JSEE Vol. 27, No. 3, 2025 DOI: 10.48303/jsee.2025.2034029.1107

Research Note

Application of Wavelet Transform in Frequency Analysis of Near, Medium and Far Field Accelerometers

Amir Shamgani[1] and Hamid Saffari[2*]

1. M.Sc., Faculty of Civil, Water & Environmental Engineering, Shahid Beheshti University, Tehran, Iran
2. Associate Professor, Department of Structure & Earthquake, Faculty of Civil, Water & Environmental Engineering, Shahid Beheshti University, Tehran, Iran, *Corresponding Author; email: h_saffari@sbu.ac.ir

Received: 27/06/2024
Revised: 30/10/2024
Accepted: 05/03/2025

Keywords:
Earthquake accelerogram; Near-field earthquake; Time history analysis; Frequency content analysis; Wavelet transform; Continuous wavelet transform

ABSTRACT

Earthquakes are one of the natural phenomena that threaten human life in areas around faults. Mathematical transformations are tools that can provide useful information to researchers in generating synthetic accelerograms, analyzing signals and accelerograms. With the help of mathematical transformations, recorded accelerograms of earthquakes can be considered as input signals and analyzed. Using the Fourier transform, the frequencies present in the signal can be obtained, but it does not provide the temporal (or spatial) information of the signal's constituent frequencies. The wavelet transform is a tool suitable for analyzing non-stationary waves and compensates for this weakness by providing not only the frequencies present in the signal but also the time (or location) of the frequencies. By changing the scale and translation parameters in the wavelet transform, the constituent frequencies of the earthquake wave and their occurrence time can be obtained. In this research, frequency analysis using wavelet transform was performed on the accelerograms of the 2020 Mosha earthquake in near, intermediate, and far fields. Based on this analysis, the dominant frequencies of each wave and their occurrence moments were obtained, showing that in the near field, the dominant frequency of the earthquake wave is larger compared to the far field. In other words, as the distance from the fault increases, the dominant frequency of the earthquake wave decreases, which is dangerous for tall structures with high natural periods due to the resonance phenomenon. The results indicate the high capability of the continuous wavelet transform method in calculating the constituent frequencies of earthquake waves and their occurrence times. With this analysis, structures whose period matches the dominant period of the earthquake wave can be identified, and the effects of waves on structures can be examined more accurately.

How to cite the article:
Shamgani, A. & Saffari, H. (2025). Application of wavelet transform in frequency analysis of near, medium and far field accelerometers. *Journal of Seismology and Earthquake Engineering*, 27(3), 71-79. https://doi.org/10.48303/jsee.2025.2034029.1107

Copyright © 2024 IIEES. Published by International Institute of Earthquake Engineering and Seismology.
This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International license (https://creativecommons.org/licenses/by-nc/4.0/). Non-commercial uses of the work are permitted, provided the original work is properly cited.

1. Introduction

Earthquakes, as natural events, have been a subject of discussion and research among many scientists from the past to the present. The damage and impacts of them have on human life have made their analysis of great importance to researchers. Studying earthquakes to gain information about energy levels, location, time, and the behavior of structures in response to them is particularly significant. The likelihood of large earthquakes occurring near cities close to faults is much higher, and these events have specific characteristics that distinguish them from those occurring far from faults. Some of these characteristics, if not considered, may lead to under estimating earthquake forces in structural design. In most processing approaches, the raw signal is available as a time-domain signal. The signal's amplitude values over time are represented in the time domain, but this representation may not be sufficient for a complete description of the signal. In many cases, valuable information about the signal is hidden in its frequency domain. The spectrum of a signal represents the different frequencies present in the signal and indicates the presence and strength of each frequency. Fourier transform is a tool that can extract the frequency content of a signal. However, Fourier transform does not provide any temporal or spatial information about the signal and only gives frequency information. In other words, Fourier transform cannot determine the time or location of different frequencies in the signal. To address this limitation, wavelet transform is used as a tool for signal processing (Newland, 1993). Wavelet transform is particularly suitable for analyzing non-stationary waves. In a country like Iran, where many major cities are near large faults, a proper understanding of ground behavior and assessing its effects on structural behavior is crucial. One significant factor that can considerably affect the dynamic behavior of structures under earthquake excitation is the proximity or distance from the fault causing these events. Previous studies have shown that the nature of earthquakes near the causative fault can be significantly different from those occurring in more distant areas (Boore, 1983, 2003; Aki, 1967; Brune, 1970; Beresnev & Atkinson, 1997, 2002). Research and studies indicate that shear and moment base values for near-fault motions are significantly higher than those for far-field motions, and the distance to the fault plays a crucial role in shaping the intensity of the earthquake spectrum. Earthquakes are cate-gorized into three groups (Tehranizadeh, 2015):

- Near-field: Distance less than 20 km from the fault.
- Intermediate-field: Distance between 20 and 50 km from the fault.
- Far-field: Distance greater than 50 km from the fault.

