Research Paper

A New Variable Step Size Adaptive Blind Sources Separation for Online Structural Modal Identification

Vida Ghasemi[1] and Fereidoun Amini[2*]

1. Ph.D., School of Civil Engineering, Iran University of Science and Technology, Tehran, Iran
2. Professor, School of Civil Engineering, Iran University of Science and Technology, Tehran, Iran,
*Corresponding Author; email: famini@iust.ac.ir

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ABSTRACT

The Equivariant adaptive separation by independence (EASI) algorithm, as an online blind structural identification method, is very important not only to better understand the structural response but also to conduct an efficient maintenance and management strategy. However, the traditional EASI algorithm has some drawbacks. It uses a constant step-size parameter and requires establishing a trade-off between the misadjustment in the steady-state and the convergence rate. This paper proposes a new variable step-size equivariant adaptive source separation via independence (VS-EASI) algorithm for online blind modal identification of structures. Unlike the traditional EASI algorithm, the proposed algorithm adaptively updates its step-size based on the input signals and the unmixing matrix, through establishing a new function between the step-size and the separating indicator. This results in a better performance for the proposed method, and fast convergence speed is achieved while the steady-state error is low. Furthermore, this algorithm mitigates the irrelevant noise, making it more suitable than the EASI algorithm for practical applications. Simulation results of synthetic examples and a benchmark structure verify the superior convergence and better performance of the proposed algorithm in the steady-state over the conventional EASI with a fixed step-size in stationary environments as well as non-stationary ones.

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1. Introduction

Recently, structural identification which plays a key role in controlling structures against environmental excitations has attracted a lot of attention (Azizi, et al., 2023; Chan, et al., 2018; Ghaderi & Amini, 2019). However, although the conventional equivariant adaptive separation by independence (EASI) algorithm is efficiently implemented in online blind structural identification, it presents some challenges. The conventional EASI algorithm uses a constant step-size parameter (Ghasemi & Amini, 2020). In this case, a small step-size and its cosequent small steady-state error cause slow convergence, and a large step-size is required for fast adaptation that leads to large misadjustment in the steady-state. A proper tradeoff between the steady-state misadjust-ment and the convergence rate is required, in the conventional EASI method. Efficient methods with high convergence speed without losing accuracy are very important and have become of great interest in online applications.

The output-only system identification methods (Ghahremani & Bitaraf, 2021; Khan, et al., 2016), such as blind source separation (BSS) methods (Morovati & Kazemi, 2016; Sadhu & Goli, 2017), are more practical and more useful, due to the immeasurability of the involved forces. BSS approaches are based on the idea of recovering sources using a variety of output measurements without prior information on the mixing process or the source signals. Among different BSS methods, Independent Component Analysis (ICA) (Furukawa & Kiyono, 2007; Mahvash & Lakis, 2016) and Second-Order Blind Identification (SOBI) (Poncelet et al., 2007; Rainieri, 2014) are the most popular. There are extensive examples of BSS methods used in system identification (SI), especially in the modal identification of structural (McNeill, 2012) and mechanical systems (Cheng et al., 2014), to date.

In general, BSS algorithms can be cataloged into two categories; batch-based and adaptive (sequential) techniques. In batch-based techniques, all data are used at every step. AMUSE (Tong et al., 1990), SOBI (Rainieri, 2014) and Fast-ICA (Hyvarinen & Oja, 2000) are different types of batch-based algorithms. Besides, these algorithms require large storage capacity, whereas online algorithms store only a little data; therefore, they are very suitable for time-varying environments. The iterative inversion ICA algorithm (Cruces-Alvarez et al., 2000), the natural gradient algorithm (NGA) (Ghasemi et al., 2019; Rattray et al., 1998), and the equivariant adaptive source separation via independence (EASI) algorithm (Cardoso & Laheld, 1996) are all online algorithms.

With fast convergence rate, lower computational complexity, small storage capacity, and the ability to track time-varying environments, online algorithms are good options for civil engineering applications, particularly in online structural identification and control of structures.

In adaptive BSS, two important factors affect the performance of algorithms; the rate of convergence and the misadjustment in the steady-state. Generally, the smaller steady-state error is required to more accurate source separation, and a faster convergence speed leads to quickly track the variations of the system (Ghasemi & Amini, 2024). The step-size controls the magnitudes of the update values of the separating matrix. The performance of blind separation is significantly dependent on the magnitudes of the update values, (Nakajima et al., 2009).

However, most of the known adaptive BSS algorithms such as the natural gradient algorithm (Rattray et al., 1998) and the EASI algorithm (Amini & Ghasemi, 2018; Cardoso & Laheld, 1996) usually assume the step-size to be a fixed small positive scalar. A fixed step-size approach often results in a relatively slow convergence rate or large steady-state error (Jafari et al., 2004).

To achieve fast convergence, a large value must be chosen while for a small steady-state error, a smaller value is required. Therefore, there is a tradeoff between convergence rate and precision. In nonstationary environments, it is critical to select the step-size appropriately. A step-size suitable for one environment may prove inappropriate for another environment because poor separation or divergence in the separation coefficients might occur. To solve this problem, variable step-size algorithms that vary online depending on measurements that account for the evolution of algorithm performance have been proposed.

Douglas and Cichocki (1998) put forward an adaptive step-size method for BSS algorithms. They presented variable step-size natural gradient algorithms. Recently, in (Douglas & Gupta, 2007), a scaled natural gradient method corresponding to various adaptive step-size methods was proposed. Von Hoff and Lindgren (2002) improved the adaptive blind source separation method by usuing a step-size that was built from the square of the running average of the coefficients of the estimating function. Zhang et al. (2003) came up with the idea of updating the step-size based on changes in correlation coefficients of extracted signals.

A variable step-size EASI algorithm for adaptive blind separation was presented by Chambers and Jafari (Chambers et al., 2004). They used a gradient adaptive step size for matching the performance of the algorithm to the changes of inpus dynamic and mixing matrix. Yuan et al. (Yuan et al., 2005) employed the sign natural gradient algorithm, which utilizes a gradient adaptive step-size parameter to improve its efficiency.

