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Correction Factors Including Nonclassical Nature of Soil-Structure Interaction Spectral Analysis

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ABSTRACT

The problem of non-classical dynamic analysis of structures resting on flexible bases is studied in this paper. Because of the presence of the underlying soil in the dynamic model of structure that acts like an energy sink, the damping matrix is not proportional to structural mass and stiffness, and theoretically a non-classical approach should be followed in modal analysis. Considering one to twenty-story buildings, two types of soils, and several suits of ground motions each containing 10 earthquake records specifically selected for each building, the seismic responses are calculated using a time history modal analysis in this paper. Three cases are considered: fixed-base buildings with classical analysis, flexible-base buildings with classical and non-classical analysis. Using the non-classical analysis, it is shown that soil-structure interaction effects cannot be recommended to be taken into account for moment frame buildings with the fundamental fixed-base periods smaller than one second. Cases for which the base flexibility should be considered for the higher modes are distinguished too. Finally, it is made clear that on each soil type, when the actual non-classical nature of the SSI system must be accounted for.

Keywords:
Soil-structure interaction; Non-classical; Spectral; Correction factor

1. Introduction

Unlike fixed-base systems in which the source of vibration damping is more or less uniquely attributed to the structural system and is hence almost uniform, in a soil-structure-interaction (SSI) problem a considerable part of damping is contributed by the totally different medium of soil.

The damping matrix of such a complex system is a non-uniform combination of structure and soil damping values and therefore is not classical.

This is while in daily spectral analysis of structures, the damping matrix is always presumed to be classical, i.e., proportional to mass and stiffness matrices. When there is a doubt on validity of this basic assumption, like in SSI problems as discussed above, availability of a spectrum analysis methodology corrected for non-classical damping while retaining its simplicity will be very helpful.

The work of Veletsos and Ventura [1] was an important step forward in this regard. They simplified the non-classical modal analysis through giving insight to the physical meaning of different terms of the formulation and converted the complex-valued equations to their real counterparts. They derived equations for determining natural periods and mode shapes of non-classical systems resulting in free vibration responses and a Duhamel integral formulation for computing the dynamic response [1].

Ziyaeifar and Tavousi [2] developed formulas for calculating the modal values of the response maxima based on the work of Veletsos and Ventura [1].

Zhou and Yu [3] derived formulas for combining the maximum modal responses of non-classically damped linear systems. They used the random vibration theory and accounted for the correlation between the modal displacement and velocity responses of structure [3]. Based on a general modal response history analysis formulation for non-classical and over-damped systems developed by Song et al. [4]. They derived a response spectrum analysis approach and proposed a general modal combination rule [4].

To gain attractiveness in practical earthquake engineering, a non-classical SSI problem should be solved within the framework of a conventional design spectrum.

In the current study, the periods from which SSI should be taken into account are identified. Then it is made clear when the higher mode SSI effects are important, and finally, cases for which non-classical SSI analysis are necessary are recognized. One to 20-story buildings are considered and for each building, two suits of ground motion each containing 10 records, one being recorded at the near field and the other at the far field, selected through a special procedure specifically for each structure are used. Two types of medium and soft soils are also considered.

2. Modal Analysis of Soil-Structure Systems

2.1. Equations of Motion

A multistory structure on flexible soil subjected to a horizontal ground motion is shown in Figure (1).

The system is assumed to be a shear building having a single horizontal degree of freedom (DOF) at each floor to retain simplicity. In addition, it is supposed that the supporting medium possesses a horizontal as well as a rotational DOF, and the input motion in the presence of structure is assumed to be identical to the free-field motion. The equations of motion of the system of Figure (1) can be written as follows:

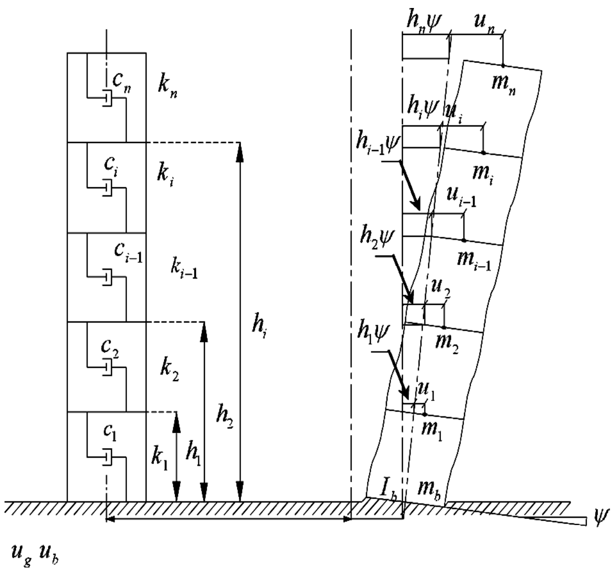


Figure 1. The multistory structure on flexible soil under lateral movement.

$$[M]\begin{Bmatrix}\{\ddot{u}\}\\ \ddot{u}_b\\ \ddot{\Psi}\end{Bmatrix}+[C]\begin{Bmatrix}\{\dot{u}\}\\ \dot{u}_b\\ \dot{\Psi}\end{Bmatrix}+[M]\begin{Bmatrix}\{u\}\\ u_b\\ \Psi\end{Bmatrix}=$$

$$-\begin{Bmatrix}\{m\}\\ m_b+\sum_1^n m_i\\ \sum_1^n m_i h_i\end{Bmatrix}\ddot{u}_g(t)=\{p(t)\} \tag{1}$$

in which:

$$[M]=\begin{bmatrix}[m] & \{m\} & \{mh\}\\ \{m\}^T & m_b+\sum_1^n m_i & \sum_1^n m_i h_i\\ \{mh\}^T & \sum_1^n m_i h_i & I+\sum_1^n m_i h_i^2\end{bmatrix}$$

$$[C]=\begin{bmatrix}[c] & \{0\} & \{0\}\\ \{0\}^T & c_{uu} & 0\\ \{0\}^T & 0 & c_{\Psi\Psi}\end{bmatrix} \tag{2}$$

$$[K]=\begin{bmatrix}[k] & \{0\} & \{0\}\\ \{0\}^T & k_{uu} & 0\\ \{0\}^T & 0 & k_{\Psi\Psi}\end{bmatrix}$$

and:

$$[m] = \begin{bmatrix} m_1 & & & & \\ & m_2 & & 0 & \\ & & & & \\ & 0 & & m_{n-1} & \\ & & & & m_n \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_1 & -c_2 & \cdots & 0 & 0 \\ -c_2 & c_1 + c_2 & -c_3 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & c_{n-1} + c_{n-2} & -c_n \\ 0 & 0 & \cdots & -c_n & c_n + c_{n-1} \end{bmatrix} \quad (3)$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\ \vdots & -k_3 & \ddots & \vdots & \vdots \\ 0 & \vdots \cdots & k_n + k_{n+1} & & -k_{n-1} \\ 0 & 0 \cdots & -k_{n-1} & & k_n \end{bmatrix}$$

and:

$$\{m\} = [m_1 \quad m_2 \quad \ldots \quad m_n]^T$$
$$\{mh\} = [m_1 h_1 \quad m_2 h_2 \quad \ldots \quad m_n h_n]^T$$
$$\{u\} = [u_1 \quad u_2 \quad \ldots \quad u_n]^T \quad (4)$$
$$I = I_b + \sum_{i=1}^{n} I_i$$

In the above equations, m_i and m_b ($i = 1, 2, \ldots, n$; n = number of stories) are mass of the i^{th} floor and the foundation, respectively, h_i is the height of the i^{th} floor from the base, I_i and I_b are respectively the mass moments of inertia of the i^{th} story and the foundation, c_i and k_i are the damping coefficient and the lateral relative stiffness of the i^{th} story, respectively, c_{jj} and k_{jj} with $j = u$ or Ψ are respectively the damping and stiffness impedances of the supporting medium in translational and rotational directions, u_i, u_b, and u_g are the horizontal displacements of the i^{th} story, the foundation, and ground with respect to a fixed reference, respectively, Ψ is the rotational component of motion of foundation, and a dot represents derivation with respect to time.