Earthquakes in the near-field can exhibit specific characteristics that result in significant changes in the behavior of structures. Therefore, it is necessary to consider these characteristics and differences in the nature of earthquakes near and far from faults during the design and construction of structures in these areas. A thorough examination and analysis of these factors enables the selection of appropriate methods and strategies to design and improve the performance of structures against earthquakes. Additionally, adherence to regulations and standards related to seismic design during the construction and operation of structures is crucial. Overall, when precise information about spectral changes in ground motions is needed, processing limited data may not provide complete information. Displaying recorded time histories of large earthquakes, especially at stations near faults, provides a more comprehensive set of these events information. Based on the complex nature of records in the near-field domain, using simple and systematic models to accurately represent these records may be challenging in some cases. In such situations, employing wavelet functions can lead to more complex models capable of capturing more effects from near-field records. Wavelet functions allow for the detection and representation of more intricate patterns in records. Through wavelet functions, models can be constructed to extract more information from near-field records. However, wavelet-based models also have their limitations. One of their main constraints is calibrating supplementary parameters in defining wavelet functions. This implies the need for precise determination and adjustment of wavelet parameters

for each specific problem (Nazari et al., 2018).

In fact, an earthquake wave is a non-stationary wave, and its constituent frequencies do not occur uniformly over the entire duration of the event. This means that an earthquake encompasses a wide spectrum of frequencies that vary at different moments (Majidi & Heidari, 2020). High frequency implies rapid changes in a mathematical variable, while low frequency indicates minimal changes in a signal. The concept of frequency denotes the rate of change within a signal and is typically measured in Hertz (cycles per second). Mathematically, a transform on a wave provides more information about the raw wave. Applying mathematical transforms to an initial raw signal attempts to extract new features that are not directly observable in the original signal.

However, it should be noted that the return period of large earthquakes is long, and many seismic-prone areas in Iran lack access to strong motion records. This shortage may be due to insufficient coverage of earthquake recording networks and, in general, a limited number of events that convey the effects of near-field seismic activity. To have a reliable seismic design plan for structures in regions near active faults, such as many large cities in Iran, a dataset is required for analysis and design. These data should either be recorded within the construction sites or be at least compatible with the site characteristics. One common method to obtain this data is through artificial accelerogram generation methods and accelerogram simulation. Mathematical transformations are tools that can provide useful information to researchers in generating synthetic accelerograms, analyzing signals and accelerograms, thereby capturing their attention and importance (Daryoushi, 2021).

2. Wavelet Transform

Wavelets are a natural extension and development of Fourier analysis (Daubechies, 1992). Other researchers such as Strang and Nguyen (1996) and Mallat (1998) have provided detailed and excellent explanations of wavelet decomposition in their respective studies. Wavelet transform serves as a powerful tool for time-frequency (or time-space) analysis of signals in the frequency domain, attracting significant research attention over the past few decades. Wavelet transform maintains dual focus in both frequency and time (or space) domains, enhancing the resolution of frequency details across different time steps (Ghodrati Amiri et al., 2006). Baker (2007) conducted a study on analyzing ground motions near faults using wavelet analysis. Yaghmaei-Sabegh (2010, 2013) utilized wavelet decomposition for examining and detecting impulsive seismic events. Ji et al. (2012) investigated changes in geomagnetic anomalies associated with some microseismic events in the Korean Peninsula using wavelet-based seismic similarity analysis. Rastegar and Kouhdaragh (2017) conducted a frequency analysis of the Bam earthquake time history using wavelet transform. Shamgani and Saffari (2024), used wavelet transform to analyze the effects of Mosha 2020 earthquake on different sites. In the context of generating synthetic accelerograms compatible with the environment, which is crucial for mitigating the lack of real data on strong motions both near and far from faults, wavelet transform plays a crucial role. Indeed, the invention of wavelet transform as a tool for time-frequency analysis has enabled more precise and suitable examinations in non-stationary frequency domains. The continuous wavelet transform is defined as follows (Mallat, 1998):

$$CWT_x^{\psi}(\tau,s) = \Psi_x^{\psi}(\tau,s) = \frac{1}{\sqrt{|s|}}\int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \quad (1)$$

In Equation (1), τ represents the translation parameter, which determines the shifting of the wavelet window and includes temporal information. In wavelet transform, unlike the direct frequency parameter, the scale parameter s is used, which is inversely related to frequency. Scaling is akin to a mathematical operator that compresses the signal for $s < 1$, revealing details, and expands the signal for $s > 1$, showing generalities. In Equation (1), ψ denotes the wavelet function, also known as the mother wavelet, and ψ^* is its complex conjugate.