The identification of systems that have sudden changes in the environment has attracted the attention of Chen et al. (Chen et al., 2013). They proposed a retrospective online EASI method to solve this problem. Wang et al. (Wang et al., 2017) proposed recursive least-squares (RLS) and Recursive-EASI algorithms for the identification of time-varying systems with time-varying sources.

However, in none of the above studies, the proposed VS-EASI algorithms have been investigated for civil engineering applications. Also, the effect of noise on the performance of the algorithms has not been studied in the mentioned articles. Since noise is always present in real systems like civil buildings; therefore, the performance of any adaptive system in the presence of noise should be studied.

This paper presents a novel variable step-size equivariant adaptive source separation via independence (VS-EASI) algorithm to overcome the conflict between convergence rate and steady-state error in conventional online blind modal identification of structures.

This paper introduces and establishes an efficient function between the step-size and the unmixing indicator. The proposed algorithm adaptively updates its step-size to adjust its performance based on the input signals and the unmixing matrix. In this algorithm, the step-size depends on both the current and the previous step values. Also, this proposed algorithm is robust against noises. This algorithm mitigates the irrelevant noise, making it more suitable for practical applications.

Some key features and abilities of the proposed method in comparison with the traditional EASI method are summarized as follows:

a) The fast convergence speed is realized,

b) The steady-state error is kept low.

c) The performance of the algorithm against noise is improved.

This paragraph introduces the organization of the rest of the paper. Section 2 briefly reviews the EASI algorithm. In Section 3, the proposed new variable step-size EASI algorithm is presented. Section 4 demonstrates the numerical simulation in stationary and non-stationary environments and in noisy cases. In section 5, the performance of the proposed method is evaluated using a benchmark structure. Finally, Section 6 presents the conclusions.

2. Adaptive Blind Source Separation and EASI Algorithm

The object of blind source separation (BSS) is to restore the source signals from output linear mixtures without having any information about the initial source signals and the mixing matrix. The BSS model, with m observed signals generated by n sources, is formulated as Equation (1).

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{1}$$

where $\mathbf{x}(t)$ is a vector of output signals, $\mathbf{A} \in R^{m \times n}$ is an unknown mixing matrix and $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_n(t)]^T \in R^n$ is a source vector.

Figure (1) illustrates an adaptive BSS where the unmixing matrix $\mathbf{B}(t)$ is updated at each step. One of the widely used iterative methods for updating the unmixing matrix is the "serial updating" technique, which is used in the Equivariant Adaptive Separation

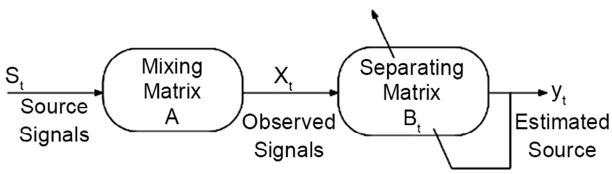

Figure 1. Depiction of an adaptive BSS algorithm.

via Independence (EASI) (Cardoso & Laheld, 1996). Essential assumptions used in the EASI algorithm are described below:

a) Matrix **A** is full rank with $n \le m$.

b) Each component of $\mathbf{s}(t)$ is a stationary zero mean process.

c) At each t, $\mathbf{s}(t)$ components are mutually statistically independent.

d) $\mathbf{s}(t)$ components have unit variance.

The EASI algorithm for adaptive source separation is given by Equation (2).

$$\mathbf{B}(t+1) = \mathbf{B}(t) + \lambda \times \left[\mathbf{I} - \mathbf{y}(t)\mathbf{y}^T(t) - g(\mathbf{y}(t))\mathbf{y}^T(t) + \mathbf{y}(t)g^T(\mathbf{y}(t))\right]\mathbf{B}(t) \tag{2}$$

where λ is a small positive constant and is called the step-size. $g(.)$ is a component-wise arbitrary nonlinear function, and $\mathbf{y}(t)$ is the output of $\mathbf{B}(t)$.

3. Novel Variable Step-Size EASI

Selecting a proper step-size in the EASI algorithm is crucial to its performance. Using a fixed step-size can limit the convergence speed, while using an adaptive step-size improves tracking efficiency in a non-stationary environment. The classical versions of adaptive blind source separation algorithms, such as the EASI algorithm, use a constant step-size parameter (Amini & Ghasemi, 2018) and need to establish a tradeoff between a high convergence rate versus low misadjustment.

3.1. Determination of Separating Indicator

In adaptive systems, the distance between the estimated parameter and its optimal value is used to control the step-size parameter. However, in the EASI method, it is impossible to measure such a distance directly, because the optimal values of the mixing and separating matrices are not known in advance. Therefore, it is needed to consider a represented function (von Hoff & Lindgren, 2002).

To do this, consider the global system $\mathbf{V}(t) = \mathbf{B}(t).\mathbf{A}(t)$ that is independent of the mixing matrix $\mathbf{A}(t)$. According to this transformation, the Equation (2) can be rewritten as:

$$\mathbf{V}(t+1) = \mathbf{V}(t) + \lambda \times \left[\mathbf{I} - \mathbf{y}(t)\mathbf{y}^T(t) - g(\mathbf{y}(t))\mathbf{y}^T(t) + \mathbf{y}(t)g^T\mathbf{y}(t)\right]\mathbf{V}(t) \tag{3}$$

Due to the equivariance property of the EASI algorithm, it is sufficient to examine the behavior of $\mathbf{V}(t)$ instead of $\mathbf{A}(t)$. With regard to the definition of the V matrix:

$$\mathbf{V} = \mathbf{V}_{eq} + \tilde{\mathbf{V}} \tag{4}$$

where $\tilde{\mathbf{V}}$ is error matrix. $\mathbf{V}_{eq}$ is optimal of value of **V** and around the equilibrium $\mathbf{V}_{eq} = \mathbf{I}$. The object of this section is to obtain represented function to calculate the error matrix. For this purpose, we first want to examine whether **F** can be considered as a measure of the distance between **A** and $\mathbf{A}_{eq}$.

