful technique [17] are studied and compared with special attention to earthquake engineering issues. As the result:

- Simple omission of inter-integration-step excitations, potentially, leads to inaccuracies, practically unacceptable. The case is worse in nonlinear problems, and the analyses might even stop during the nonlinearity iterations. Consequently, simple omission of inter-integration-step excitations is unreliable.
- Implementation of the technique proposed in [17] can be considerably more adequate, for both linear and nonlinear problems.
- The results above are independent, from system, excitation, and the integration method.
- Besides, it is worth noting that application of the technique proposed in [17] to transient analysis of semi-discretized equations of motion resulted from finite volume method was not carried out before. The results reported for the first time in this paper clearly display the adequate performance.

Accordingly, further investigation on both techniques is recommended; on simple omission of inter-integration-step excitations, to determine a specific range of reliability, and on the technique proposed in [17], for more advancement. An area for research on the latter, is the comparative distribution of errors in different oscillatory modes.

Finally, in view of the progresses in seismological instrumentation, the enhancements in integration methods, and the trend of establishing larger and more complex structural systems, further research towards more efficient time integration analyses against digitized excitations is essential and strongly recommended.

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References

1. Argyris, J. and Mlejnek, J.P. (1991) *Dynamics of Structures.*, Elsevier, Amsterdam, The Netherlands.

2. Belytschko, T., Liu, W.K., and Moran B. (2000) *Non-linear Finite Elements for Continua and Structures*. John Wiley & Sons, New York, USA.

3. BCJ (2001) *Structural Provisions for Building Structures*. Building Centre of Japan, Tokyo.

4. BHRC (2015) *Iranian Code of Practice for Seismic Resistant Design: Standard No. 2800*. Building and Housing Research Center, Tehran (in Persian).

5. ICC (2003) *IBC-International Building Code*. International Code Council, Club Hills, USA.

7. Soroushian, A. (2003) *New Methods to Maintain Responses' Convergence and Control Responses' Errors in the Analysis of Nonlinear Dynamic Models of Structural Systems*. Ph.D. Dissertation, University of Tehran, Tehran (in Persian).

6. NZS (2004) *Structural Design Actions, Part 5: Earthquake Actions-New Zealand*. New Zealand Standard, New Zealand.

8. Wriggers, P. (2002) *Computational Contact Mechanics*. John Wiley & Sons, New York, USA.

9. Hughes, T.J.R., Pister, K.S., and Taylor, R.L.

(1979) Implicit-Explicit finite elements in nonlinear transient analysis. *Computer Methods in Applied Mechanics and Engineering*, **17-18**(1), 159-182.

10. Gavin, H. (2001) *Structural Dynamics*. Class Notes CE 283. Duke University, USA.

11. Chung, J. and Hulbert, G.M. (1994) A family of single-step Houbolt time integration algorithms for structural dynamics. *Computer Methods in Applied Mechanics and Engineering*, **118**(1-2), 1-11.

12. Rossi, D.F., Ferreira, W.G., Mansur, W.J. and Calenzani, A.F.G. (2014) A review of automatic time-stepping strategies on numerical time integration for structural dynamics analysis. *Engineering Structures*, **80**, 118-136.

13. Soroushian, A. and Farjoodi, J. (2008) A unified starting procedure for the Houbolt method. *Communications in Numerical Methods in Engineering*, **24**(1), 1-13.

14. Belytschko, T. and Hughes, T.J.R. (1983) *Computational Methods for Transient Analysis*. Elsevier, Amsterdam, The Netherlands.

15. Allgower, E.L. and Georg, K. (1990) *Numerical Continuation Methods: An Introduction*. Springer, New York, USA.

16. Bathe, K.J. (1996) *Finite Element Procedures*. Prentice-Hall, Upper Saddle River, USA.

17. Soroushian, A. (2008) A technique for time integration with steps larger than the excitation steps. *Communications in Numerical Methods in Engineering*, **24**(12), 2087-2111.

18. Henrici, P. (1962) *Discrete Variable Methods in Ordinary Differential Equations*. Prentice-Hall, Upper Saddle River, USA.

19. Strikwerda, J.C. (1989) *Finite Difference Schemes and Partial Differential Equations*. Wadsworth & Books/Cole, Pacific Grove, USA.

20. Soroushian, A. and Farjoodi, J. (2003) More reliable responses for time integration analyses. *Structural Engineering and Mechanics*, **16**(2), 219-240.

21. Zhou, K.K. and Tamma K.K. (2004) A new unified theory underlying time dependent first-order systems: a prelude to algorithms by design. *International Journal for Numerical Methods in Engineering*, **60**(10), 1699-1740.

22. Katsikadelis, J.T. (2013) A new direct time integration method for the equations of motion in structural dynamics, ZAMM. *Journal of Applied Mathematics and Mechanics*, **94**(9), 757-774.

23. KaiPing, Y. (2008) A new family of generalized-α time integration algorithms without overshoot for structural dynamics. *Earthquake Engineering and Structural Dynamics*, **37**(12), 1389-1409.