The term "mother wavelet" is used because the wavelet function serves as a template or prototype for generating scaled and translated versions that are used in wavelet transform analysis. The term "mother" signifies that all scaled and translated versions are constructed based on this initial

function. In other words, the mother wavelet is the pattern for generating other wavelet windows in the wavelet transform. One example of a mother wavelet function that is also used in this research is the Morlet wavelet. The complex Morlet wavelet includes both real and imaginary coefficients. These coefficients are used to calculate the amplitude and phase information of a signal, which highlights the analytical advantage of complex wavelet transforms in frequency content analysis. This ability to extract both magnitude and phase information provides a more comprehensive understanding of the signal's characteristics, making the complex Morlet wavelet particularly effective for analyzing non-stationary signals like seismic waves. The complex Morlet wavelet function is defined as follows (Teolis, 1998):

$$\Psi(x) = \frac{1}{\pi f_b} e^{2\pi i f c x} e^{-x^2/f_b} \tag{2}$$

In Equation (2), the parameter f_b controls the bandwidth of the wavelet, while f_c represents the central frequency of the wavelet. The complex Morlet wavelet, illustrated in Figure (1), is characterized by these parameters. To ensure that the scale parameter corresponds to the wavelength, f_c is typically set to 1 (Cooper & Cowan, 2008).

In this section, the inverse relationship of the wavelet transform and the conditions under which this relationship is invertible from a mathematical perspective is briefly examined. Equation (1) is said to be invertible if:

$$\int \psi(t)\, d\, t = 0 \tag{3}$$

In all functions, regardless of whether the basic functions are orthogonal or not, to establish Equation (3), which is the condition for the invertibility of the wavelet transform, it is necessary that the selected mother wavelet function be oscillatory. This can be easily verified. The inverse relation of the wavelet transform is calculated as follows:

$$x(t) = \frac{1}{C_\psi^2} \int_s \int_\tau \Psi_x^\psi(\tau, s) \psi\left(\frac{t-\tau}{s}\right) d\pi ds \tag{4}$$

C_ψ in Equation (4) is the admissibility constant, which is a fixed value that ensures the invertibility and complete reconstruction of the transform,

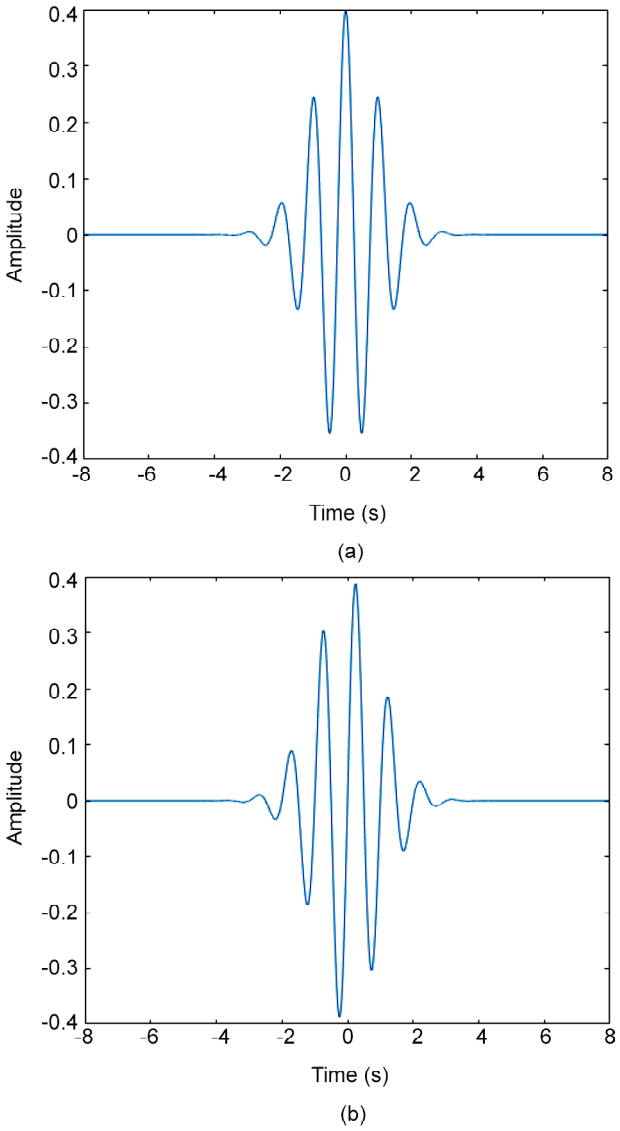

Figure 1. (a) Real part of the complex Morlet wavelet and (b) Imaginary part of the complex Morlet wavelet.

depending on C_ψ and the chosen mother wavelet. Therefore, the admissibility condition is presented as Equation (5):

$$C_\psi = \left(2\pi \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\xi)|^2}{|\xi|} d\xi \right)^{\frac{1}{2}} < \infty \tag{5}$$

$\hat{\psi}$ in the above relation is the Fourier transform of the mother wavelet function.