Describe the separating indicator **F** to first order around equilibrium $\mathbf{V}_{eq}$.

$$\mathbf{F} == \mathbf{I} - \mathbf{y}\mathbf{y}^T - \mathrm{g}(\mathbf{y})\mathbf{y}^T + \mathbf{y}\mathrm{g}^T(\mathbf{y}) \tag{5}$$

$$\begin{aligned}&\approx \mathbf{I} - \mathbf{V}_{eq}\mathbf{s}\mathbf{s}^T\mathbf{V}_{eq}{}^T - \mathrm{g}(\mathbf{V}_{eq}\mathbf{s})\mathbf{s}^T\mathbf{V}_{eq}{}^T + \\ &\mathbf{V}_{eq}\mathbf{s}\mathrm{g}^T(\mathbf{V}_{eq}\mathbf{s}) - \tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T\mathbf{V}_{eq}{}^T - \mathbf{V}_{eq}\mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T - \\ &\mathbf{D}_{g'}(\mathbf{V}_{eq}\mathbf{s})\tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T\mathbf{V}_{eq}{}^T - \mathrm{g}(\mathbf{V}_{eq}\mathbf{s})\mathbf{s}^T\tilde{\mathbf{V}}^T + \\ &\tilde{\mathbf{V}}\mathbf{s}\mathrm{g}^T(\mathbf{V}_{eq}\mathbf{s}) + \mathbf{V}_{eq}\mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T\mathbf{D}_{g'}{}^T(\mathbf{V}_{eq}\mathbf{s})\end{aligned} \tag{6}$$

As mentioned before, $\mathbf{V}_{eq} = \mathbf{I}$ therefore:

$$\begin{aligned}\mathbf{F} &\approx \mathbf{I} - \mathbf{s}\mathbf{s}^T - \mathrm{g}(\mathbf{s})\mathbf{s}^T + \mathbf{s}\mathrm{g}^T(\mathbf{s}) - \tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T - \\ &\mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T - \mathbf{D}_{g'}(\mathbf{s})\tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T - \mathrm{g}(\mathbf{s})\mathbf{s}^T\tilde{\mathbf{V}}^T + \\ &\tilde{\mathbf{V}}\mathbf{s}\mathrm{g}^T(\mathbf{s}) + \mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T\mathbf{D}_{g'}{}^T(\mathbf{s})\end{aligned} \tag{7}$$

Where

$$\mathbf{D}_{g'} = diag\left(\left[g_1'(s_1), \ldots, g_N'(s_N)\right]\right) \tag{8}$$

Assuming $\tilde{\mathbf{V}}(t)$ is independent of $\mathbf{s}(t)$ and in the equilibrium condition

$$E\left\{\mathbf{I} - \mathbf{s}\mathbf{s}^T - \mathrm{g}(\mathbf{s})\mathbf{s}^T + \mathbf{s}\mathrm{g}^T(\mathbf{s})\right\} = 0 \tag{9}$$

Then, the expectation value of Equation (7) is obtain

$$\begin{aligned}\{\mathbf{F}\} &= -\left\{\tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T\right\} - E\left\{\mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T\right\} - E\left\{\mathbf{D}_{g'}\tilde{\mathbf{V}}\mathbf{s}\mathbf{s}^T\right\} - \\ &E\left\{\mathrm{g}(\mathbf{s})\mathbf{s}^T\tilde{\mathbf{V}}^T\right\} + E\left\{\tilde{\mathbf{V}}\mathbf{s}\mathrm{g}^T(\mathbf{s})\right\} + E\left\{\mathbf{s}\mathbf{s}^T\tilde{\mathbf{V}}^T\mathbf{D}_{g'}{}^T(\mathbf{s})\right\}\end{aligned} \tag{10}$$

The Equation (10) can be rewritten as Equations (11) and (12)

$$\begin{aligned}\left\{f_{ij}(t)\right\} &= -E\left\{s_j^2\right\}\tilde{v}_{ij} - E\left\{s_i^2\right\}\tilde{v}_{ji} - \\ &E\left\{g_i'(s_i)\right\}E\left\{s_j^2\right\}\tilde{v}_{ij} - E\left\{g_i(s_i)s_i\right\}\tilde{v}_{ji} + \\ &E\left\{s_j g_j(s_j)\right\}\tilde{v}_{ij} + E\left\{s_i^2\right\}E\left\{g_j'(s_j)\right\}\tilde{v}_{ji} \qquad i \neq j\end{aligned} \tag{11}$$

$$E\{f_{ii}(t)\} = -E\{2s_i^2\}\tilde{v}_{ii} \qquad i = j \tag{12}$$

From Equations (11) and (12), it is seen that E{**F**(t)} can be considered as measure of the error matrix $\tilde{\mathbf{V}}(t)$. Therefore, the $\mathbf{F}(t)$ can be used for step-size control in the EASI algorithm (Ghasemi & Amini, 2024).

3.2. Variable-Step Size Algorithm

In the previous section, it was proved that the separating indicator $\mathbf{F}(t)$ can be utilized as an error matrix. In addition, in order to mitigate the adverse effect of noises, the smooth form of the separating indicator is considered as the cost function. Therefore, in this paper, an appropriate error measure function as given in Equation (13), is defined to control the step-size parameter.

$$\bar{\mathbf{F}}(t) = \beta_1\bar{\mathbf{F}}(t-1) + (1-\beta_1)\mathbf{F}(t) \tag{13}$$

where $0 << \beta_1 < 1$, is forgotten factor.

In the course of adaptation, the algorithm step-size is changed in the form of Equation (14), which depends on $\bar{\mathrm{F}}(t)$.

$$\lambda(t) = \beta_2\lambda(t-1) + (1-\beta_2)\bar{\mathbf{F}}(t), \bar{\mathbf{F}}(t) \tag{14}$$

where $0 << \beta_2 < 1$ is a weighting factor controlling the averaging process. The inner product of matrices is defined as:

$$\langle \mathbf{C}, \mathbf{D} \rangle = tr\left(\mathbf{C}^T\mathbf{D}\right) \tag{15}$$

where $\langle , \rangle$ and *tr*(.) denote the inner product and the trace operator, respectively, and $C, D \in \mathbb{R}^{m\times n}$.