24. Tang, X.W., Zhang, X.W., and Uzuoka, R. (2015) Novel adaptive time stepping method and its application to soil seismic liquefaction analysis. *Soil Dynamics and Earthquake Engineering*, **71**, 100-113.

25. Clough, R.W. and Penzien, J. (1993) *Dynamics of Structures*. McGraw-Hill, Singapore.

26. McNamara, J.F. (1974) Solution schemes for problems of nonlinear structural dynamics. *Journal of Pressure Vessel Technology*, **96**(2), 147-155.

27. Havskov, J. and Alguacil, G. (2005) *Instrumentation in Earthquake Seismology (Modern Approaches in Geophysics)*. Springer, Amsterdam, Netherlands.

28. Chang, S.Y. (2002) Integrated equations of motion for direct integration methods. *Structural Engineering and Mechanics*, **13**(5), 569-589.

29. Hosseini, M. and Mirzaei, I. (2012) Simplification of earthquake accelerograms for rapid time history analysis based on the impulsive load concept. *Proceedings of the 10th International Congress on Computational Mechanics*, Sao Paulo, Brazil.

30. Soroushian, A. (2009) *Time Integration with Step Sizes Less Depending on the Steps of Excitation: I.* SDOF Systems, Structural Engineering Research Centre (SERC) IIEES, Report No. 7509-P89-1 (in Persian).

31. Saaed, A. (2012) *A Technique for Faster Time*

Integration of MDOF Structural Systems Based on Using Larger Time Interval in Step by Step Integration. M.Sc. Dissertation, International Institute of Earthquake Engineering and Seismology, Tehran. (in Persian)

32. Soroushian, A. (2016) *Integration Step Size and its Adequate Selection in Analysis of Structural Systems Against Earthquakes*. In Computational Methods in Earthquake Engineering Vol. 3, edited by Paparakakis, M. and Lagaros, N., and Plevris, V., Springer, The Netherlands. (accepted for publication)

33. Soroushian, A. (2010) On practical performance of a technique recently proposed for time integration analysis with less computational cost. *Proceedings of the 17th International Congress on Sound & Vibration*, Cairo, Egypt.

34. Soroushian, A. and Aziminejad, A. (2011) A more efficient seismic analysis of tall buildings by implementing a recently proposed technique. *Proceedings of the 6th International Conf. on Seismology and Earthquake Engineering*, Tehran, Iran.

35. Bahar, O. and Ramezani, S. (2011) Faster time integration analysis for building structures subjected to 3-component earthquakes. *Proce. of the 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Corfu, Greece.

36. Nateghi, F. and Yakhchalian, M. (2011) An investigation into the effectiveness of a technique proposed for reducing computational cost of time integration in the analysis of silos seismic behaviors. *Proceeding of the 11th US National Congress on Computational Mechanics*, Minneapolis, Minnesota.

37. Soroushian, A., Saaed, A., Arghavani, M., Rajabi, M., and Sharifpour, M.M. (2011b) Less computational costs in the analysis of reservoirs seismic behaviors by time integration. *Proce. 10th Biennial Conference on Vibration Problems, Prague*, Czech Republic.

38. Soroushian, A. (2011) On the performance of a recent technique for more efficient time integration in severe seismic conditions. *Proceedings of the 1st World Congress on Advances in Structural Engineering and Mechanics*, Seoul, South Korea.

39. Soroushian, A., Hosseini, M., and Vasseghi, A. (2011) On the performance of a technique for more efficient time integration when applied to bridge structures seismic analysis. *Proceedings of the 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Corfu, Greece.

40. Papadrakakis, M., Fragiadakis, M., and Plevris, V. (2013) *Computational Methods in Earthquake Engineering*. Springer, New York, USA.

41. Sabzei, A. (2013) *On the Performance of a Recent Technique for Seismic Analyses Computational Cost Reduction when Applied to Buildings Structural Systems*. M.Sc. Dissertation, International Institute of Earthquake Engineering and Seismology, Tehran (in Persian).

42. Bastami, M. (2014) A technique for more efficient time integration applied to seismic analysis of power substation equipment. *Proceedings of the 11th World Congress on Computational Mechanics*, Barcelona, Spain.

43. Garakaninezhad, A. and Moghadas, R.K. (2015) On the performance of a technique to accelerate time integration when applied to space structures analyses. *Proceedings of the 5th ECCOMAS Thematic Conf. on Computational Methods in Structural Dynamics and Earthquake Engineering*, Kos, Greece.

44. Azad, S. (2015) *A Study on Accelerating Time History Analysis of Bridges*. M.Sc. Dissertation, International Institute of Earthquake Engineering and Seismology, Tehran (in Persian).

45. Hadad, A.A. (2015) *Reducing Computational Costs in Time Integration Analyses of Buildings with Irregularities in Height Because of Mass*. M.Sc. Dissertation, International Institute of Earthquake Engineering and Seismology, Tehran (in Persian).

47. Hughes, T.J.R. (1987) *The Finite Element*

Method: Linear Static and Dynamic Finite Element Analysis. Prentice-Hall, Upper Saddle River, USA.