3. Analyzing the Earthquake Signals Using Wavelet Transform

In previous studies, such as the work by Rastegar and Kouhdaragh (2017), only the time history of the Bam earthquake was analyzed. However, in this research, the time history of an event at various

distances from the source has been analyzed and examined. This allows for a clearer understanding of the effect of distance on the dominant frequency of an accelerogram. By considering different distances from the earthquake source, the variation in dominant frequencies can be observed more distinctly, providing valuable insights into how seismic waves behave across different locations. In this study, using the wavelet transform, the analysis of near, mid, and far-field accelerograms for the Mosha earthquake, which occurred on May 7, 2020, at 20:18 with a magnitude of $M_W = 5$, was performed to calculate the timing of the earthquake's frequency components. By using the scale parameter, various frequencies present in the signal can be calculated. In this method, by computing Equation (1) for different shift and scale parameters, the effect of the desired scale (frequency) on the main wave is assessed. If the resulting value is large, it indicates that the desired scale (frequency) is one of the main components of the accelerogram. Otherwise, the desired scale (frequency) is not present in the wave (Gurly & Kareem, 1999).

For this purpose, the Mosha-3 station with an epicentral distance of 13 km was chosen as the near-field site, the Rudehen station with an epicentral distance of 22 km as the mid-field site, and the Tehran 303 (Shahid Rajaee University) station with an epicentral distance of 57 km as the far-field site from the 2020 Mosha earthquake. The accelerograms related to this event were obtained from the Iranian Strong Motion Network (ISMN, 2024). In this study, the preprocessing of the accelerogram and baseline correction were performed using the SeismoSignal software. After completing the baseline correction, the corrected accelerogram was utilized for analysis in this research. Figure (2a) shows the recorded accelerogram of the near-field Mosha 2020