In the following, to accelerate the convergence rate $\rho(t)$ is introduced. This function decreases exponentially with time.

$$\rho(t) = \rho_0 e^{-bt} \tag{16}$$

ρ_0 and *b* are positive constants. Therefore, ρ(.) is positive function. Substituting Equations (5), (13), (15) and (16) into Equation (14) yields:

$$\lambda(t) = \beta_2\lambda(t-1) + (1-\beta_2)\,\rho_0 e^{-bt}\, tr\left(\bar{\mathbf{F}}^T(t)\bar{\mathbf{F}}(t)\right) \tag{17}$$

Equations (2) and (17) together represent the proposed VS-EASI algorithm.

4. Simulation Study

This section investigates the performance of the proposed variable step-size EASI algorithm for stationary and non-stationary environments and its robustness against noise. The convergence rate and the errors in the steady state of the proposed VS-EASI algorithm are compared with the fixed step-size EASI algorithm and the variable step-size EASI algorithm that wasproposed by Chambers et al. (Chambers et al., 2004). The criterion for choosing the better algorithm is based on comparing the values of performance indexes (PI) of all algorithms.

4.1. Stationary Environment

To simulate a stationary environment, two uniformly distributed signals with zero mean and unity variance, which were sub-Gaussian, were considered as sources and mixed with a 2×2 mixing matrix, and the simulation was run for 50 trials to calculate the performance index (PI) resulting from mentioned approaches. A cubic nonlinearity *g*(.) is employed for all simulations.

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 1 & 0.5 \end{bmatrix} \tag{18}$$

The performance index is computed as a function of the system matrix $\mathbf{V} = \mathbf{B}(t)\mathbf{A}$, $(\mathbf{B} \in \mathbb{R}^{n\times n})$, i.e. and employed to assess the proposed algorithm.

$$PI(\mathbf{V}) = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{|v_{ij}|}{max_j |v_{ij}|} - 1\right) + \sum_{j=1}^{n}\left(\sum_{i=1}^{n}\frac{|v_{ij}|}{max_i |v_{ij}|} - 1\right) \tag{19}$$

where v_{ij} is the ij^{th} element of **V**.

The value of $\lambda(t)$ for all algorithms is initialized at 0.0002 and $\mathbf{B}_0 = 0.5\times I$ (I is the identity matrix). The parameters of the proposed method are set to $\beta_1 = \beta_2 = 0.98$, $\rho_0 = 0.01$, $b = 0$ and for VS-EASI by Chambers et al. (Chambers et al., 2004) $\rho_0 = 1\times10^{-5}$. Figure (2) shows the average values of PI obtained from the simulations for all algorithms.

It can be observed that the proposed algorithm generates the fast convergence rate compared to the other two algorithms. Also, both VS-EASI algorithms keep the misadjustment low in the steady state.

The average evolution of $\lambda(t)$ for the proposed algorithm is depicted in Figure (3). As shown in

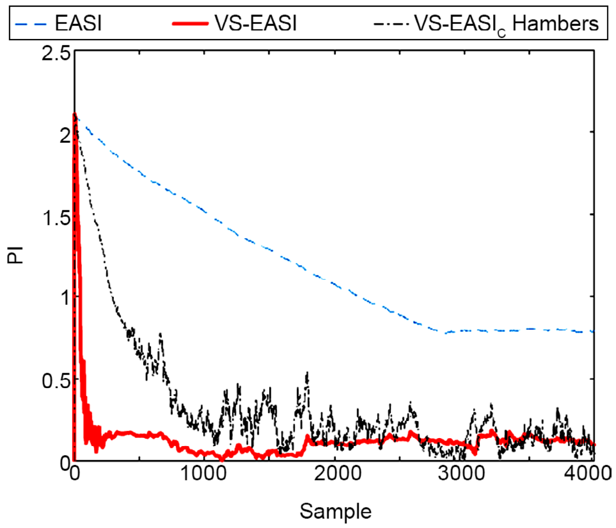

Figure 2. Assessment of the PI for proposed VS-EASI, VS-EASI (Chambers 2004) and EASI algorithms for the stationary environment.

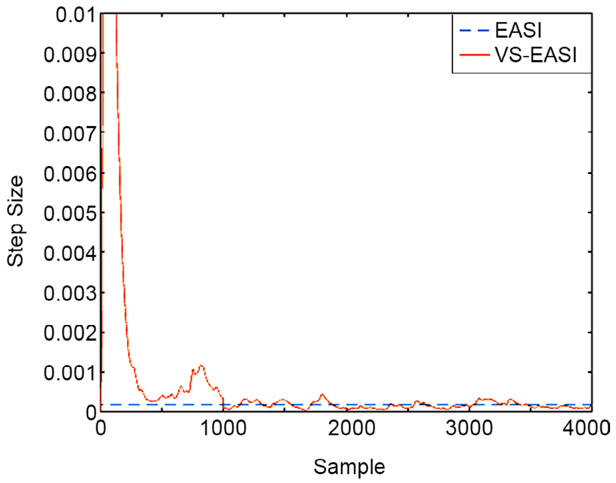

Figure 3. Evolution of step-size for EASI and proposed VS-EASI algorithms for the stationary environment.

this figure, the value of the designed step-size is adaptively modulated corresponding to the time-varying changes of the input signals and the separation matrix. It grows quickly in the beginning and keeps varying over time while being smoothly reduced in the steady-state. Finally, in the steady-state, $\lambda(t)$ of the proposed VS-EASI converges to $\lambda_0 = 0.0002$.

4.2. Non-Stationary Environment

This section compares the performance of the conventional EASI algorithm with that of the new VS-EASI algorithm and the VS-EASI algorithm proposed by Chambers et al. in a non-stationary environment. To simulate this, the following time-varying mixing matrix is chosen as:

$$\mathbf{A}(t) = \mathbf{A} + \mathcal{H}(t) \tag{20}$$

$$\mathcal{H}(t) = \eta \mathcal{H}(t-1) + \kappa \mathrm{randn}(t) \tag{21}$$

The $\lambda(t)$ parameter was initialized for all the algorithms at 0.0002. Also, the other parameters are the same as the stationary case. Here, randn(.) is a function that produces an n×n matrix with zero mean and one variance random element, and A is as defined in Equation (18).