As seen with the damping matrix in Eq. (2), the additional two last rows and columns pertaining to soil dampings, makes the total system damping matrix to be in essence non-classical, i.e., non-proportional to mass and stiffness matrices. Of course, it can be assumed to be proportional just as an approximate presumption.

2.2. The Modal Analysis

Eq. (1) can be rewritten as:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = p(t) \quad (5)$$

in which:

$$\{U\} = \begin{Bmatrix} \{u\} \\ u_b \\ \Psi \end{Bmatrix}$$

$$\{U(t)\} = \left\{ \{m\} \quad m_b + \sum_1^n m_i \quad \sum_1^n m_i h_i \right\}^T \ddot{u}_g(t) \quad (6)$$

It has been shown in reference [5] that when the damping matrix in Eq. (5) is not proportional to mass and stiffness, response to the base acceleration is as follows:

$$\{U\} = \sum_{j=1}^{N} \left[\{\alpha_j\} V_j(t) + \{\beta_j\} \dot{D}_j(t) \right] \quad (7)$$

in which $V_j(t)$ and $\dot{D}_j(t)$ are the pseudo-velocity and the relative velocity of the j^{th} mode equivalent SDF system with natural frequency p_j and damping ratio ξ_j, and $\{\alpha_j\}$ and $\{\beta_j\}$ are vectors of modal response distributions. Besides, the base shear is computed as the summation of lateral story forces as:

$$V^b(t) = \sum_{j=1}^{N} \left[(m_j^\alpha) p_j V_j(t) + (m_j^\beta) p_j \dot{D}_j(t) \right] \quad (8)$$

in which m_j^α and m_j^β are modal mass factors corresponding to the pseudo-velocity and the relative velocity of the j^{th} mode.

2.3. The Spectrum Analysis

One may use the SRSS rule to combine the maximum responses as:

$$\{U\}_{j\max} = \sqrt{\left(\{\alpha_j\} V_{j\max}\right)^2 + \left(\{\beta_j\} \dot{D}_{j\max}\right)^2}$$
$$V_{j\max}^b = \sqrt{\left((m_j^\alpha\} p_j V_{j\max}\right)^2 + \left((m_j^\beta\} p_j \dot{D}_{j\max}\right)^2} \quad (9)$$

The following relation can be assumed between the maximum values of $V_j(t)$ and $\dot{D}_j(t)$:

$$\dot{D}_j(t)_{\max} = \eta_j V_j(t)_{\max} \tag{10}$$

There have been different proposals for the conversion factor η_j in the above equation [5]. Pekcan suggested that:

$$\eta_j = f(T_j, \xi_j) = 0.8 - 0.6\xi_j + 0.17T_j + 0.4\xi_j T_j \tag{11}$$

Equation (11) is based on the random vibration analysis of an earthquake ground motion assumed to be a stationary function, and is used in this study [6].

3. Design Assumptions

For the purposes of this study, special steel moment frame structures being 1, 2, 4, 6 ... 18 and 20 story buildings are designed. All of the frames are supposed to be seismic resistant. The frames have three bays both ways each bay spanning 5m. The floor to floor heights are 3m. The residential buildings are designed according to ASCE 7-10 [7] and AISC-ASD [8]. The seismicity of the region is considered to be very high with the effective peak acceleration at the ground surface to be 0.35g. Two types of underlying soils are considered: a soft soil (soil type *D* [7]) and a very soft soil (soil type *E*). Their characteristics are given in Table (1).

Eleven buildings with the mentioned number of stories are designed with fixed bases for each soils type. Therefore, totally 22 buildings are considered in this study. For 1 to 4-story buildings, single footings, and for 6 to 20-story structures strip foundations are designed. Natural periods of the buildings in their first three modes of lateral displacement are shown in Table (2).

The design spectra are shown in Figure (2) for each soil type [7].