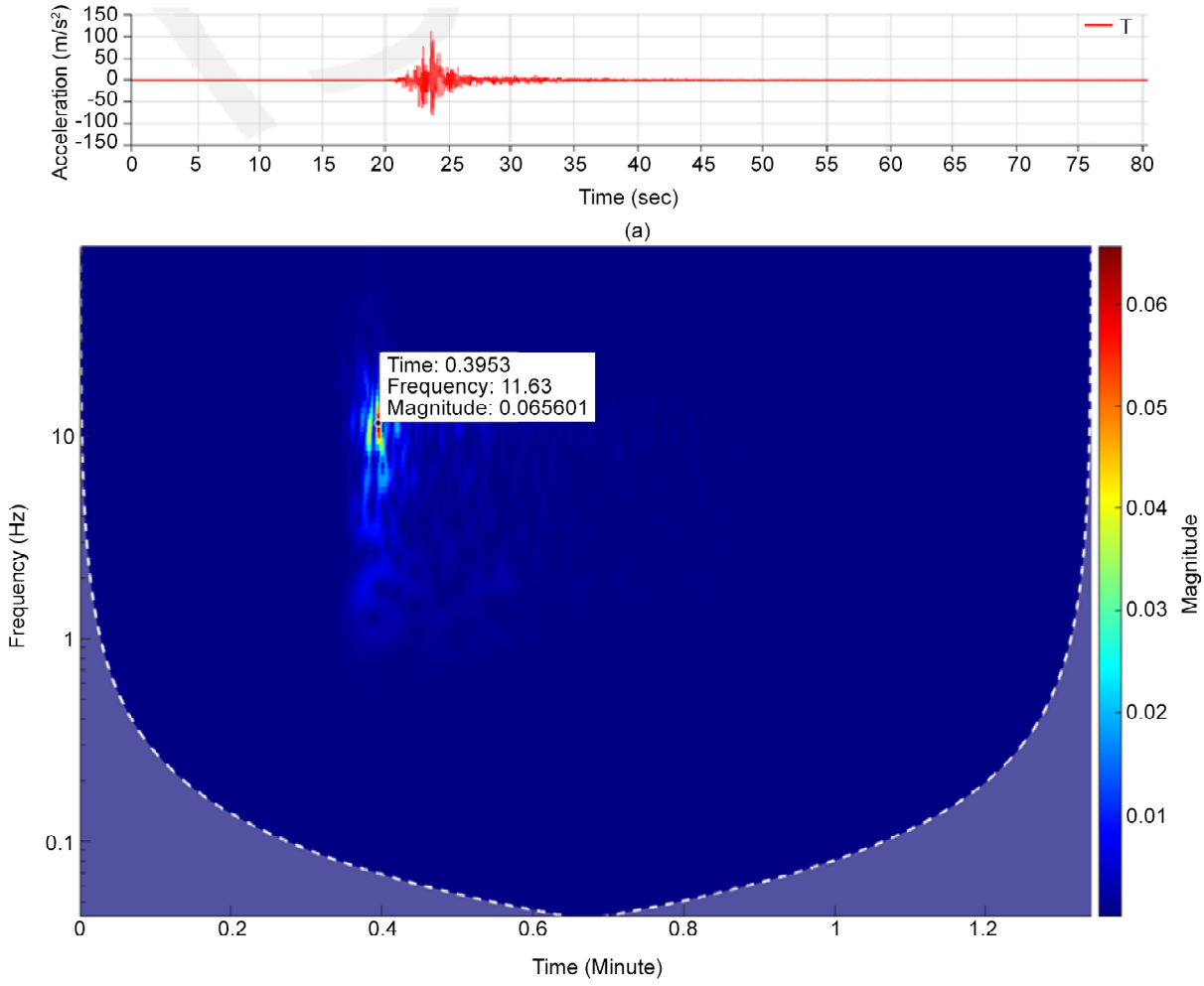

Figure 2. (a) The accelerogram recorded for the near-field of the 2020 Mosha earthquake at the Mosha-3 station (13 km distance); (b) The wavelet transform of the accelerogram recorded for the near-field of the 2020 Mosha earthquake at the Mosha-3 station (13 km distance).

earthquake at the Mosha-3 station, and Figure (2b) illustrate the wavelet transform of this accelerogram.

As shown in Figure (2b), the highest value of the Continuous Wavelet Transform (CWT) is 0.066, which occurs at a frequency of 11.63 Hz and at a time of 23.718 seconds (0.3953 minutes). In other words, the frequency of 11.63 Hz has the most significant impact on this accelerogram, corresponding to a period of 0.086 seconds.

In Figure (3a), the accelerogram recorded for the medium-field of the 2020 Mosha earthquake at the Rudehen station is shown, and in Figure (3b), the wavelet transform of this accelerogram is depicted.

According to Figure (3b), the highest value of the Continuous Wavelet Transform (CWT) is 0.068, which occurs at a frequency of 10.85 Hz and at a time of 17.72 seconds. In other words, the frequency with the most significant impact on this accelerogram is 10.85 Hz.

Figure (4a) shows the accelerogram recorded in the far-field zone of the Mosha 2020 earthquake at the Tehran 303 station (Shahid Rajaee University), and Figure (4b) shows the wavelet transform of this accelerogram.

As shown in Figure (4b), the maximum value of the Continuous Wavelet Transform (CWT) function is 0.013, which occurs at a frequency of 3.84 Hz and at the time 24.282 seconds (0.4047 minutes). In other words, the frequency that has the most significant impact on this accelerogram is 3.84 Hz, equivalent to a period of 0.26 seconds, which has increased compared to the previous two cases.