The initial $\mathcal{H}_0$ in Equation (21) is set to a null matrix and the parameters μ and κ are set to 0.9 and 0.03, respectively. Figures (4) and (5) present the average values of PI and λ(t) for traditional EASI and VS-EASI algorithms, respectively. The results show that the proposed algorithm performs well in this non-stationary environment.

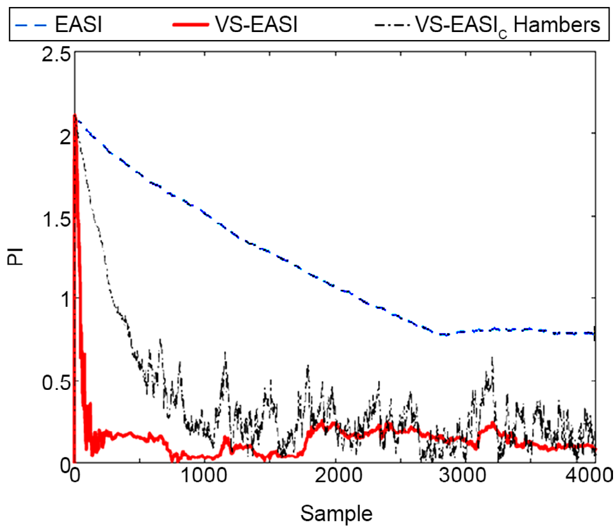

Figure 5. Assessment of the PI for EASI, VS-EASI (Chambers 2004) and proposed VS-EASI algorithms for the non-stationary environment.

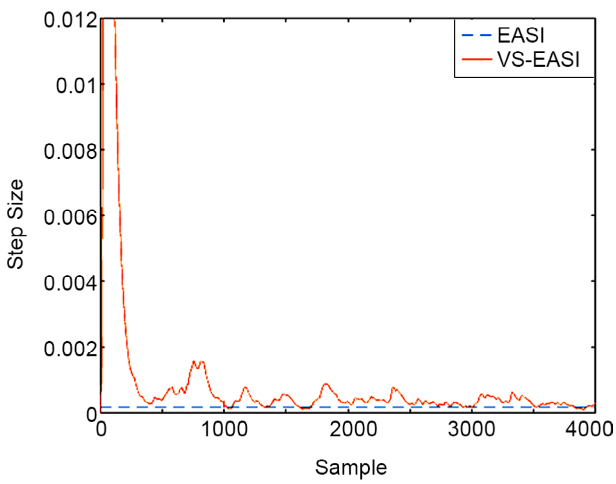

Figure 5. Evolution of step-size for EASI and proposed VS-EASI algorithms for nonstationary environment.

The convergence speed of this algorithm is high compared to other algorithms and it also causes fewer errors in the steady state.

For stationary and non-stationary environments, when **A**(t) varies and the matrix **V**(t) moves away from the optimal matrix $V_{eq}(t)$, the error increases and is reflected in $\bar{\mathbf{F}}(t)$ then λ(t) increases in turn. With a large λ(t), the VS-EASI algorithm adapts **B**(t) more quickly to move **V**(t) toward $\mathbf{V}_{eq}$ (t). Afterward, when **V**(t) is close to $\mathbf{V}_{eq}(t)$ the error and $\bar{\mathbf{F}}(t)$ decrease. Reducing $\bar{\mathbf{F}}(t)$ reduces λ(t), as shown in Figures (3) and (5).

4.3. Robustness of Proposed VS-EASI Algorithm Against Noise

In this section, the robustness of the VS-EASI algorithm to noise is investigated. To do this, the stationary sources are polluted by white Gaussian noise. Because of the non-deterministic characteristic of noise, 100 distinct separation are performed. The signal-to-noise ratios (SNRs) of 6.12, 8.95, 11.83, and 12.66 dB are considered for each output channel for the simulation study. The mean values of PI calculated for EASI, VS-EASI proposed by Chambers, and proposed VS-EASI in this article are demonstrated in Figure (6).

Figure (6) also illustrates that both the proposed VS-EASI and Chamber's algorithm have good performance in the presence of noise, but the proposed algorithm is faster and and has a smoother behavior in the steady state. It is worth noting that the obtained results were similar for the non-stationary environment.

5. Numerical validation of proposed VS-EASI Algorithm in Structures

This section is devoted to the validation of proposed VS-EASI algorithm in structural buildings.