For time-history analysis of the buildings under study, earthquake records with the following

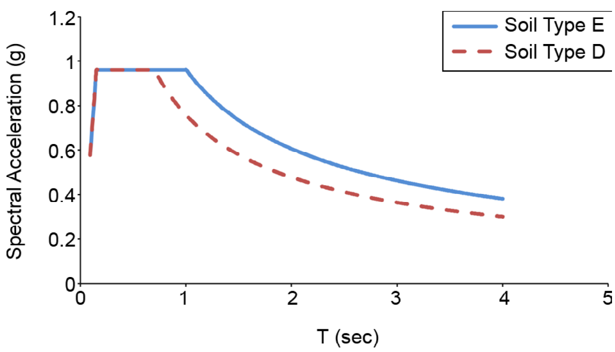


Figure 2. The design spectra for the soil types D and E.

Table 1. Characteristics of the soil types.

Soil Type	Shear Wave Velocity V_s (m/s)	Unit Mass ρ (kg/m^3)	Poisson's Ratio N	Bearing Capacity (kgf/cm^2)
D	250	1800	0.4	2
E	125	1700	0.45	1.5

Table 2. First three natural periods of the fixed-base buildings in lateral motion (s).

Soil Type	Type D			Type E		
No. of Stories	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
1	0.555	-----	-----	0.555	-----	-----
2	0.768	0.315	-----	0.768	0.315	-----
4	0.971	0.399	0.257	0.971	0.399	0.257
6	1.19	0.490	0.313	1.19	0.490	0.313
8	1.42	0.567	0.352	1.36	0.548	0.347
10	1.72	0.699	0.438	1.52	0.603	0.385
12	1.93	0.770	0.481	1.74	0.688	0.428
14	2.06	0.840	0.529	1.94	0.777	0.485
16	2.34	0.939	0.578	2.12	0.828	0.515
18	2.56	1.02	0.627	2.35	0.980	0.620
20	2.81	1.10	0.682	2.49	0.972	0.600

characteristics are selected out of the PEER NGA database [9]: 6.5 ≤ Magnitude ≤ 7.5, soil type is whether *D* or *E*. Two groups of earthquakes are selected for each building on each soil type regarding the epicentral distance, *R*. Group one, the near-field earthquakes with $R \leq 20$ km, and group two, the far-field earthquakes with 20 < R ≤ 50 km. Then the records are scaled according to ASCE7-10 [7], such that their response spectra does not fall below the design spectrum of Figure (2) between $0.2T$ and $1.5T$, where T is the fundamental period of the fixed-base buildings. Then 10 records in each distance group with scale factors closer to unity are retained for dynamic analysis of the same buildings. Modeling of soil-structure interaction

A stick model of the flexible-base 2D frames of the buildings is developed in Matlab. This model is necessary for non-classical analysis because such an analysis is not possible in commercial engineering software. The flexible base is modeled using a rigid foundation being in dimensions equivalent to the actual foundations of the 2D frames. For strip foundations (in 6 to 20-story buildings), the same dimensions are used. For single footings (in 1 to 4-story buildings), the length of the foundation element is taken to be equal to sum of the single foundation lengths in the corresponding 2D frames. Width of the foundation in this case is mean of the foundation widths in the corresponding frame. It can be shown that the mentioned assumptions are appropriate for the responses targeted in this study[5].

The base of the model rests on springs and dampers along the two degrees of freedom in plane as shown in Figure (3). Characteristics of the springs and dampers are given in the following sections.

4. Stiffness of the Soil Springs

The stiffness of soil springs is a function of the dynamic shear modulus of soil, *G*. The dynamic shear modulus can be much smaller than the static shear modulus, G_0, because of the large strains that develop in soil during an earthquake. The ratio G/G_0 depends on the soil type and the effective peak acceleration of ground motion at the ground surface. Values of G/G_0 for the two soil types are used according to ASCE 7-10 [7].