4. Discussion

The wavelet transform is a powerful tool in

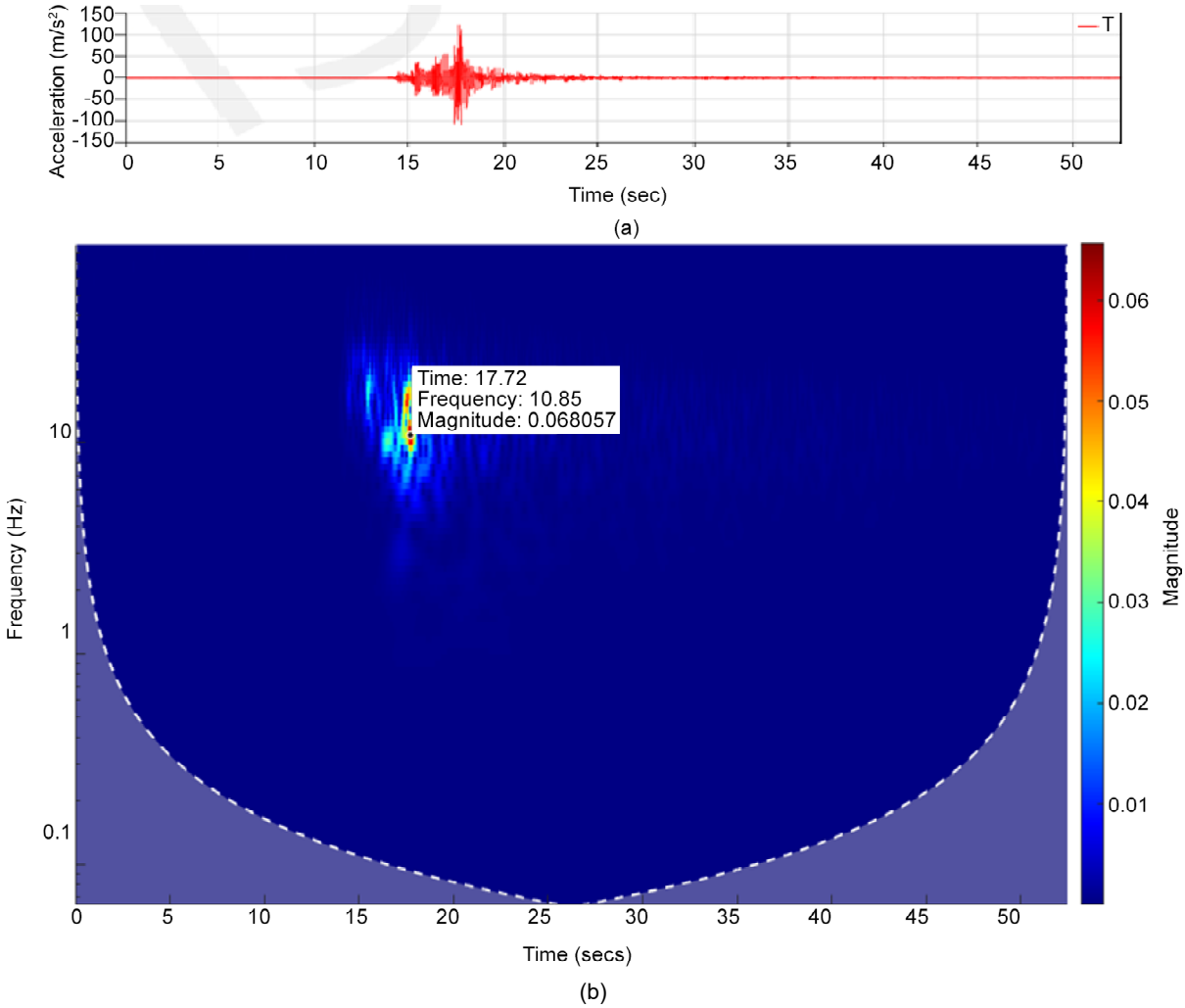

Figure 3. (a) The accelerogram recorded for the medium-field of the 2020 Mosha earthquake at the Rudehen station (22 km distance); (b) The wavelet transform of the accelerogram recorded for the medium-field of the 2020 Mosha earthquake at the Rudehen station (22 km distance).