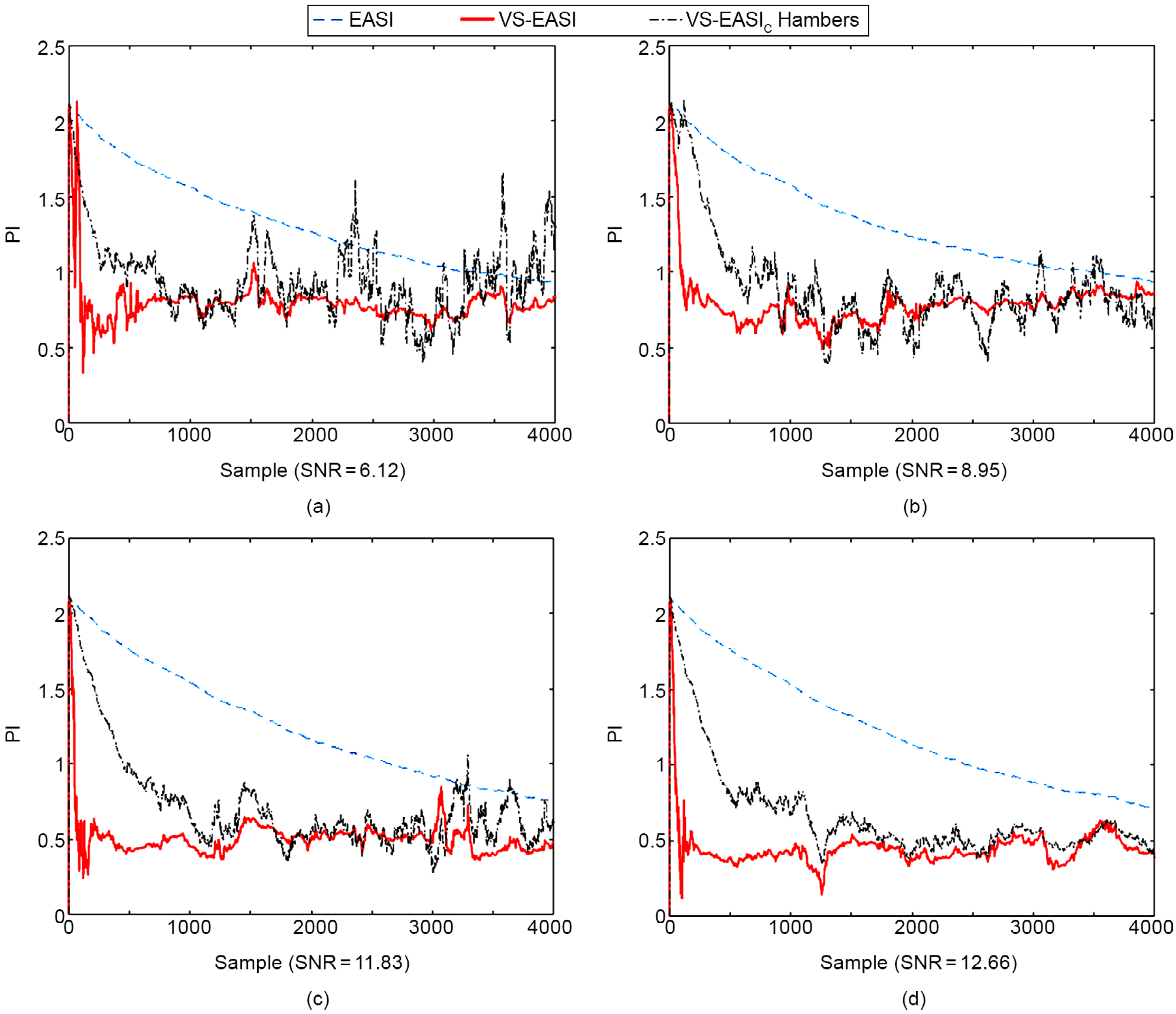

Figure 6. Effectiveness of the proposed VS-EASI algorithm against noise compered to EASI and VS-EASI (Chambers et al., 2004).

5.1. Model Expansion

For a linear and lumped-mass n-degree-of-freedom (n-DOF) structural system excited under a force vector **f**(t), equations of motion can be expressed as Equation (22).

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{22}$$

where **M**, **K** and **C** are symmetric mass, stiffness damping and damping matrices, respectively. **x**(t), $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ are the displacement, velocity and acceleration vectors, respectively. For proportionally damped systems, the displacement responses can be expressed in modal space as Equation (23).

$$\mathbf{x}=\mathbf{\Phi q} \tag{23}$$

where $\mathbf{\Phi}\in\mathbb{R}^{n\times n}$ and $\mathbf{q}\in\mathbb{R}^{n\times 1}$ are the modal matrix and modal coordinate vector, respectively. Excitations are considered as stationary white noise. Therefore, the $q_j(t)$ modal coordinate can be written as Equation (24).

$$q_j(t)=\alpha_j e^{\zeta_j\omega_j t}\sin\left(\omega_{dj}t+\theta_j\right),\qquad \omega_{dj}=\omega_j\left(1-\zeta_j^2\right)^{1/2} \tag{24}$$

where $\zeta_j, \omega_j, \omega_{dj}, \theta_j$ and α_j are the damping ratio, natural modal frequency, damped modal frequency, phase lag and constant parameter, respectively.

Given the similarity between Equations (1) and (23), one can say that in civil buildings, modal coordinates can be treated as independent sources when they are uncorrelated. Consequently, the modal identification of structures will be based on the frame work of BSS problem.

5.2. Benchmark Simulation

To assess the efficiency of the proposed VS-EASI algorithm in structural buildings, the benchmark problem shown in Figure (7) is considered, see references (Azam, et al., 2017; De Callafon, et al., 2008). This shear building has eight stories with a floor mass of 625 tons and an inter-story stiffness of 10^6 kN/m. The measured responses in two states, free vibration and forced vibration, will be used for system identification purposes. For the free vibration response, the benchmark building model is subjected to a short impulse, and for random vibration responses, ambient vibrations are modeled by Gaussian white noise. All external forces are applied on the first story. The floor's absolute displacement responses are considered as output data and measured at a sample rate of 10 Hz. Also, the random excitation records are 800 seconds long. Although in this benchmark, the displacement responses are batch, in order to simulate online identification, it is assumed that data is recorded for each step in real time. For simplicity and considering the limits of the EASI algorithm, two assumptions have been made: 1) normal modes exist, 2) the damping ratio is zero for all modes.

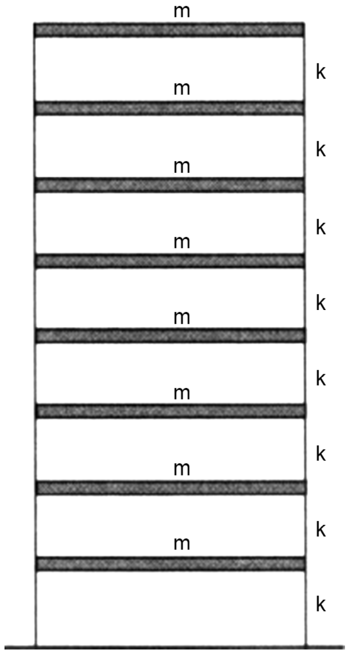

Figure 7. Eight-story benchmark model.