According to ASCE41-13[10], before determination of the spring stiffnesses, condition of the foundation, being flexible or rigid with regard to the underlying soil must be determined. The above criterion results in all of the foundations of this study resting on the soil type *E* to be rigid. For the soil type *D*, only foundations of 1, 2 and 4-story buildings prove to be rigid. For flexible foundations, a uniformly distributed vertical spring is utilized ASCE41-13 [10]. For rigid foundations, coupling of vertical and rocking degrees of freedom is taken into account using a non-uniform distribution of vertical springs.

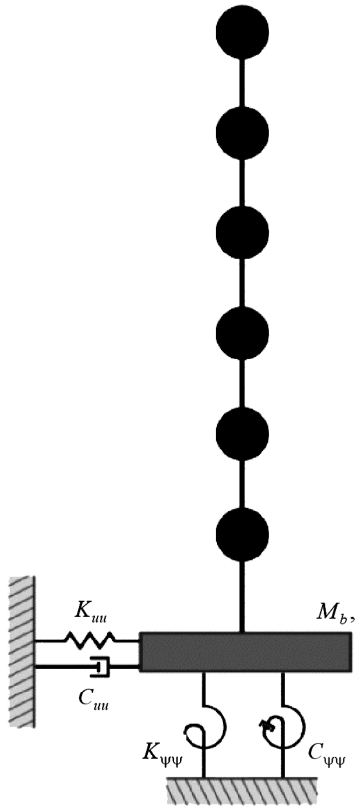


Figure 3. Configuration of the stick model.

For this purpose, each foundation is divided to interior and exterior zones. The exterior zones are two rectangles, one at each end of the foundation, with a length of *B*/6 (*B* = foundation width) and a width equal to that of the foundation [10].

5. Damping

Design spectra, as shown in Fig. 2, are given for a damping ratio of 0.05, as common. For SSI applications, the spectral values will be needed for other damping ratios. This is usually done using a spectral reduction factor RF [5].The damping coefficients in the horizontal and rocking degrees of freedom have been given as [11]:

$$C_{uu} = pV_{la}\,A_b$$
$$C_{\psi\psi} = pV_{la}\,I_{by}\,\tilde{c}_{ry} \tag{12}$$

where p is the unit mass of soil, A_b is the area of foundation in plan, I_{by} is the moment of inertia of the foundation in plan about the transverse axis, V_{la} is a wave velocity equal to $\frac{3.4V_S}{\pi(1-\nu)}$ with V_S being the shear wave velocity, and $\tilde{c}_{ry}$ is a coefficient beginning from zero for the static case and tending to unity for very large excitation frequencies. It is tuned to the fundamental frequency having no sensible effects on the total response if tuned to other lower modes in this study.