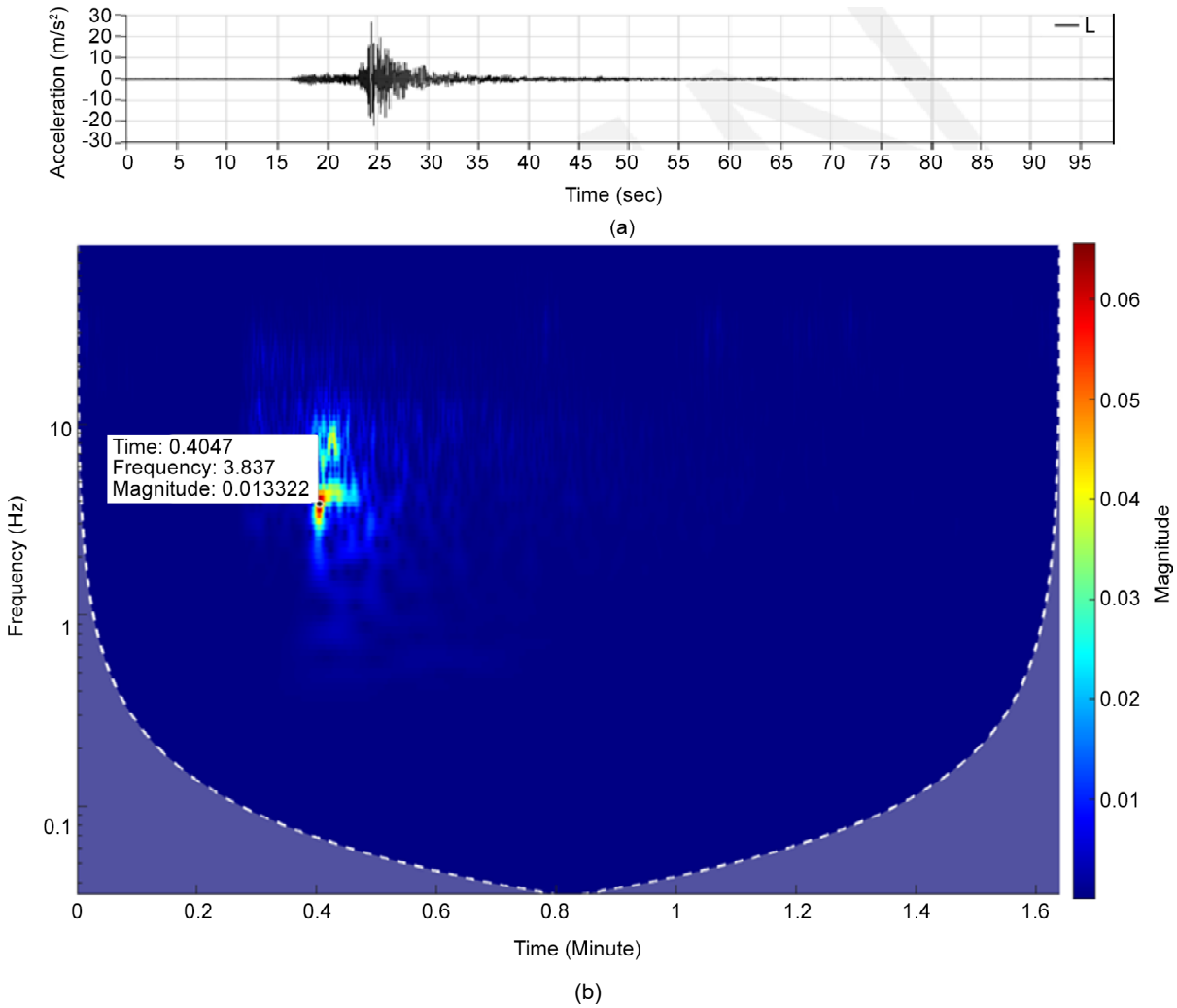

Figure 4. (a) The accelerogram recorded for the far-field of the 2020 Mosha earthquake at the Tehran 303 station (Shahid Rajaee University, 57 km distance); (b) The wavelet transform of the accelerogram recorded for the far-field of the 2020 Mosha earthquake at the Tehran 303 station (Shahid Rajaee University, 57 km distance).

decision analysis. Among its applications in civil and earthquake engineering is the similarity analysis of wavelet transform earthquakes. It is also possible to mention the creation of artificial ground motions and the recognition of the pulse-like accelerograms. The summary of the three aforementioned analyses shows that by applying the continuous wavelet transform on the recorded accelerograms of the 2020 Mosha earthquake from the near, intermediate, and far fields, the dominant frequencies were found to be 11.63 Hz, 10.85 Hz, and 3.84 Hz, respectively. This indicates a decrease in dominant frequency and conversely, an increase in the dominant period of the earthquake wave. Note that the periods mentioned in this study refer to the period of the ground surface (the site of the accelerometer), and the period of the structure will reflect its dynamic properties. The calculated dominant periods or frequencies should not be directly considered as the dominant period or frequency of the structure. According to structural dynamics principles, the closer the natural frequency of the structure is to the frequency of the site, the greater the resonance effect and seismic damage will be.

In the studies mentioned in previous sections, the effect of distance was not considered. As is evident, this parameter was taken into account in this research, and the results clearly indicate that as the distance from the earthquake source increases, the dominant frequency decreases.

5. Conclusion

Using Fourier transform, the frequencies present in a signal can be extracted, but it does not provide temporal (or spatial) information about the frequencies within the signal. The wavelet

transform, however, is a tool that can offer valuable information to researchers for generating synthetic accelerograms, signal analysis, and accelerogram analysis. This transform is well-suited for analyzing non-stationary waves and compensates for the shortcomings of the Fourier transform. Given that the output of continuous wavelet analysis is a curve in the time-frequency-amplitude domain, a clearer and more detailed view of earthquake accelerograms can be obtained.

The main advantage of the wavelet transform is its ability to identify the timing of each frequency within an earthquake wave, allowing for the determination of the moment when the acceleration has the greatest impact on the ground. Additionally, the wavelet transform can reveal how often a specific frequency occurs throughout the duration of an earthquake. The most significant outcome from frequency analysis using the wavelet transform is identifying the dominant frequency in an earthquake wave and the timing of its occurrence. Structures whose natural frequency matches the dominant frequency of the earthquake wave can experience resonance, leading to potential structural failure.

In this study, it was shown that the dominant frequency in accelerograms from the 2020 Mosha earthquake decreases with increasing distance from the fault. This finding is crucial for structures in distant areas, as they generally have higher periods and are more susceptible to resonance during strong ground motions.

Specifically, the dominant frequency of the 2020 Mosha earthquake was 11.63 Hz at 23.718 seconds for the near field (13 km from the source), 10.85 Hz at 17.72 seconds for the intermediate field (22 km from the source), and 3.84 Hz at 24.282 seconds for the far field (57 km from the source).

As a general conclusion, the wavelet transform is a useful tool for frequency analysis of earthquake waves. By examining the continuous wavelet transform (CWT) at various frequencies and times, the impact of earthquake acceleration on different structures with varying natural periods can be assessed.

This includes investigating phenomena such as resonance at the moment of maximum earthquake-induced force.