Using a correlation coefficient known as the Modal Assurance Criterion (MAC), the obtained mode shapes are compared (Maia & Silva, 1997). MAC values vary between 0 and 1, when the MAC value is close to 1, the separation will be more accurate. MAC is defined as Equation (25)

$$\mathrm{MAC}_i=\frac{\left(\boldsymbol{\varphi}_i^T\hat{\boldsymbol{\varphi}}_i\right)^2}{\left(\boldsymbol{\varphi}_i^T\boldsymbol{\varphi}_i\right)\left(\hat{\boldsymbol{\varphi}}_i^T\hat{\boldsymbol{\varphi}}_i\right)} \tag{25}$$

where $\hat{\boldsymbol{\varphi}}_i$ and $\boldsymbol{\varphi}_i$ represent i^{th} estimated and theoretical mode shape vectors, respectively. To assess the accuracy of separation, Euclidian distance (*Er*) of two mode shape vectors is used.

$$Er_i=\boldsymbol{\varphi}_i-\hat{\boldsymbol{\varphi}}_{i\,2} \tag{26}$$

5.2.1. Free Vibration

To achieve the free vibration state, an external short impulse force of $f(t)=1.1*10^9$ N is applied on the first floor. Figure (8) illustrates the PI for the EASI and the proposed VS-EASI algorithms for 20 trials. The step-size for both algorithms was initialized at 0.002 and for the proposed VS-EASI algorithm, $\beta_1 = 0.999$, $\beta_2 = 0.998$, $b = 1.0$ and $\rho_0 = 10^6$. As shown in this figure, the convergence speed of the proposed VS-EASI algorithm is significantly improved compared with the conventional EASI method.

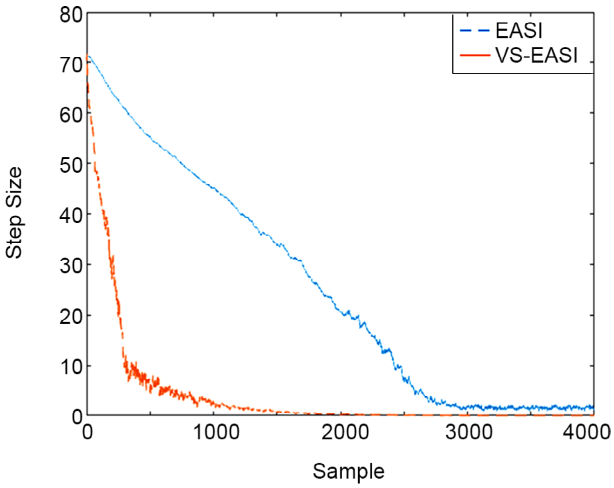

Figure 8. Comparison of the PI for EASI and proposed VS-EASI algorithms for free vibration state.

The convergence of solutions which is crucial in adaptive algorithms is investigated for the proposed VS-EASI algorithm and the results are depicted in Figure (9). It can be seen that by increasing time and the number of samples, MAC values converge to the unit. It demonstrates that proposed VS-EASI performs effectively in separation. The modal identification results of both methods, EASI and the proposed VS-EASI, are compared and shown in Table (1). As seen in the table, both methods have estimated the modes well, but the proposed VS- EASI algorithm has been a better performance.

5.2.2. Forced Vibration

This section studies the performance of the proposed algorithm in a forced vibration state. The Gaussian white noise is considered as the stationary random excitations applied to the system. The identification is performed for 20 different $f(t)$ samples. Therefore, the MACs, error values and estimated damping ratios and natural frequencies are the mean values of the acceptable identifications when the MAC value of each identified mode is greater than 0.90. The performance index of the

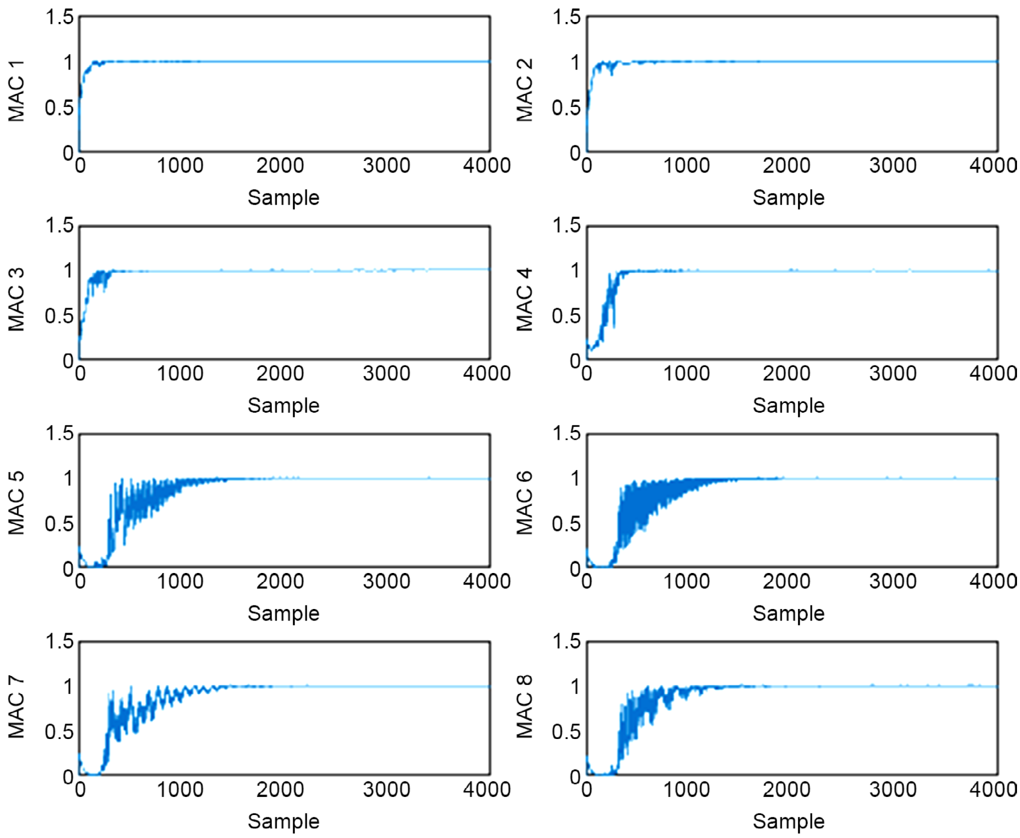

Figure 9. Convergence of MAC values for free vibration.

Table 1. Evaluation [MAC and Er] of separation performed by proposed VS- EASI for free vibration.

Method		Modes							
		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8
EASI	MAC	1.0000	0.9999	1.0000	0.9997	0.9693	0.9465	0.9930	0.9947
	Er.	0.0056	0.0097	0.0033	0.0167	0.1759	0.2328	0.0837	0.0073
VS-EASI	MAC	1.0000	1.0000	1.0000	1.0000	0.9997	0.9995	1.0000	1.0000
	Er.	0.0002	0.0004	0.0002	0.0007	0.0165	0.0221	0.0044	0.0069

Table 2. Estimated modal parameters using proposed VS-EASI algorithm for benchmark building.