6. The Analysis Results

6.1. Responses in the Fundamental Mode

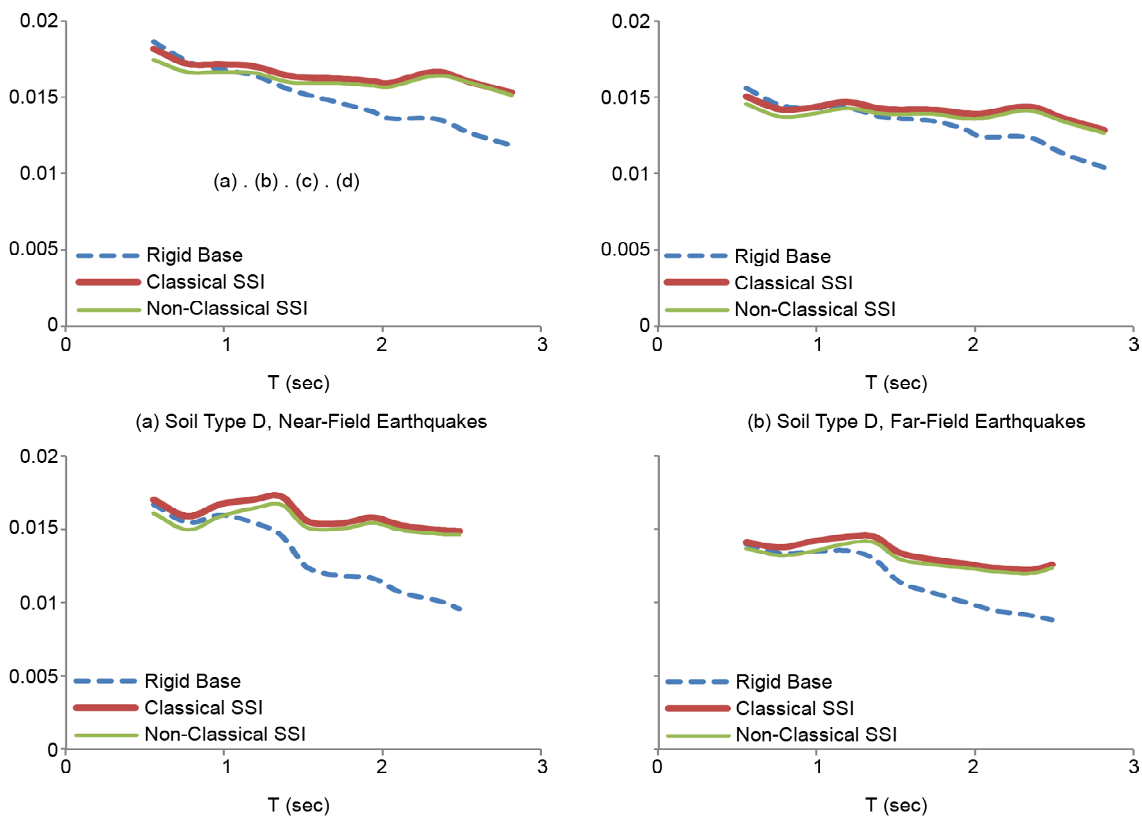
A modal time history analysis is accomplished in the section. All of the response parameters are shown versus the fixed-base period of each building. Figure (4) shows the averaged maximum story drift ratios.

According to Figure (4), from a period of about 1 second, story drifts of the flexible-base models overtake those of the fixed-base model considerably, for both categories of earthquakes. The relative difference of drifts between models increases with the height such that for the soil types D and E it reaches about 33% and 56%, respectively, for larger periods. For periods smaller than 1 s, difference between the drifts calculated by the three models is small, and effect of SSI on displacements is negligible.

It is interesting that in the same period range, the non-classical analysis results in drifts smaller than those of the fixed-base model.

The maximum base shear of each system normalized to its weight is shown in Figure (5).

Figure (5) shows that SSI decreases the base shear on both soil types. The reduction is important from the same 1 s period mentioned in drift analysis. The more rigorous non-classical analysis procedure is similar in results to the fixed-base case for periods smaller than 1 s and to the classical procedure for larger periods. The base shear reduction is up to 23% and 38% for taller buildings. For periods smaller than 1 s, soil-structure interaction effects should be disre-



(a) Soil Type D, Near-Field Earthquakes

(b) Soil Type D, Far-Field Earthquakes

(c) Soil Type E, Near-Field Earthquakes

(d) Soil Type E, Far-Field Earthquakes

Figure 4. The averaged maximum story drift ratios of each building.