48. Ralston, A. and Rabinowitz, P. (1978) *First Course in Numerical Analysis*. McGraw-Hill, USA.

49. Soroushian, A., Wriggers, P., and Farjoodi, J. (2013) On practical integration of semi-discretized nonlinear equations of motion: proper convergence for systems with piecewise linear behavior. *Journal of Engineering Mechanics* (ASCE), **139**(2),114-145.

50. Newmark, N.M. (1959) A method of computation for structural dynamics. *Journal of Engineering Mechanics* (ASCE), **85**(3), 67-94.

51. Houbolt, J.C. (1950) A recurrence matrix solution for the dynamic response of elastic aircraft. *Journal of the Aeronautical Sciences*, **17**(9), 540-550.

52. Katona, M.G. and Zienkiewicz, O.C. (1985) A unified set of single step algorithms part 3: the beta-m method, a generalization of the Newmark scheme. *International Journal for Numerical Methods in Engineering*, **21**(7), 1345-1359.

53. Nau, J.M. (1983) Computation of inelastic spectra. *Journal of Engineering Mechanics,* (ASCE), **109**(1), 279-288.

54. Mahin, S.A. and Lin, J. (1983) *Construction of Inelastic Response Spectra for Single Degree-of-Freedom Systems*. Report UCB/EERC-83/17, Earthquake Engineering Research Center (EERC), University of California, Berkeley, USA.

55. Soroushian, A. and Farjoodi, J. (2002) An improvement in non-linear analysis. *Proc. 5th ASCE Engineering Mechanics Division Conf.*, ASCE, Reston, VA.

56. Ibrahim, R.A. (2005) *Liquid Sloshing Dynamics Theory and Applications*. Cambridge University Press, New York, USA.

57. Godderidge, B., Turnock, S., Tan, M., and Earl, Ch. (2009) An investigation of multiphase CFD modelling of a lateral sloshing tank. *International Journal of Computational Fluid Dynamics*, **38**(2), 183-193.

58. Hairer, E. (2000) *Solving Ordinary Differential Equations I: Nonstiff Problems*. Springer, New York, USA.

59. Farshadmanesh, P. (2011) *Optimizing The Placement and Depth of Multiple Vertical Baffles in Liquid Storage Tanks*. M.Sc. Dissertation, South Tehran Branch of the Islamic Azad University, Tehran (in Persian).

60. Newmark, N.M. and Rosenblueth, E. (1971) *Fundamentals of Earthquake Engineering*. Prentice-Hall, Englewood Cliffs, NJ.

61. Saatcioglu, M. and Humar, J. (2003) Dynamic analysis of buildings for earthquake resistant design. *Canadian Journal of Civil Engineering*, **30**(2), 338-359.

62. Soroushian, A. (2015) A general practical procedure for a recently proposed seismic analysis computational cost reduction technique. *Proce. of the 7th International Conf. on Seismology and Earthquake Engineering*, Tehran, Iran.

63. Sabzei, A., Reziakolaei, A.Y., and Soroushian, A. (2013) On more versatility for an integration step size enlargement technique in time integration analysis. *Proc., 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2013)*, Kos Island, Greece.

64. Soroushian, A. (2012) On the accuracy of accelerations in general implementation of a recently proposed seismic analysis computational cost reduction technique. *Proc., 5th International Conference from Scientific Computing to Computational Engineering (5th IC-SCCE)*, Athens, Greece

65. Soroushian, A., Garakaninezhad, A., Yahyapour, A., and Asgarihadad, A. (2014) Performance of a computational cost reduction technique in lengthy time interval analyses. *Proc., 11th World Conference on Computational Mechanics (WCCM XI)*, Barcelona, Spain.

66. Shearer, M.C. and Cesnik, C.E.S. (2006) Modified Generalized-α method for integrating governing equations of very flexible aircraft. *Proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Newport, Rhode Island.

67. Zhou, X., Har, J., Tamma, K.K., and Shimada, M. (2010). On the numerical discretization in space and time: Part 2-total energy framework and satisfaction of conservation properties of discretized equations of motion for linear dynamics problems. *International Journal for Computational Methods in Engineering Science and Mechanics*, **11**(5), 280-298.

68. Kuo, S.R., Yau, J.D., and Yang, Y.B. (2012) A robust time-integration algorithm for solving nonlinear dynamic problems with large rotations and displacements. *International Journal of Structural Stability and Dynamics*, **12**(06),

69. Gholampour, A.A. and Ghassemieh, M. (2013) Nonlinear structural dynamics analysis using weighted residual integration. *Mechanics of Advanced Materials and Structures* , **20**(3), 199-216.

70. Rezaiee-Pajand, M. and Hashemian, M. (2015) Time integration method based on discrete transfer function. *International Journal of Structural Stability and Dynamics*

71. de Béjar, L.A. and Danielson, K.T. (2016) Critical time-step estimation for explicit integration of dynamic higher-order finite-element formulations. *Journal of Engineering Mechanics*, 04016043.

72. Soares Jr, D. (2015). A second-order time-marching procedure with enhanced accuracy. CMES. *Computer Modeling in Engineering and Sciences*, **105**(5), 341-360.