Reference

Aki, K. (1967). Scaling law of seismic spectrum. *Journal of Geophysical Research: Solid Earth*, *72*(4), 1217-1231. https://doi.org/10.1029/JZ072i004p01217

Baker, J. W. (2007). Quantitative classification of near-fault ground motions using wavelet analysis. *Bulletin of the Seismological Society of America*, *97*(5), 1486-1501. https://doi.org/10.1785/0120060255

Beresnev, I. A., & Atkinson, G. M. (2002). Source parameters of earthquakes in eastern and western North America based on finite-fault modeling. *Bulletin of the Seismological Society of America*, *92*(2), 695-710. https://doi.org/10.1785/0120010101

Beresnev, I. A., & Atkinson, G. M. (1997). Modeling finite-fault radiation from the ω^n spectrum. *Bulletin of the Seismological Society of America*, *87*(1), 67-84. https://doi.org/10.1785/bssa0870010067

Boore, D. M. (1983). Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra. *Bulletin of the Seismological Society of America*, *73*(6), 1865-1894. https://doi.org/10.1785/bssa07306a1865

Boore, D. M. (2003). Simulation of ground motion using the stochastic method. *Pure and Applied Geophysics*, *160*(3-4), 635-676. https://doi.org/10.1007/PL00012553

Brune, J. N. (1970). Tectonic stress and the spectra of seismic shear waves from earthquakes. *Journal of Geophysical Research: Solid Earth*, *75*(26), 4997-5009. https://doi.org/10.1029/JB075i026p04997

Cooper, G. R. J., & Cowan, D. R. (2008). Comparing time series using wavelet-based semblance analysis. *Computers & Geosciences*, *34*(2), 95-102. https://doi.org/10.1016/j.cageo.2007.03.009

Daryoushi, M. (2021). *Presenting a Model to Convert Far and Near Field Acceleration Maps to Each Other Using Real and Artificial Records [Doctoral Dissertation, Shahid Beheshti University]* (in Persian).

Daubechies, I. (1992). Ten lectures on wavelets. *CBMS-NSF Conference Series in Applied Mathematics, Montpelier,* Vermont. Society for Industrial

and Applied Mathematics. https://doi.org/10.1137/1.9781611970104

Ghodrati Amiri, G., Ashtari, P., & Rahami, H. (2006). New development of artificial record generation by wavelet theory. *Structural Engineering and Mechanics*, *22*(2), 185-195. https://doi.org/10.12989/sem.2006.22.2.185

Gurley, K., & Kareem, A. (1999). Applications of wavelet transforms in earthquake, wind, and ocean engineering. *Engineering Structures*, *21*(2), 149-167. https://doi.org/10.1016/S0141-0296(97)00139-9

Iran Strong Motion Network (ISMN), *Building and Housing Research Center*. Available: http://smd.bhrc.ac.ir. Retrieved August 20, 2024.

Ji, Y., Oh, S., & Kim, K. (2012). Wavelet-based semblance and eigenvalue analysis for geomagnetic variation related to micro-earthquakes in the Korean Peninsula. *The Journal of the Korean Earth Science Society*, *33*(5), 408-421. https://doi.org/10.5467/JKESS.2012.33.5.408

Majidi, N. & Heidari, A. (2020). Analysis of structures using wavelet theory and duration of strong ground motion. *Journal of AmirKabir Civil Engineering*, *52*(7), 1685-1704 (in Persian).

Mallat, S. (1998). *A Wavelet Tour of Signal Processing*. Academic Press, New York, 577pp.

Nazari, Sh., Mashkohaldini, A., & Keyvani, J. (2018). Simulation of near field earthquake records using wavelet functions. *Journal of Engineering Geology*, *12*(4), 707-726 (in Persian).

Newland, D. E. (1993). *An Introduction to Random Vibrations, Apectral and Wavelet Analysis (3rd Ed.)*. Longman Scientific & Technical.

Rastegar, A. & Kouhdaragh, M. (2017). Frequency analysis of earthquake time history with wavelet transform. *International Conference on Xivil Engineering, Architecture and Urban Planning of Contemporary Iran,* Tehran (in Persian).

Shamgani, A. & Saffari, H. (2024). Frequency analysis of accelerometers of near, medium and far field of Mosha earthquake 2020 using wavelet transform. *9th International Seismology and Earthquake Engineering Conference (SEE9)*, Tehran (in Persian).

Strang, G. & Nguyen, T. (1996). *Wavelets and Filter Banks (2nd ed.)*. Wellesley-Cambridge Press, Wellesley, MA, 490pp.

TehraniZadeh, M. (2015). *Characteristics of Earthquakes in the Near Field and Its Effect on Response and Design Spectra*, Tehran: AmirKabir University of Technology (in Persian).

Teolis, A. (1998). *Computational Signal Processing with Wavelets*. Birkha¨ user Boston Inc., Boston, MA, 352pp.

Yaghmaei-Sabegh, S. (2010). Detection of pulse-like ground motions based on continues wavelet transform. *Journal of Seismology*, *14*(4), 715-726. https://doi.org/10.1007/s10950-010-9193-8

Yaghmaei-Sabegh, S. (2013). Wavelet-based analysis for pulse period of earthquake ground motions. *International Journal of Engineering*, *26*(10A), 1135-1144. https://doi.org/10.5829/idosi.ije.2013.26.10a.04