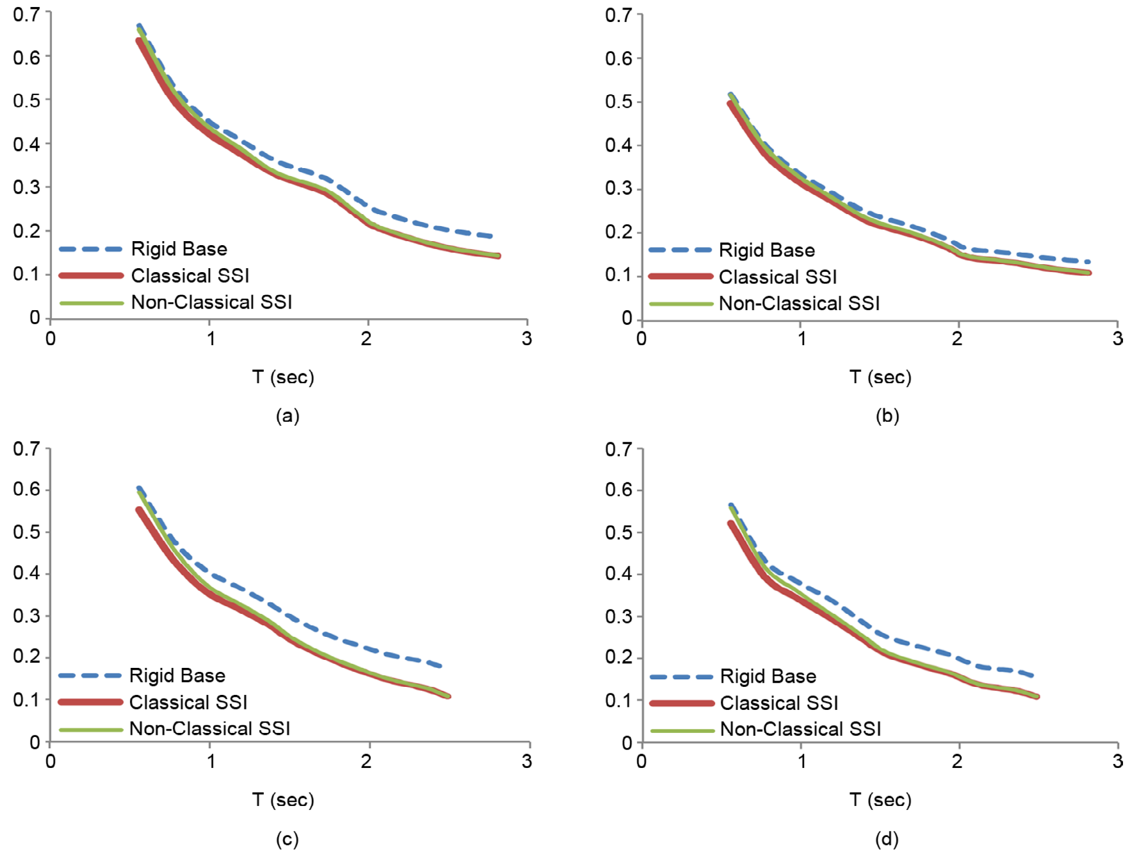


Figure 5. Averaged maximum base shear normalized to the building weight.

garded for seismic analysis of structures similar to the ones in this study.

6.2. Responses in the Higher Modes

In the current section, sum of the response corresponding to all other modes (called the higher modes) is illustrated. The averaged maximum values of story drifts and base shear normalized to the building height and building weight, are shown in Figures (6) and (7), respectively, for the three analysis cases. The values have also been averaged between near and far-field earthquakes.

Based on Figures (6) and (7), it can be said that the response in the higher modes can equally be calculated using classical or non-classical analysis. Therefore, the use of non-classical analysis is not necessary for the higher modes. On the other hand, accounting for SSI in the higher modes is important for systems with fixed-base fundamental periods larger than 2.5 s when calculating displacements