Modes	MAC	Er	Frequency (Hz)		Damping Ratio (%)	
			Theoretical	Estimated	Theoretical	Estimated
1	0.9968	0.0566	1.17	1.17	0.0000	0.0065
2	0.9996	0.0195	3.48	3.50	0.0000	0.0080
3	0.9997	0.0161	5.67	5.66	0.0000	0.0072
4	0.9994	0.0249	7.67	7.59	0.0000	0.0081
5	0.9872	0.0308	9.41	9.46	0.0000	0.0012
6	0.9973	0.0523	10.82	10.78	0.0000	0.0022
7	0.9886	0.1067	11.87	11.86	0.0000	0.0082
8	0.9800	0.1416	12.51	12.51	0.0000	0.0058

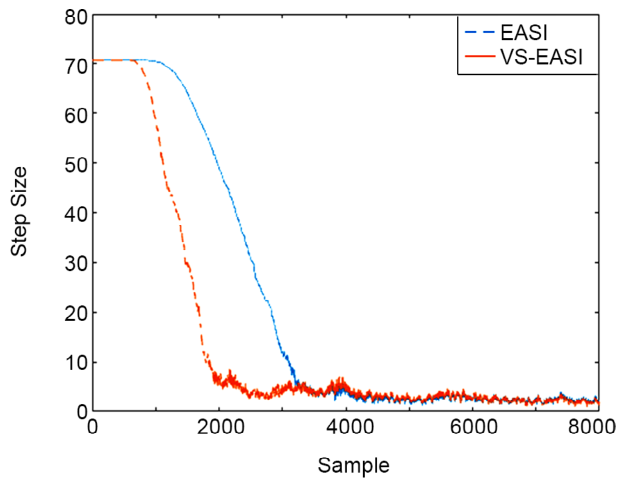

Figure 10. Comparison of the PI for EASI and proposed VS-EASI algorithms for forced vibration state.

EASI and proposed VS-EASI algorithms are shown in Figure (10). The step-size for both algorithms was started at 0.0025 and the constants for the proposed VS-EASI are $\beta_1 = 0.998$, $\beta_2 = 0.9999$, $b = 1.0$ and $\rho_0 = 10^6$. It is clear from Figure (10) that for forced vibration, the proposed VS-EASI has a higher convergence speed than the conventional EASI algorithm as well.

Table (2) presents the results of modal identification, including natural frequencies and damping ratios. As is evident in the table, the proposed VS-EASI algorithms has a good performance in identifying mode shapes; the extracted mode shapes are illustrated in Figure (11). After separation, in order to estimate the modal parameters from the extracted sources, online system identification methods are applied (Maragos, Kaiser, & Quatieri, 1993).

Ultimately, the estimated results of the proposed algorithm are in agreement with exact parameters.

Conclusions

As a solution for the problem of the compromise between convergence speed and steady state misadjustment error in the traditional equivariant adaptive source separation via independence (EASI) algorithm with a fixed step-size, this paper has proposed a novel variable step-size EASI (VS-EASI) algorithm for on-line blind modal identification of structures. The step-size is adaptively updated in response to changes in the dynamics of the input signals and the separating matrix. Thus, the

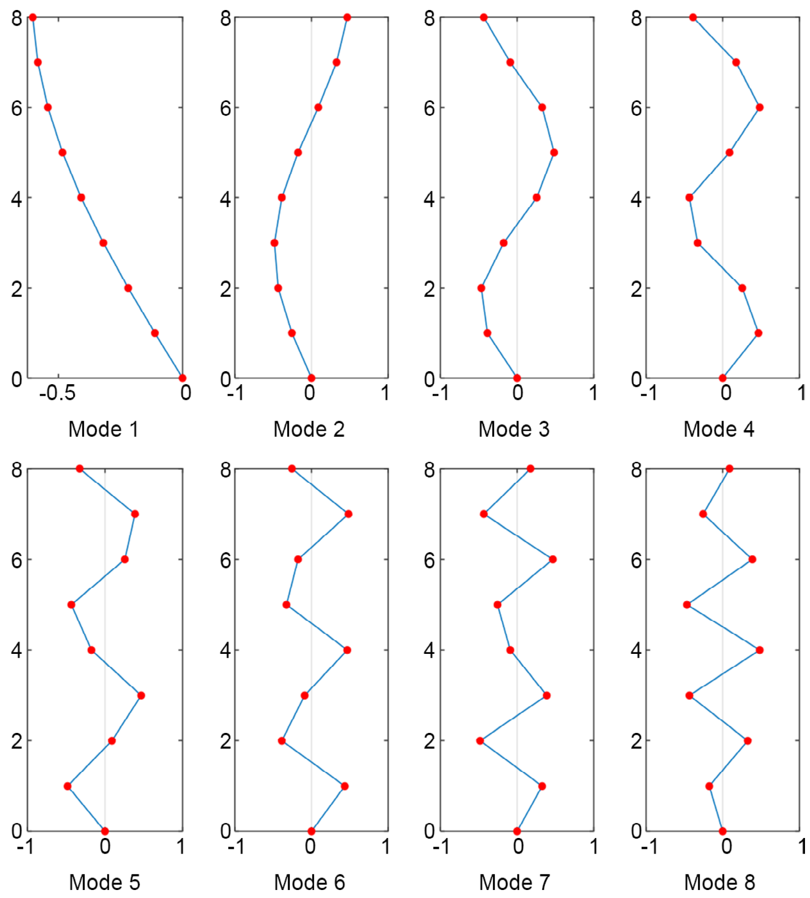

Figure 11. Representation of the extracted mode shaped (the solid lines) and the theoretical mode shapes (the red dots).

performance of the conventional EASI algorithm is improved by applying this technique, which obtains a fast convergence rate while maintaining a low steady-state error. Compared with the conventional EASI and VS-EASI proposed by Chambers algorithms, this proposed VS-EASI algorithm is faster with low errors in steady-state and also more robust against noise. Simulation results demonstrate that the VS-EASI algorithm is particularly well-suited for separating sources mixed by time-varying environments.

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