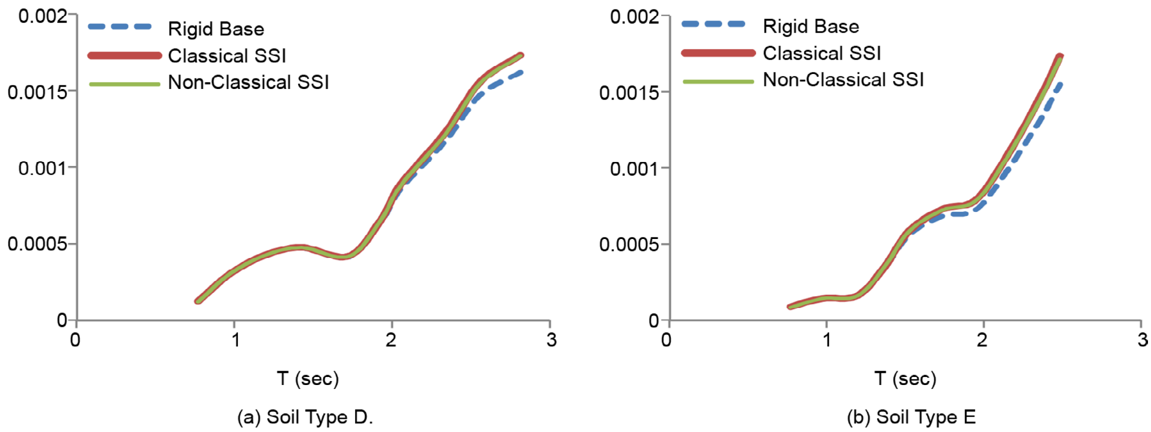


Figure 6. The averaged maximum story drifts corresponding to the higher modes, normalized to the building height.

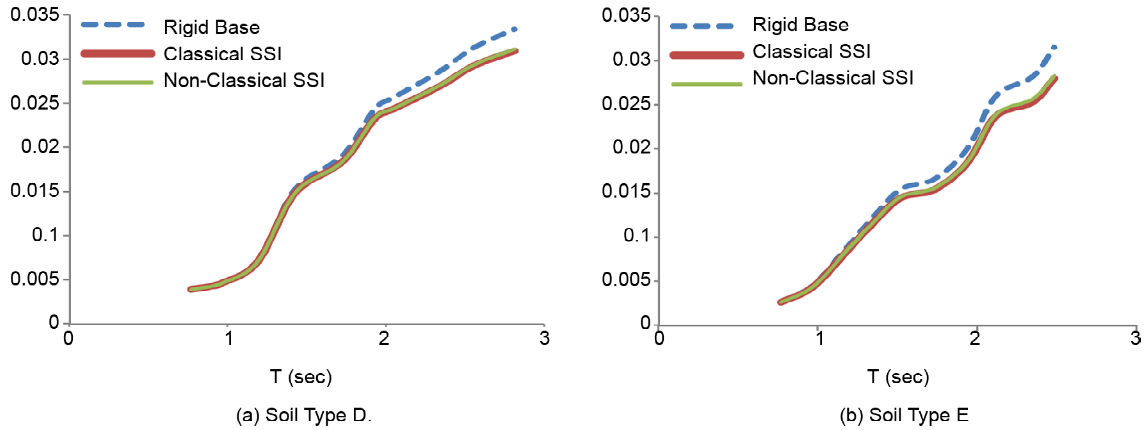


Figure 7. The averaged maximum base shear corresponding to the higher modes, normalized to the building weight.

and larger than 2 s when deriving the base shear. In such ranges, higher mode displacements increase and base shears decrease considerably due to SSI.

7. Conclusions

In this study, the theory of the modal analysis of non-classical systems was illustrated for systems resting on flexible bases, representative of soil-structure interaction problems. Then, several structures having one to twenty stories resting on two types of soils, being medium and soft, were analyzed each one under two suits of ten consistent and scaled earthquake motions specific to that structure recorded at near and far distances. Modal time history analysis was accomplished in its classical and non-classical versions. It was shown that the relative difference of displacements between the associated fixed-base and flexible-base models became important from a period of about 1 s and increased with height, and was larger for the softer soil. For periods smaller than 1 s, the non-classical analysis resulted in displacements somewhat smaller than those of the fixed-base model. SSI decreased the base shear on both soil types. The reduction was important again from a period of 1 s.

Results of SSI analysis with the non-classical procedure were similar to the fixed-base case for periods smaller than 1 s and to the classical SSI procedure for larger periods. The base shear reduction was up to 23% and 38% for taller buildings. For periods smaller than 1 s, soil-structure interaction effects should be ignored for seismic analysis of moment frame structures resting on surface foundations. Besides, the use of non-classical SSI analysis is not necessary for the higher modes. Accounting for SSI in the higher modes is important for systems with fixed-base fundamental periods larger than 2.5 s when calculating drifts, and larger than 2 s when deriving the base shear.

The above conclusions were based on the study, as outlined in this paper, on buildings consisting of moment frames resting on surface foundations. This study can further be extended to other structural and foundation systems in order to make general conclusions.

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